CS 188 Spring 2025 — Intro to Artificial Intelligence Midterm

Print Your Name: ______________________________

Print Your Student ID: ______________________________

Print Student name to your left: ______________________________

Print Student name to your right: ______________________________

You have 110 minutes. There are 6 questions of varying credit. (100 points total)

Question:	1	2	3	4	5	6	Total
Points:	20	16	21	19	14	10	100

For questions with **circular bubbles**, you may select only one choice.

○ Unselected option (Completely unfilled)

✓ Don't do this (it will be graded as incorrect)

● Only one selected option (completely filled)

For questions with **square checkboxes**, you may select one or more choices.

■ You can select

■ multiple squares

☑ Don't do this (it will be graded as incorrect)

Anything you write outside the answer boxes or you ~~cross out~~ will not be graded. If you write multiple answers, your answer is ambiguous, or the bubble/checkbox is not entirely filled in, we will grade the worst interpretation.

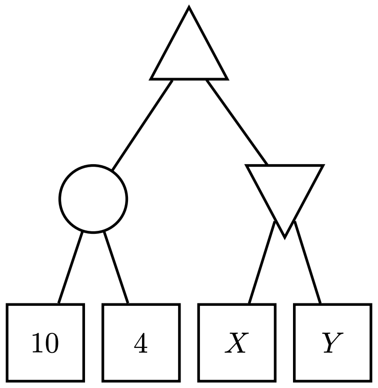
Read the honor code below and sign your name.

By signing below, I affirm that all work on this exam is my own work. I have not referenced any disallowed materials, nor collaborated with anyone else on this exam. I understand that if I cheat on the exam, I may face the penalty of an "F" grade and a referral to the Center for Student Conduct.

Sign your name: ______________________________

Q1 *Potpourri* **(20 points)**

In the next 3 subparts, consider the following game tree, representing a two-player game where the players take turns. Assume that the children of the expectation node have equal probability.



Assume nodes are pruned from left to right, and we prune on equality.

Q1.1 (3 points) Can you ever prune X?

If you select "Yes", write an inequality that describes when X will be pruned. For example, you could write "$X \leq 5$" or "$X + Y > 30$".

If you select "No", leave the box blank.

○ Yes ○ No

Q1.2 (3 points) Can you ever prune Y?

If you select "Yes", write an inequality that describes when Y will be pruned. For example, you could write "$X \leq 5$" or "$X + Y > 30$".

If you select "No", leave the box blank.

○ Yes ○ No

Q1.3 (2 points) Suppose the maximizer node represents Pacman, and both the minimizer node and the chance node represent Blinky. What kind of situation does this specific game tree represent?

○ Pacman acts randomly in some states, but adversarially in others.

○ Blinky acts randomly in some states, but adversarially in others.

○ Pacman and Blinky use different utility functions.

○ Pacman and Blinky act adversarially in all states.

Q1.4 (4 points) Select all true statements about game trees.

☐ Increasing the depth of a minimax game tree may result in a smaller value at the root node.

☐ Decreasing the depth of a minimax game tree may result in a smaller value at the root node.

☐ Increasing the depth of a minimax game tree may result in a larger value at the root node.

☐ Decreasing the depth of a minimax game tree may result in a larger value at the root node.

○ None of the above

Q1.5 (3 points) Select all true statements about policy iteration.

☐ Policy iteration always converges in strictly fewer iterations than value iteration.

☐ During policy evaluation, we can compute V^{π_i} using a system of linear equations.

☐ If V^{π_i} is optimal, then the policy extraction step will not change the policy in the next iteration.

