CS 188 Spring 2025 — Introduction to Artificial Intelligence — Written HW3

Due: Wednesday 2/19 at 11:59pm.

Policy: Can be solved in groups (acknowledge collaborators) but must be submitted individually.

Make sure to show all your work and justify your answers.

Note: This is a typical exam-level question. On the exam, you would be under time pressure, and have to complete this question on your own. We strongly encourage you to first try this on your own to help you understand where you currently stand. Then feel free to have some discussion about the question with other students and/or staff, before independently writing up your solution.

Note: Leave the self-assessment sections blank for the original submission of your homework. After the homework deadline passes, we will release the solutions. At that time, you will review the solutions, self-assess your initial response, and complete the self-assessment sections below. The deadline for the self-assessment is 1 week after the original submission deadline.

Your submission on Gradescope should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question begins on page 2.). **Do not reorder, split, combine, or add extra pages**. The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	
Last name	
SID	
Collaborators	

Q1. [19 pts] Games

Alice, Eve, and Bob are playing a multiplayer game. Each game state consists of three numbers where the left value represents Alice's score, the middle value represents Eve's score, and the right value represents Bob's score. Alice makes the first move, followed by Eve, and finally Bob. All scores for a single player are **between 1 and 9 inclusive**. In all pruning scenarios, **remember that we do not prune on equality.**

Rather than trying to maximize their individual scores, Alice and Bob decide to work together to maximize their combined score, hoping that this will allow them to score higher. At each of Alice's and Bob's nodes, they will choose the option that maximizes **left value + right value**.

(a) Eve overhears their plan and decides that instead of maximizing her own score, she will try to minimize Alice and Bob's combined score. Alice and Bob are aware of Eve's strategy. Let the value of a node be the sum of the left and right scores of the node. Answer the following questions based on the game tree shown below.

