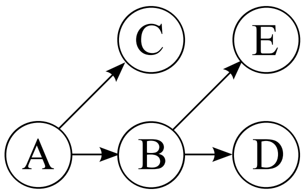
Q1. Bayes Nets: Variable Elimination



	$P(A)$
$+a$	0.25
$-a$	0.75

$P(B\|A)$	$+b$	$-b$
$+a$	0.5	0.5
$-a$	0.25	0.75

$P(C\|A)$	$+c$	$-c$
$+a$	0.2	0.8
$-a$	0.6	0.4

$P(D\|B)$	$+d$	$-d$
$+b$	0.6	0.4
$-b$	0.8	0.2

$P(E\|B)$	$+e$	$-e$
$+b$	0.25	0.75
$-b$	0.1	0.9

(a) Using the Bayes' Net and conditional probability tables above, calculate the following quantities:

(i) $P(+b| + a) =$

(ii) $P(+a, +b) =$

(iii) $P(+a| + b) =$

(b) Now we are going to consider variable elimination in the Bayes' Net above.

(i) Assume we have the evidence $+c$ and wish to calculate $P(E \mid +\text{c})$. What factors do we have initially?

(ii) If we eliminate variable B, we create a new factor. What probability does that factor correspond to?

(iii) What is the equation to calculate the factor we create when eliminating variable B?

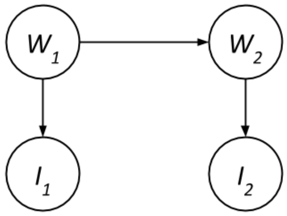
(iv) After eliminating variable B, what are the new set of factors? As in (ii), write the probabilities that the factors represent. For each factor, also provide its size.

(v) Now assume we have the evidence $-c$ and are trying to calculate $P(A| - c)$. What is the most efficient elimination ordering? If more than one ordering is most efficient, provide any one of them.

(vi) Once we have run variable elimination and have $f(A, -c)$ how do we calculate $P(+a \mid -c)$?

Q2. Sampling in Bayes Nets

We would like to analyze people's ice cream eating habits on sunny and rainy days. Suppose we consider the weather, along with a person's ice-cream eating, over the span of two days. We'll have four random variables: W_1 and W_2 stand for the weather on days 1 and 2, which can either be rainy R or sunny S, and the variables I_1 and I_2 represent whether or not the person ate ice cream on days 1 and 2, and take values T (for truly eating ice cream) or F. We can model this as the following Bayes Net with these probabilities.



W_1	$P(W_1)$
S	0.6
R	0.4

W_1	W_2	$P(W_2 \mid W_1)$
S	S	0.7
S	R	0.3
R	S	0.5
R	R	0.5

W	I	$P(I \mid W)$
S	T	0.9
S	F	0.1
R	T	0.2
R	F	0.8

Suppose we produce the following samples of (W_1, I_1, W_2, I_2) from the ice-cream model:

R, F, R, F   R, F, R, F   S, F, S, T   S, T, S, T   S, T, R, F
R, F, R, T   S, T, S, T   S, T, S, T   S, T, R, F   R, F, S, T

(a) Using these samples, what is our estimate of $P(W_2 = \mathtt{R})$?

(b) Cross off samples above which are rejected by rejection sampling if we're trying to estimate $P(W_2 \mid I_1 = \mathtt{T}, I_2 = \mathtt{F})$

Rejection sampling seems to be wasting a lot of effort, so we decide to switch to likelihood weighting. Assume we generate the following six samples for (W_1, I_1, W_2, I_2), given the evidence $I_1 = \mathtt{T}$ and $I_2 = \mathtt{F}$:

(S, T, R, F)   (R, T, R, F)   (S, T, R, F)   (S, T, S, F)   (S, T, S, F)   (R, T, S, F)

(c) Calculate the weight of each sample.

(d) Estimate $P(W_2 \mid I_1 = \mathtt{T}, I_2 = \mathtt{F})$ using our likelihood weights from the previous part.

Q3. Bayes Nets: Inference

Consider the following Bayes Net, where we have observed that $D = +d$.

$P(A)$	
$+a$	0.5
$-a$	0.5

$P(B \mid A)$		
$+a$	$+b$	0.5
$+a$	$-b$	0.5
$-a$	$+b$	0.2
$-a$	$-b$	0.8

$P(C \mid A, B)$			
$+a$	$+b$	$+c$	0.8
$+a$	$+b$	$-c$	0.2
$+a$	$-b$	$+c$	0.6
$+a$	$-b$	$-c$	0.4
$-a$	$+b$	$+c$	0.2
$-a$	$+b$	$-c$	0.8
$-a$	$-b$	$+c$	0.1
$-a$	$-b$	$-c$	0.9

$P(D \mid C)$		
$+c$	$+d$	0.4
$+c$	$-d$	0.6
$-c$	$+d$	0.2
$-c$	$-d$	0.8

(a) Below is a list of samples that were collected using prior sampling. Mark the samples that would be **rejected** by rejection sampling.

☐ $+a \quad -b \quad +c \quad -d$
☐ $+a \quad -b \quad +c \quad +d$
☐ $-a \quad +b \quad -c \quad -d$
☐ $+a \quad +b \quad +c \quad +d$

(b) To decouple from the previous part, you now receive a *new* set of samples shown below:

$+a \quad +b \quad +c \quad +d$
$-a \quad -b \quad -c \quad +d$
$+a \quad +b \quad +c \quad +d$
$+a \quad -b \quad -c \quad +d$
$-a \quad -b \quad -c \quad +d$

Estimate the probability $P(+a \mid +d)$ if these new samples were collected using...

(i) ... rejection sampling: ____________________

(ii) ... likelihood weighting: ____________________

(c) Instead of sampling, we now wish to use **variable elimination** to calculate $P(+a \mid +d)$. We start with the factorized representation of the joint probability:

$$P(A, B, C, +d) = P(A)P(B \mid A)P(C \mid A, B)P(+d \mid C)$$

(i) We begin by eliminating the variable B, which creates a new factor f_1. Complete the expression for the factor f_1 in terms of other factors.

$f_1($____________________$) =$ ____________________

(ii) After eliminating B to create a factor f_1, we next eliminate C to create a factor f_2. What are the remaining factors after both B and C are eliminated?

☐ $p(A)$ ☐ $p(B \mid A)$ ☐ $p(C \mid A, B)$ ☐ $p(+d \mid C)$ ☐ f_1 ☐ f_2

(iii) After eliminating both B and C, we are now ready to calculate $P(+a \mid +d)$. Write an expression for $P(+a \mid +d)$ in terms of the remaining factors.

$P(+a \mid +d) =$ ____________________