○ None of the above

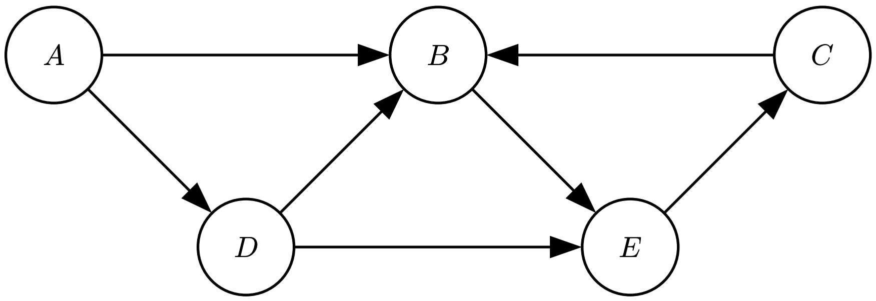
Q1.6 (2 points) Consider policies π_1, π_2 for the same MDP. Select all true statements about their value functions.

☐ If $\pi_1 = \pi_2$, then $Q^{\pi_1}(s,a) = Q^{\pi_2}(s,a)$ for all (s,a).

☐ If $\pi_1 = \pi_2$, then $V^{\pi_1}(s) = V^{\pi_2}(s)$ for all s.

○ None of the above

Q1.7 (3 points) Consider the graph below:



Select the edges that, when removed **in isolation** (i.e., independently of the other answer choices), make the graph a valid Bayes Net.

☐ A → B ☐ B → E ☐ D → B

☐ A → D ☐ C → B ☐ E → C

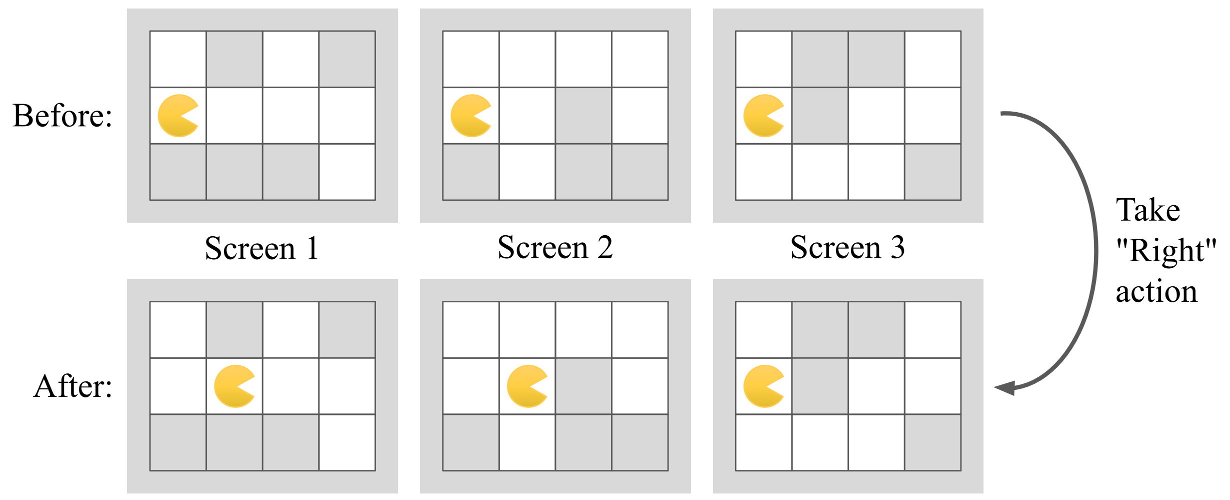
○ None of the above

Q2 *Pac Trifecta* **(16 points)**

You are playing a new arcade game called Pac Trifecta.

- There are three screens, with one Pacman on each screen. Pacmen cannot move between screens.
- Each screen is an $M \times N$ grid. Each screen may have different walls.
- When you press a button (Up, Down, Left, or Right), each Pacman simultaneously takes that action. For example, when you press "Right", all Pacmen will attempt to move right on their screen.
- If moving in a direction causes a Pacman to hit a wall, then that Pacman stays in its current position.
- All actions cost 1.

Here is an example. Your answers should work for any arbitrary problem, **not** just the example shown.



For the next 3 subparts, suppose that our goal is to get all Pacmen in the top-left corner at the same time.

Q2.1 (3 points) What is the size of the smallest state space representation for this problem?

