CS 188 Introduction to Artificial Intelligence
Spring 2024 Note 11

Author (all other notes): Nikhil Sharma

Author (Bayes' Nets notes): Josh Hug and Jacky Liang, edited by Regina Wang

Author (Logic notes): Henry Zhu, edited by Peyrin Kao

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Bayesian Network Representation

While inference by enumeration can compute probabilities for any query we might desire, representing an entire joint distribution in the memory of a computer is impractical for real problems — if each of n variables we wish to represent can take on d possible values (it has a **domain** of size d), then our joint distribution table will have d^n entries, exponential in the number of variables and quite impractical to store!

Bayes nets avoid this issue by taking advantage of the idea of conditional probability. Rather than storing information in a giant table, probabilities are instead distributed across a number of smaller conditional probability tables along with a **directed acyclic graph** (DAG) which captures the relationships between variables. The local probability tables and the DAG together encode enough information to compute any probability distribution that we could have computed given the entire large joint distribution. We will see how this works in the next section

We formally define a Bayes Net as consisting of:

- A directed acyclic graph of nodes, one per variable X.
- A conditional distribution for each node $P(X|A_1 \dots A_n)$, where A_i is the i^{th} parent of X, stored as a **conditional probability table** or CPT. Each CPT has $n+2$ columns: one for the values of each of the n parent variables $A_1 \dots A_n$, one for the values of X, and one for the conditional probability of X given its parents.

The structure of the Bayes Net graph encodes conditional independence relations between different nodes. These conditional independences allow us to store multiple small tables instead of one large one.

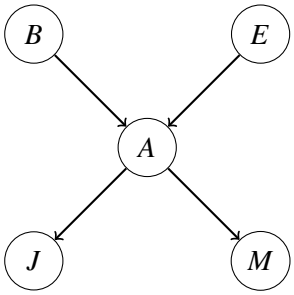
It is important to remember that the edges between Bayes Net nodes do not mean there is specifically a *causal* relationship between those nodes, or that the variables are necessarily dependent on one another. It just means that there may be *some* relationship between the nodes.

As an example of a Bayes Net, consider a model where we have five binary random variables described below:

- B: Burglary occurs.

- A: Alarm goes off.
- E: Earthquake occurs.
- J: John calls.
- M: Mary calls.

Assume the alarm can go off if either a burglary or an earthquake occurs, and that Mary and John will call if they hear the alarm. We can represent these dependencies with the graph shown below.



In this Bayes Net, we would store probability tables $P(B), P(E), P(A|B,E), P(J|A)$ and $P(M|A)$.

Given all of the CPTs for a graph, we can calculate the probability of a given assignment using the following rule:

$$P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | parents(X_i))$$

For the alarm model above, we can actually calculate the probability of a joint probability as follows: $P(-b,-e,+a,+j,-m) = P(-b) \cdot P(-e) \cdot P(+a|-b,-e) \cdot P(+j|+a) \cdot P(-m|+a)$.

We will see how this relation holds in the next section.

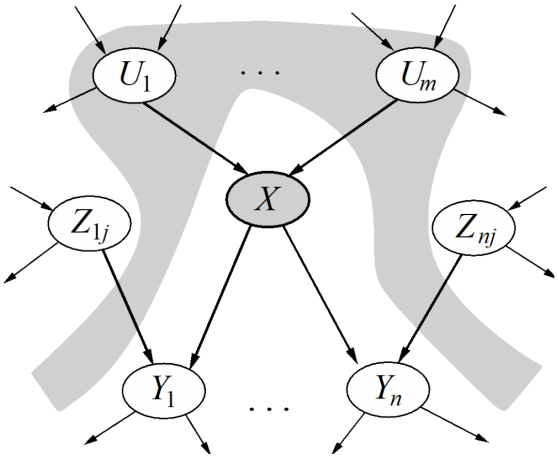
As a reality check, it's important to internalize that Bayes Nets are only a type of model. Models attempt to capture the way the world works, but because they are always a simplification they are always wrong. However, with good modeling choices they can still be good enough approximations that they are useful for solving real problems in the real world.