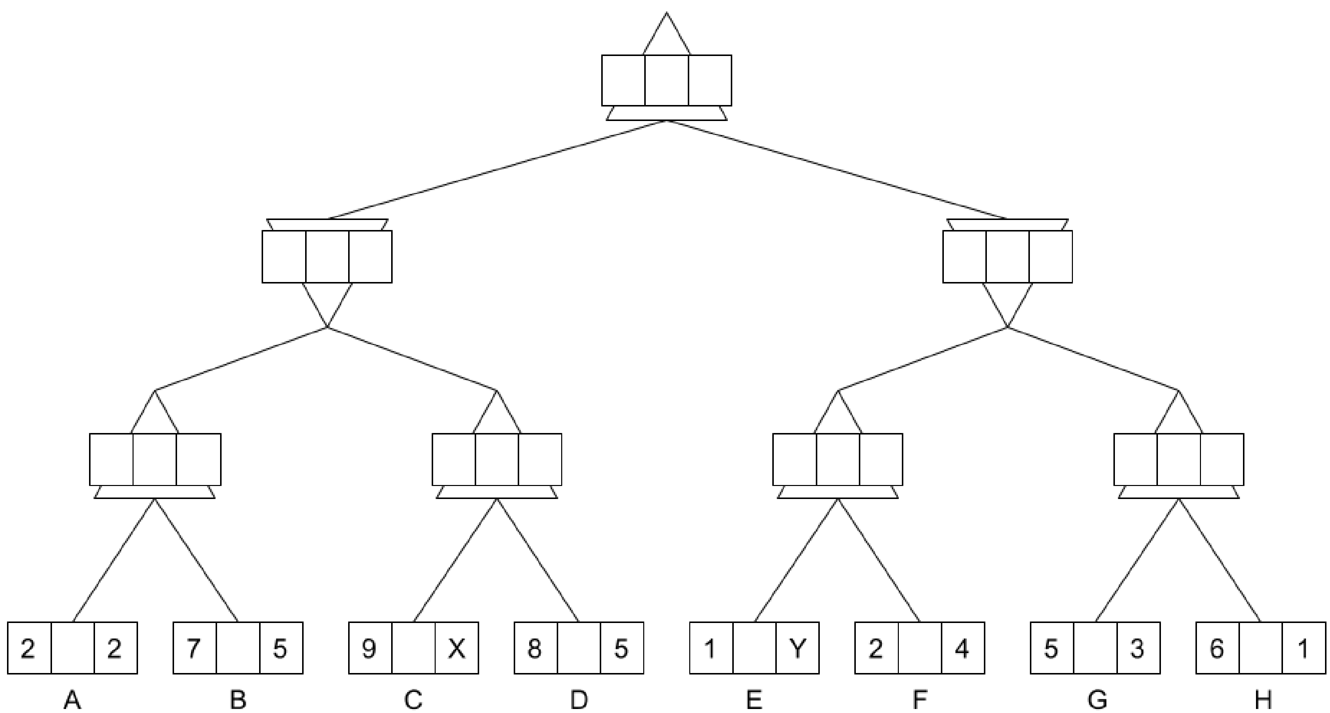

Figure 1: Game tree where Alice is the root maximizer, Eve is the minimizer, and Bob is the bottom maximizer. Eve's score at each node (center cell) is not shown for simplicity.

(i) [1 pt] Solve the game tree shown in figure 1. What is the value of the root node?

- ○ 6
- ● 12
- ○ 13
- ○ Depends on the value of X only.
- ○ Depends on the value of Y only.
- ○ Depends on the values of both X and Y.
- ○ None of the above.

See diagram below.

(ii) [2 pts] Without pruning, which of the following are possible values for the right minimizer?

☐ < 6 ■ 6 ■ 7 ■ 8 ☐ > 8

Consider each child of the right minimizer. The right child has a value of 8 ($\max(5+3, 6+1) = 8$). For the left child, we must consider the bounds on Y. Given that $1 \le Y \le 9$, we know that at minimum, the left child is $\max(1+1, 2+4) = 6$ and at most it is $\max(1+9, 2+4) = 10$. So the value of the left child is between 6 and 10. The minimizer is guaranteed to choose the lower of its two children's values so the bounds on its values are between 6 and 8.

(iii) [2 pts] Which of the following nodes are **guaranteed** to be pruned when running alpha beta pruning on the game tree above? If there are nodes that may or may not be pruned depending on the values of X and Y, do not select them.

☐ A ☐ D ■ G
☐ B ☐ E ■ H
☐ C ☐ F ○ None of the above.

The root maximizer gets a value of 12 from its left subtree. When visiting the right subtree, we know that the **maximum** value that the maximizer of E and F can be is $\max(1+9, 2+4) = 10$ (if Y is the highest value possible of 9). Since even $10 < \alpha = 12$, we can prune the entire right child of the right minimizer. Therefore, G and H are both guaranteed to be pruned regardless of the value of Y.

(iv) [3 pts] Which of the following nodes **may or may not** be pruned depending on the values of X and Y? Do not select any nodes from the previous part which are guaranteed to be pruned regardless of the values for X and Y.

☐ A ■ D ☐ G
☐ B ☐ E ☐ H
☐ C ☐ F ○ None of the above.

From the previous part, we know that the value of Y does not affect whether nodes are pruned or not, so here we consider only the possible values of X. In the left subtree, the left child of the minimizer has a value of 12 (from $\max(2+2, 7+5) = 12$). Therefore, if $C > \beta = 12$, then we can prune D. This case occurs if $9 + X > 12$ or when $X > 3$.