◯ $3 \times M \times N$	◯ $(M \times N)^3$	◯ $M^3 \times N$
◯ $M \times N$	◯ $M \times N^3$	◯ $3^{M \times N}$

Q2.2 (2 points) Regardless of your answer to the previous subpart, assume that you use the following state space representation: three $M \times N$ boolean arrays, where a 1 denotes Pacman's location.

What is the asymptotic runtime to run the goal test on a given state?

◯ $O(1)$	◯ $O(M)$	◯ $O((M \times N)^3)$
◯ $O(N)$	◯ $O(M \times N)$	◯ $O(3^{M \times N})$

Q2.3 (4 points) Let D_i be the Manhattan distance from the Pacman on the ith screen to the top-left corner of the ith screen.

Select all admissible heuristics.

- ☐ $\max(D_1, D_2, D_3)$
- ☐ $\min(D_1, D_2, D_3)$
- ☐ $D_1 + D_2 + D_3$
- ☐ $\frac{D_1 + D_2 + D_3}{3}$
- ○ None of the above

Now, consider this modified problem: The goal is to bring all Pacmen to the same position on their respective screens, not necessarily the top-left corner. The specific position doesn't matter, as long as they are all in the same position at the same time step.

Q2.4 (2 points) Assume that you use the following state representation: three $M \times N$ boolean arrays, where a 1 denotes Pacman's location.

For the modified problem, what is the asymptotic runtime to run the goal test on a given state?

- ○ $O(1)$
- ○ $O(M)$
- ○ $O((M \times N)^3)$
- ○ $O(N)$
- ○ $O(M \times N)$
- ○ $O(3^{M \times N})$

Oh no! Blinky has taken control of the third screen! Now, when you press a button, only the Pacmen on screens 1 and 2 follow your action, while screen 3 remains unchanged. Then, Blinky presses a button and controls the Pacman on screen 3, leaving screens 1 and 2 unchanged. You and Blinky take turns.

Q2.5 (3 points) For this subpart, your goal is to get all three Pacmen to the same position, but Blinky doesn't necessarily have this goal.

We decide to model this using a game tree.

For a depth of one (where you take a turn, then Blinky takes a turn), what is the maximum number of leaves in the game tree?

Q2.6 (2 points) For this subpart, suppose that you both wish to bring all three Pacmen to the top-left corner in as few time steps as possible. Blinky still has separate control of the third screen.

We can formulate a single search problem and compute the optimal solution.

True or false: We can also formulate two separate search problems with strictly smaller state spaces (compared to the single search problem) and still compute the optimal solution.

- ○ True
- ○ False

Q3 *Constrained Student Problem* **(21 points)**

We want to assign 100 students ($s_1, s_2, ..., s_{100}$) to either Dwinelle (D) or VLSB (V). We'd like to model this as a CSP. For the first 4 subparts, suppose each room is infinitely large.

Assume that only the first 10 students that are assigned ($s_1, ..., s_{10}$) are left-handed. VLSB has at least 10 left-handed desks and Dwinelle has no left-handed desks. Left-handed students must be assigned to left-handed desks.

Q3.1 (2 points) Is this CSP easy or hard to solve, and why?

○ Easy, because it is under-constrained. ○ Easy, because it is over-constrained.

○ Hard, because it is under-constrained. ○ Hard, because it is over-constrained.

Q3.2 (2 points) How many possible assignments exist (including invalid assignments)?

○ 10 ○ 100 ○ 100^2 ○ 2^{100} ○ 100!

Recall the naive DFS-based algorithm from lecture. In particular, notice that we only check constraints when all variables have been assigned values, and we do not pre-process by applying unary constraints.

```
1 dfs(assignment, csp):
2     if assignment is complete and satisfies all constraints:
3         print(assignment) # found solution
4         exit # terminate program
5     else:
6         for each value in domain of the next unassigned variable:
7             new_assignment ← assignment with (variable, value) assigned
8             dfs(new_assignment, csp)
```

Q3.3 (2 points) If you always assign students to Dwinelle first on line 6, how many complete assignments does DFS examine?