In general, a good model may not account for every variable or even every interaction between variables. But by making modeling assumptions in the structure of the graph, we can produce incredibly efficient inference techniques that are often more practically useful than simple procedures like inference by enumeration.

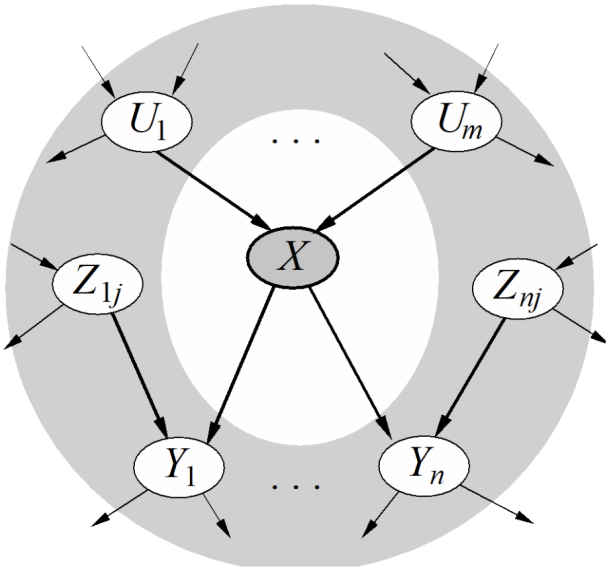
Structure of Bayes Nets

In this class, we will refer to two rules for Bayes Net independences that can be inferred by looking at the graphical structure of the Bayes Net:

- **Each node is conditionally independent of all its ancestor nodes (non-descendants) in the graph, given all of its parents**.



- **Each node is conditionally independent of all other variables given its Markov blanket.** A variable's Markov blanket consists of parents, children, children's other parents.

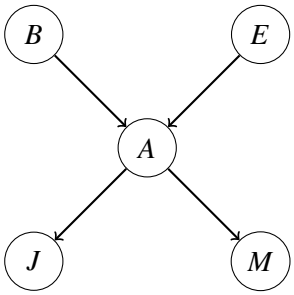


Using these tools, we can return to the assertion in the previous section: that we can get the joint distribution of all variables by joining the CPTs of the Bayes Net.

$$P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | parents(X_i))$$

This relation between the joint distribution and the CPTs of the Bayes net works because of the conditional independence relationships given by the graph. We will prove this using an example.

Let's revisit the previous example. We have the CPTs $P(B), P(E), P(A|B,E), P(J|A)$ and $P(M|A)$, and the following graph:



For this Bayes net, we are trying to prove the following relation:

$$P(B,E,A,J,M) = P(B)P(E)P(A|B,E)P(J|A)P(M|A) \tag{1}$$

We can expand the joint distribution another way: using the chain rule. If we expand the joint distribution with topological ordering (parents before children), we get the following equation

$$P(B,E,A,J,M) = P(B)P(E|B)P(A|B,E)P(J|B,E,A)P(M|B,E,A,J) \tag{2}$$

Notice that in Equation (1) every variable is represented in a CPT $P(var|Parents(var))$, while in Equation (2), every variable is represented in a CPT $P(var|Parents(var),Ancestors(var))$.

We rely on the first conditional independence relation above, that **each node is conditionally independent of all its ancestor nodes in the graph, given all of its parents**[1].

Therefore, in a Bayes net, $P(var|Parents(var),Ancestors(var)) = P(var|Parents(var))$, so Equation (1) and Equation (2) are equal. The conditional independences in a Bayes Net allow for multiple smaller conditional probability tables to represent the entire joint probability distribution.

[1]Elsewhere, the assumption may be defined as "a node is conditionally independent of its *non-descendants* given its parents." We always want to make the minimum assumption possible and prove what we need, so we will use the ancestors assumption.