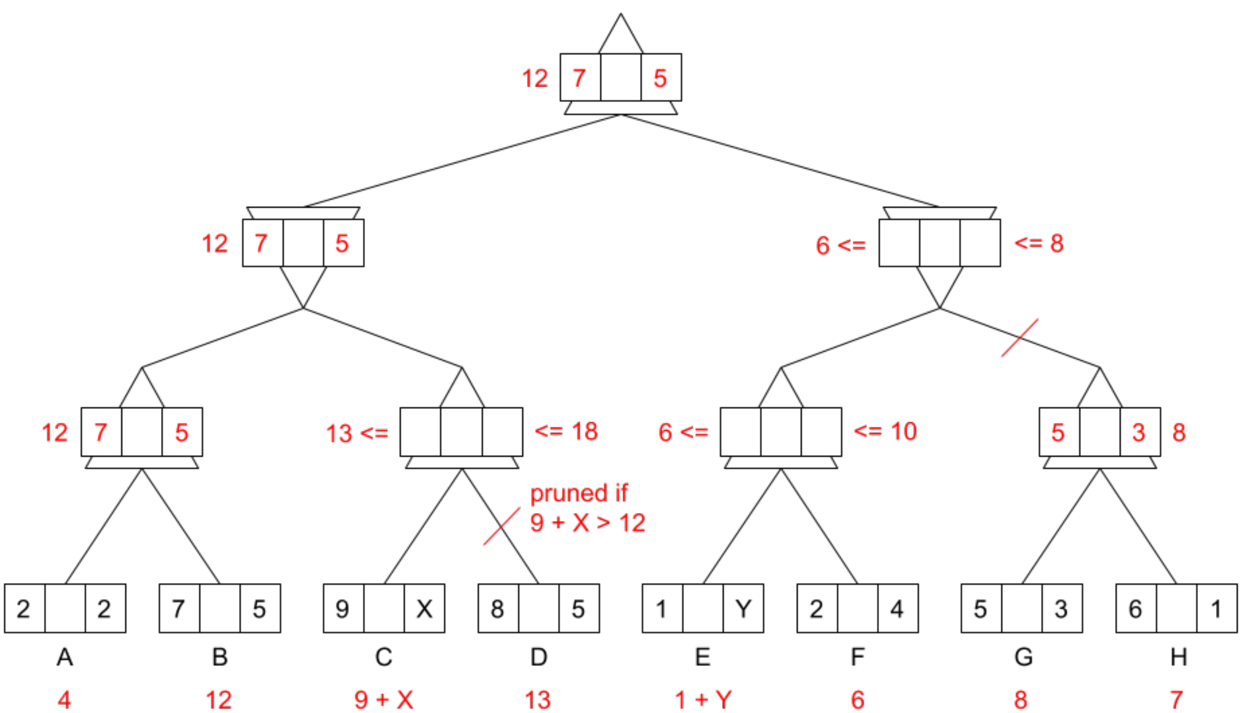

Figure 2: Part a solutions

Q1(a) Self-Assessment - leave this section blank for your original submission. We will release the solutions to this problem after the deadline for this assignment has passed. After reviewing the solutions for this problem, assess your initial response by checking one of the following options:

- ◯ I fully solved the problem correctly, including fully correct logic and sufficient work (if applicable).
- ◯ I got part or all of the question incorrect.

If you selected the second option, explain the mistake(s) you made and why your initial reasoning was incorrect (do not reiterate the solution. Instead, reflect on the errors in your original submission). Approximately 2-3 sentences for *each* incorrect sub-question.

(b) Eve now decides that in addition to minimizing Alice and Bob's scores, she also wants to maximize her own score. Her new strategy is to choose the option that maximizes her own score minus Alice and Bob's combined score. That is, at Eve's turn, she will choose the option that maximizes **middle value − (left value + right value)**. Alice and Bob are aware of this new strategy. Using the same game tree shown above, assume that we can choose any number between 1 and 9 (inclusive) for X, Y, and Eve's score at each leaf node.

(i) [1 pt] *True/False*: Compared to Eve's strategy in part (a), Eve's new strategy will result in an equal or higher final score **for Eve** in any leaf node configuration.

● True ○ False

Let V_1 and V_2 be the combined score for Alice and Bob in the previous strategy. In part a, Eve is trying to minimize Alice and Bob's scores, so this is equivalent to choosing the maximum between $-V_1$ and $-V_2$. In part b, Eve factors in her own score so her new strategy is to choose the maximum between $E_1 - V_1$ and $E_2 - V_2$. WLOG, assume Eve chose option 1 in the former case. Then $-V_1 > -V_2$ and she would only change her choice using strategy b if E_2 was significantly greater than E_1 such that $E_1 - V_1 < E_2 - V_2$. Therefore, Eve's score can only be higher.

(ii) [1 pt] *True/False*: Compared to Eve's strategy in part (a), Eve's new strategy will result in an equal or higher final combined score **for Alice and Bob** in any leaf node configuration.

● True ○ False

Eve's strategy in part a was the optimal adversarial strategy against Alice and Bob. By considering other factors like her own score, Eve becomes an un-optimal adversary since she doesn't always choose the option that most minimizes Alice and Bob's scores. Therefore, Alice and Bob's score can only be the same or increase.

(iii) [3 pts] Which of the following leaf nodes could possibly be the game outcome if all players play optimally according to their strategy?