○ 1 ○ Between 2 and 100 ○ More than 100

Q3.4 (2 points) If you always assign students to VLSB first on line 6, how many complete assignments does DFS examine?

○ 1 ○ Between 2 and 100 ○ More than 100

For the rest of the question, we add a new constraint: Each room can hold at most 50 students.

Q3.5 (3 points) Suppose we enforce Dwinelle's room capacity using only one higher-order constraint. In the worst case, how many variables do you need to check to determine if this constraint is satisfied?

Q3.6 (2 points) If you always assign students to VLSB first on line 6 with this new room constraint, how many complete assignments does DFS examine?

○ 1 ○ Between 2 and 100 ○ More than 100

Recall backtracking search from lecture. In particular, notice that unlike DFS, we now check constraints immediately following each variable assignment. We do not pre-process by applying unary constraints.

```
1 backtracking(assignment, csp):
2     if assignment is complete:
3         print(assignment) # found solution
4         exit # terminate program
5     else:
6         for each value in domain of the next unassigned variable:
7             new_assignment ← assignment with (variable, value) assigned
8             if new_assignment does not violate any constraints:
9                 backtracking(new_assignment, csp)
```

Q3.7 (2 points) Suppose you always assign students to VLSB first. During backtracking search, how many `new_assignment`s violate a constraint on line 8?

○ 1 ○ Between 2 and 100 ○ More than 100

Q3.8 (2 points) Consider two unassigned students s_1 (left-handed) and s_{88} (non-left-handed).

If you enforce arc consistency between s_1 and s_{88}, what happens?

○ At least one value gets removed from s_1's domain.

○ At least one value gets removed from s_{88}'s domain.

○ At least one value gets removed from both variables' domains.

○ No values get removed from either variable's domain.

Q3.9 (2 points) Suppose for this subpart only, we pre-process by applying unary constraints. What happens if you use the MRV (minimum remaining values) heuristic on this CSP?

○ Students are assigned to VLSB first.

○ Students are assigned to Dwinelle first.

○ Left-handed students are assigned first.

○ Non-left-handed students are assigned first.

Q3.10 (2 points) Pacman suggests using local search to solve this CSP.

Local search $\underset{(1)}{\underline{\qquad}}$ guaranteed to find a solution, and when local search finds a solution, the runtime is $\underset{(2)}{\underline{\qquad}}$ than exponential time on average.

○ (1) is (2) slower ○ (1) is NOT (2) slower

○ (1) is (2) faster ○ (1) is NOT (2) faster

Q4 *Modified MDP Equation* **(19 points)**

Recall the standard Bellman equation discussed in lecture:

$$Q(s,a) = \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma \max_{a'} Q(s',a')\right]$$

In this equation, $\max\limits_{a'} Q(s',a')$ is calculated by computing $Q(s',a')$ for every a', and then taking the **maximum** of all the resulting Q-values.

Consider a modified Bellman equation, where we replace the max function with an average function:

$$Q_\mu(s,a) = \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma \operatorname*{avg}_{a'} Q_\mu(s',a')\right]$$

In this equation, $\operatorname*{avg}\limits_{a'} Q_\mu(s',a')$ is calculated by computing $Q_\mu(s',a')$ for every a', and then taking the **average** of all the resulting Q_μ values.

Q4.1 (1 point) It is always possible to solve the **standard** Bellman equation (shown above) using a system of linear equations.

- ○ True
- ○ False

Q4.2 (1 point) It is always possible to solve the **modified** Bellman equation (shown above) using a system of linear equations.

- ○ True
- ○ False

Q4.3 (2 points) The optimal policy extracted from the Q_μ values (in the modified equation) is ____ the same as the optimal policy extracted from the Q-values (in the standard equation).

- ○ never
- ○ sometimes
- ○ always

Q4.4 (2 points) For this subpart only, consider a state s with many successor states, all of which are terminal states. Recall that once you enter a terminal state, no future rewards are available.