☐ A	■ C	☐ E	☐ G
■ B	■ D	☐ F	☐ H

The bottom level of maximizers will still be the same as in part a since Bob's strategy is not changing, so we can still use the same bounds on those values that we determined in part a. Additionally, we know that the root maximizer will only choose the path that maximizes the values from the bottom maximizers. Regardless of what Eve chooses, we know based on the bounds of the depth 2 maximizers that the best value that the root can achieve from the right subtree is 10 (left maxmimizer can provide between 6 and 10 while right maximizer is 8). 10 is less than all possible options from the left subtree (12 or between 13 and 18) so none of the leaf values of the right subtree are possible outcomes since the root would never choose the right action.

Now that we have narrowed down our options to A, B, C, or D, we know that A will never be chosen because the leftmost maximizer is guaranteed to choose B. Since we don't know the value of the right maximizer, it's possible that the right maximizer could choose either C or D. From the left minimizer's perspective, it is possible to select values for Eve's score that would make it choose either left or right child. Therefore, B, C, and D are all valid possible outcomes.

(iv) [2 pts] Is it possible to prune in this scenario?

● Yes because scores in each cell are bounded between 1 and 9.

○ Yes but not for the reason above.

○ No because Alice, Bob, and Eve are all acting as maximizers.

○ No but not for the reason above.

There are various possibilities here, but since the values of each leaf score are bounded between 1 and 9, it is possible to come up with a scenario where we are guaranteed to be able to prune. Consider the following example which focuses only on the left subtree:

Let $E_B = 9$ and $E_C = 1$ be Eve's value at node B and node C respectively. Let $X = 8$. The leftmost maximizer will, like before, choose node B, giving it the scores $[7, 9, 5]$. From Eve's perspective, she will see the value $9-(7+5) = -3$ from her left subtree. Next, the rightmost maximizer will look at node C with scores $[9, 1, 8]$, giving it a value of $9 + 8 = 17$. At the same time, Eve sees that node C returns $1 - 17 = -16 < \beta = -3$, so node D can be pruned.

Q1(b) Self-Assessment - leave this section blank for your original submission. We will release the solutions to this problem after the deadline for this assignment has passed. After reviewing the solutions for this problem, assess your initial response by checking one of the following options:

- ◯ I fully solved the problem correctly, including fully correct logic and sufficient work (if applicable).
- ◯ I got part or all of the question incorrect.

If you selected the second option, explain the mistake(s) you made and why your initial reasoning was incorrect (do not reiterate the solution. Instead, reflect on the errors in your original submission). Approximately 2-3 sentences for *each* incorrect sub-question.

(c) Eve is fed up with Alice and Bob teaming up and quits the game. Alice and Bob continue playing and decide to use brand new strategies that incorporate Eve's score for fun. This new game setup can be represented in the diagram below. In each of the following scenarios, Alice and Bob are aware of each other's strategies.

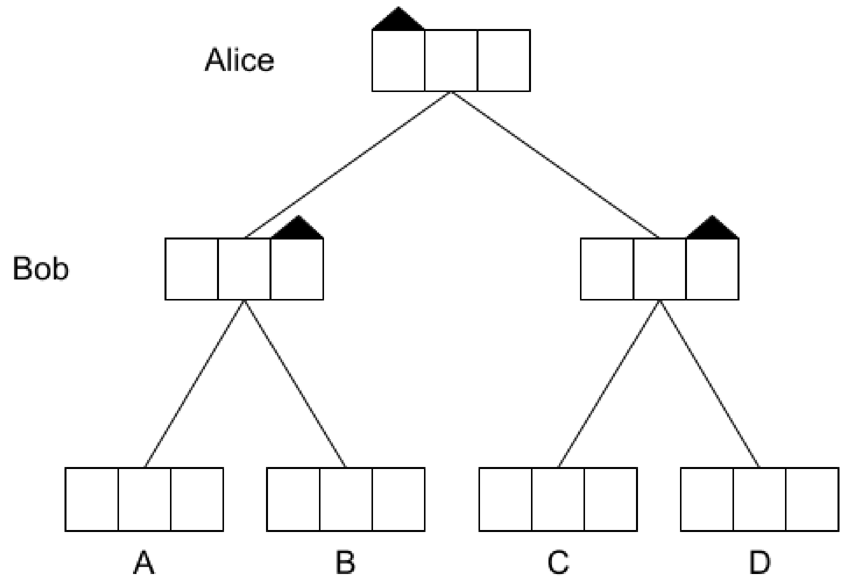

Figure 3: Game tree where Alice is the root and Bob controls the nodes in the middle level. Black triangles above a cell indicate that the cell's value contains the current player's score. (As a reminder, Bob's score is the right value and Alice's score is the left value.)

(i) [2 pts] Alice and Bob agree to use the following strategy: each player maximizes their own score **plus** the average of the remaining two scores at each node. Assume that you can assign any value between 1 to 9 (inclusive) to all the leaf node scores. Is it possible to prune in this scenario?

- ○ Yes.
- ● No because Alice and Bob are both acting as maximizers.
- ○ No because Alice and Bob are both acting as expectimax nodes.
- ○ No but not for the above reasons.

Let $[S_1, S_2, S_3]$ refer to the scores in a node. Alice is trying to maximize $S_1 + \frac{1}{2}\left(S_2 + S_3\right)$ and Bob is trying to maximize $S_3 + \frac{1}{2}\left(S_1 + S_2\right)$. Jointly, they both end up maximizing $\frac{1}{2}\left(S_1 + S_2 + S_3\right)$ and the extra terms (Alice maximizing an additional $\frac{1}{2}S_1$ and Bob maximizing an additional $\frac{1}{2}S_3$) are not adversarial in any way.

(ii) [2 pts] Alice and Bob decide to follow a new strategy: each player maximizes their own score **minus** the average of the remaining two scores at each node. Assume that you can assign any value between 1 to 9 (inclusive) to all the leaf node scores. Is it possible to prune in this scenario?

- ● Yes.
- ○ No because Alice and Bob are both acting as maximizers.
- ○ No because Alice and Bob are both acting as expectimax nodes.
- ○ No because Alice and Bob are maximizing different values which are not directly adversarial.
- ○ No but not for the above reasons.

Let $[S_1, S_2, S_3]$ refer to the scores in a node. Alice is trying to maximize $S_1 - \frac{1}{2}\left(S_2 + S_3\right)$ and Bob is trying to maximize $S_3 - \frac{1}{2}\left(S_1 + S_2\right)$. We can disregard the S_2 term here since it only serves as an equal shift in both players' values. What we can see then is any increase to Alice's score (either due to increase in S_1 or decrease in S_3) will always result in a decrease in Bob's score. Therefore, Alice and Bob are playing an adversarial game and it is possible to prune. The following graph provides a specific example.

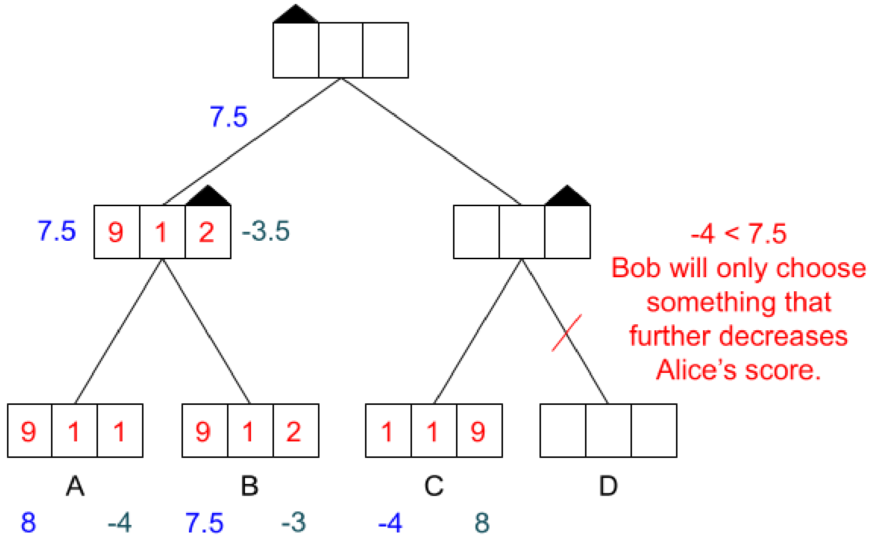

Figure 4: Part c solutions

Q1(c) Self-Assessment - leave this section blank for your original submission. We will release the solutions to this problem after the deadline for this assignment has passed. After reviewing the solutions for this problem, assess your initial response by checking one of the following options:

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