Which of these statements is always true?

- ○ $Q_\mu(s,a) < Q(s,a)$ for all actions a
- ○ $Q_\mu(s,a) > Q(s,a)$ for all actions a
- ○ $Q_\mu(s,a) = Q(s,a)$ for all actions a
- ○ None of the above

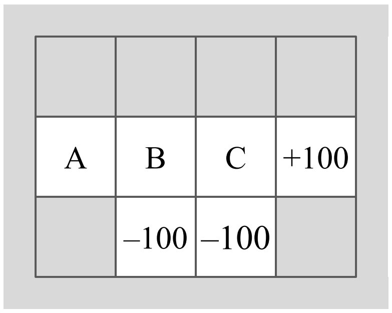
Q4.5 (2 points) Given a set of Q_μ values for all state-action pairs, which of these equations extracts a policy from the Q_μ values?

- ○ For each state s, compute $\arg\max\limits_{a} Q_\mu(s,a)$.
- ○ For each state s, compute $\operatorname*{avg}\limits_{a} Q_\mu(s,a)$.
- ○ For each action a, compute $\max\limits_{s} Q_\mu(s,a)$.
- ○ For each action a, compute $\arg\max\limits_{s} Q_\mu(s,a)$.

For the rest of this question, consider the Gridworld shown. Some states are labeled with letters (A, B, C).

Assume all actions succeed with 100% probability, $\gamma = 1$, and there is 0 living reward.

Reminder: In a Gridworld, there is only one action, "Exit", available from the squares labeled $+100$ and -100, that gives the rewards of $+100$ and -100, respectively.



Q4.6 (6 points) For this Gridworld, fill in the tables for $Q(s,a)$, the Q-values from the **standard** equation.

Write one number per box. The last row has been filled in for you as an example.

(s,a)	$Q(s,a)$
$(A,\rightarrow)$	
$(B,\leftarrow)$	
$(B,\downarrow)$	
$(B,\rightarrow)$	

(s,a)	$Q(s,a)$
$(C,\leftarrow)$	
$(C,\downarrow)$	
$(C,\rightarrow)$	100

For the remaining subparts, select the correct expression for $Q_\mu(s,a)$, the Q_μ values from the **modified** equation.

Q4.7 (1 point) $Q_\mu(C,\rightarrow)$

○ -100

○ 100

○ $\frac{1}{3}[Q_\mu(A,\downarrow) + Q_\mu(B,\downarrow) + Q_\mu(C,\downarrow)]$

○ $\frac{1}{3}[Q_\mu(C,\rightarrow) + Q_\mu(C,\downarrow) + Q_\mu(C,\leftarrow)]$

○ $\frac{1}{3}[Q_\mu(B,\rightarrow) + Q_\mu(B,\downarrow) + Q_\mu(B,\leftarrow)]$

○ $\frac{1}{3}[Q_\mu(A,\rightarrow) + Q_\mu(B,\rightarrow) + Q_\mu(C,\rightarrow)]$

Q4.8 (1 point) $Q_\mu(B,\downarrow)$

○ -100

○ 100

○ $\frac{1}{3}[Q_\mu(A,\downarrow) + Q_\mu(B,\downarrow) + Q_\mu(C,\downarrow)]$

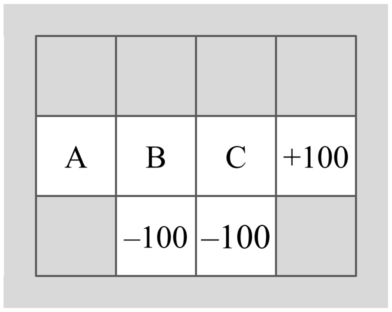
○ $\frac{1}{3}[Q_\mu(C,\rightarrow) + Q_\mu(C,\downarrow) + Q_\mu(C,\leftarrow)]$

○ $\frac{1}{3}[Q_\mu(B,\rightarrow) + Q_\mu(B,\downarrow) + Q_\mu(B,\leftarrow)]$

○ $\frac{1}{3}[Q_\mu(A,\rightarrow) + Q_\mu(B,\rightarrow) + Q_\mu(C,\rightarrow)]$

(Question 4 continued...)

The Gridworld, reprinted for your convenience:



Q4.9 (1 point) $Q_\mu(A, \rightarrow)$

- ○ -100
- ○ 100
- ○ $\frac{1}{3}\left[Q_\mu(A,\downarrow) + Q_\mu(B,\downarrow) + Q_\mu(C,\downarrow)\right]$
- ○ $\frac{1}{3}\left[Q_\mu(C,\rightarrow) + Q_\mu(C,\downarrow) + Q_\mu(C,\leftarrow)\right]$
- ○ $\frac{1}{3}\left[Q_\mu(B,\rightarrow) + Q_\mu(B,\downarrow) + Q_\mu(B,\leftarrow)\right]$
- ○ $\frac{1}{3}\left[Q_\mu(A,\rightarrow) + Q_\mu(B,\rightarrow) + Q_\mu(C,\rightarrow)\right]$

Q4.10 (1 point) $Q_\mu(B, \rightarrow)$

- ○ -100
- ○ 100
- ○ $\frac{1}{3}\left[Q_\mu(A,\downarrow) + Q_\mu(B,\downarrow) + Q_\mu(C,\downarrow)\right]$
- ○ $\frac{1}{3}\left[Q_\mu(C,\rightarrow) + Q_\mu(C,\downarrow) + Q_\mu(C,\leftarrow)\right]$
- ○ $\frac{1}{3}\left[Q_\mu(B,\rightarrow) + Q_\mu(B,\downarrow) + Q_\mu(B,\leftarrow)\right]$
- ○ $\frac{1}{3}\left[Q_\mu(A,\rightarrow) + Q_\mu(B,\rightarrow) + Q_\mu(C,\rightarrow)\right]$

Q4.11 (1 point) $Q_\mu(C, \leftarrow)$

- ○ -100
- ○ 100
- ○ $\frac{1}{3}\left[Q_\mu(A,\downarrow) + Q_\mu(B,\downarrow) + Q_\mu(C,\downarrow)\right]$
- ○ $\frac{1}{3}\left[Q_\mu(C,\rightarrow) + Q_\mu(C,\downarrow) + Q_\mu(C,\leftarrow)\right]$
- ○ $\frac{1}{3}\left[Q_\mu(B,\rightarrow) + Q_\mu(B,\downarrow) + Q_\mu(B,\leftarrow)\right]$
- ○ $\frac{1}{3}\left[Q_\mu(A,\rightarrow) + Q_\mu(B,\rightarrow) + Q_\mu(C,\rightarrow)\right]$

Q5 *Reinforcement Learning* **(14 points)**

All subparts of this question are **independent**.

Pacman is running approximate Q-learning with two features. Recall that we can calculate features for a given state-action pair, and we have a weight for each feature.

$$f(s,a) = \begin{bmatrix} f_1(s,a) \\ f_2(s,a) \end{bmatrix}, \text{ and } w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

Suppose Pacman introduces a third new feature, f_3, with corresponding weight w_3. The existing features and weights don't change.

Q5.1 (2 points) Before introducing f_3, a given state-action pair (s,a) had an approximate Q-value of 10.

After introducing f_3, what is the new approximate Q-value of (s,a)?

○ $10 + f_3(s,a) + w_3$

○ $(10 \cdot w_3) + f_3(s,a)$

○ $10 + (w_3 \cdot f_3(s,a))$

○ $10 \cdot (w_3 + f_3(s,a))$

Q5.2 (2 points) Suppose we had already determined w_1 and w_2 such that the policy extracted using approximate Q-learning was optimal.

After introducing f_3, what should we set w_3 to be such that the policy remains optimal?

Express your answer as a single decimal number.

Q5.3 (2 points) In this subpart, consider this scenario:

1. We run approximate Q-learning with only the original two features f_1 and f_2. We extract an *old policy* π from the converged approximate Q-values.
2. Then, we introduce the new feature f_3, and continue running approximate Q-learning (with all three features) until convergence.
3. Finally, we extract a *new policy* π' from the converged approximate Q-values.

At the end of Step 3, it is ____ the case that $\pi(s) = \pi'(s)$ for all states s.

○ never

○ sometimes

○ always

Q5.4 (4 points) We extract a feature vector $f(s_x, a_x)$ from a state-action pair (s_x, a_x).

We extract another feature vector $f(s_y, a_y)$ from another state-action pair (s_y, a_y).

We notice that $f(s_x, a_x)$ and $f(s_y, a_y)$ are identical feature vectors.

What can we conclude about the states and actions that generated these features?
Select all that apply.

- ☐ s_x and s_y must be the same state.
- ☐ a_x and a_y must be the same action.
- ☐ After convergence, the approximate Q-values computed for (s_x, a_x) and (s_y, a_y) are equal.
- ☐ The optimal actions from s_x and s_y are the same.
- ○ None of the above

In the next 2 subparts, consider the exploration function $K(s, a) = \frac{1}{N(s,a)}$, where $N(s, a)$ is the number of times we have seen the state-action pair (s, a).

Blinky suggests modifying approximate Q-learning by incorporating $K(s, a)$.

Notation:
- $|S|$ is the number of states.
- $|A|$ is the number of actions available per state. Assume all states have the same actions available.
- $|F|$ is the number of features. Assume $|F|$ is much less than $|S|$.

Q5.5 (2 points) What is the space complexity of standard approximate Q-learning from lecture, without $K(s, a)$?

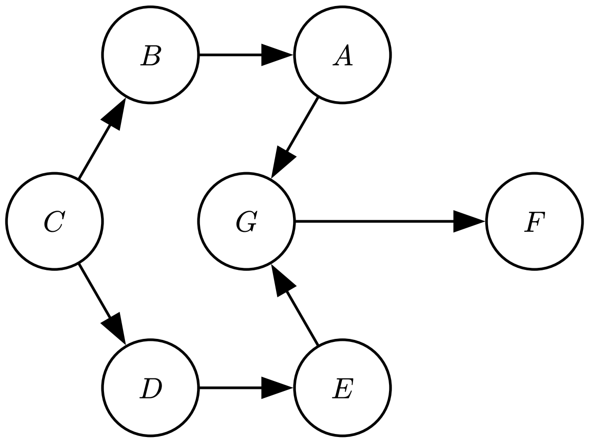
- ○ $O(|S| \times |A|)$
- ○ $O(|F|)$
- ○ $O(|S|^2 \times |A|)$
- ○ $O(|F| \times |S|)$

Q5.6 (2 points) What is the space complexity of modified approximate Q-learning, with $K(s, a)$?

- ○ $O(|S| \times |A|)$
- ○ $O(|F|)$
- ○ $O(|S|^2 \times |A|)$
- ○ $O(|F| \times |S|)$

Q6 *Pac Net* **(10 points)**

Consider the Bayes Net below. All random variables are binary (each variable has two possible values).



Q6.1 (2 points) C is conditionally independent of F given A and E.

○ True ○ False

Q6.2 (2 points) B is conditionally independent of D given F.

○ True ○ False

Q6.3 (2 points) Which CPT (conditional probability table) has the fewest number of entries?

○ A ○ C ○ E ○ G

○ B ○ D ○ F

Q6.4 (2 points) What is the sum of all values in D's CPT?

○ 0 ○ 1/2 ○ 1 ○ 2 ○ 4

○ Not enough information

Q6.5 (2 points) What is the sum of all values in G's CPT?

○ 0 ○ 1/2 ○ 1 ○ 2 ○ 4

○ Not enough information

Comment Box

Congrats for making it to the end of the exam! Leave any thoughts, comments, feedback, or doodles here.

Nothing in the comment box will affect your grade.