CS 188 Spring 2025	Introduction to Artificial Intelligence	Written HW6

Due: Wednesday 3/12 at 11:59pm.

Policy: Can be solved in groups (acknowledge collaborators) but must be submitted individually.

Make sure to show all your work and justify your answers.

Note: This is a typical exam-level question. On the exam, you would be under time pressure, and have to complete this question on your own. We strongly encourage you to first try this on your own to help you understand where you currently stand. Then feel free to have some discussion about the question with other students and/or staff, before independently writing up your solution.

Note: Leave the self-assessment sections blank for the original submission of your homework. After the homework deadline passes, we will release the solutions. At that time, you will review the solutions, self-assess your initial response, and complete the self-assessment sections below. The deadline for the self-assessment is 1 week after the original submission deadline.

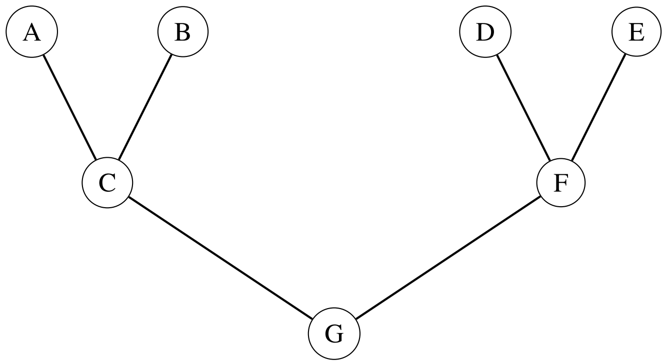
Your submission on Gradescope should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question begins on page 2.). **Do not reorder, split, combine, or add extra pages**. The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	
Last name	
SID	
Collaborators	

Q1. [9 pts] Tracing Contacts

There is a rumor circulating that a past CS 188 head TA is submitting fake pet pics while disguising as a student. You are trying to retrace the origins of this rumor but do not know who spread the story to whom. However, you know the contacts between each pair of people and are able to create the network below.

(a) Assume that the direction of each edge is uniformly random and that the direction of any two edges is independent.



(i) [1 pt] What is the probability that a member of the group ABC spread the rumor to group DEF through G? (Hint: Don't overthink this question. What must the direction of certain edges be in the graph?) $\frac{1}{4}$

Only the CG and GF edges are relevant. There are 4 equally likely combination of arrow directions, and the only arrow direction that suggests a member of group ABC spread the viral variant to group DEF is with $C \to G$ and $G \to F$. Thus, $\frac{1}{4}$

(ii) [2 pts] What is the probability that $B \perp\!\!\!\perp F$ is guaranteed via the independence relations encoded by the Bayes Net? $\frac{1}{2}$

This is actually very similar to the first part of problem b(i), but applied to 4 nodes. We actually only need to focus on the four variables B, C, G, F since that's the only path from B to F. There are 8 possible bayes net edge configurations. Only 4 of them give us a configuration where we cannot guarantee $B \perp\!\!\!\perp F$:

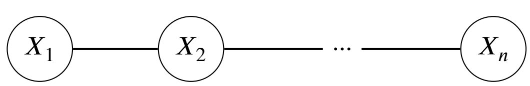
- 2 configurations from fixing the edges $B \to C$ and $C \leftarrow G$ to create inactive common effect BCG.
- 2 configurations from fixing edges $C \to G$ and $G \leftarrow F$ to create inactive common effect CGF.

With 3 edges, we have 2^3 possible configurations of the bayes net. And we have 4 possible configurations where $B \perp\!\!\!\perp F$ is guaranteed. So the probability of $B \perp\!\!\!\perp F$ is $\frac{4}{8} = \frac{1}{2}$

(iii) [2 pts] If we add an edge between nodes B and D, what is the probability that the above Bayes Net is undefined? $\frac{1}{16}$

A Bayes Net must be an acyclic DAG by definition. There are two arrangements of edges that cause the above bayes net to be cyclic: $B \to D \to F \to G \to C \to B$, or $B \leftarrow D \leftarrow F \leftarrow G \leftarrow C \leftarrow B$. Thus, $\frac{2}{2^5} = \frac{1}{2^4}$

(b) In another class, you recover the following chain of contacts. Continue assuming that the direction of each edge is uniformly random and that the direction of any two edges is independent.



(i) [1 pt] What is the probability that $X_1 \perp\!\!\!\perp X_n$ is guaranteed for $n = 3$? $\frac{1}{4}$

With no given nodes, you must have a common effect for the triple to be inactive. So $X_1 \to X_2 \leftarrow X_3$. There are 2^2 possibilities for arrow direction combinations, so the probability of $X_1 \perp\!\!\!\perp X_n$ being guaranteed is $\frac{1}{2^2}$

(ii) [1 pt] What is the probability that $X_1 \perp\!\!\!\perp X_n$ for any $n \geq 3$?

○ $\frac{n}{4} - \frac{1}{2}$ ○ $\frac{2^{n-1}-1}{2^{n-1}}$ ○ $\frac{1}{2^{n-2}}$ ○ $\frac{2^{n-1}-1}{2^{n-1}} - \frac{1}{2}$

○ $\frac{1}{2^{n-2}} - \frac{1}{4}$ ○ $\frac{1}{2^{n-1}}$ ● $\frac{2^{n-1}-n}{2^{n-1}}$ ○ None of the above

We know that there has to be at least one common effect triple in the chain to make the entire path inactive.
It is more helpful here to think about the cases where X_1, X_n are NOT guaranteed to be independent. When does this happen? Only when all the triples are a mix of causal chain or common cause. We can actually get all of the possible bayes nets where X_1 and X_n are not guaranteed to be independent by "sliding" a common cause triple across the entire bayes net, like so.

Graph 1:

$$X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow \cdots \rightarrow X_{n-2} \rightarrow X_{n-1} \rightarrow X_n$$

Graph 2:

$$X_1 \leftarrow X_2 \rightarrow X_3 \rightarrow \cdots \rightarrow X_{n-2} \rightarrow X_{n-1} \rightarrow X_n$$

Graph 3:

$$X_1 \leftarrow X_2 \leftarrow X_3 \rightarrow \cdots \rightarrow X_{n-2} \rightarrow X_{n-1} \rightarrow X_n$$

Graph $n-1$:

$$X_1 \leftarrow X_2 \leftarrow X_3 \leftarrow \cdots \leftarrow X_{n-2} \leftarrow X_{n-1} \rightarrow X_n$$

Graph n:

$$X_1 \leftarrow X_2 \leftarrow X_3 \leftarrow \cdots \leftarrow X_{n-2} \leftarrow X_{n-1} \leftarrow X_n$$

We can think about each of these n graphs, and that if any other edges are flipped in direction in any of these n graphs, the resulting bayes net will have some inactive common effect triple, creating an inactive path.
Thus, there are n different possible bayes net configurations where X_1 and X_n are guaranteed independent. This gives us $2^{n-1} - n$ bayes net configurations where we are guaranteed $X_1 \perp\!\!\!\perp X_n$. There are 2^{n-1} possible bayes net edge configurations. So the probability is $\frac{2^{n-1}-n}{2^{n-1}}$.

(iii) [2 pts] Conditioned on $X_1 \perp\!\!\!\perp X_n$ guaranteed true, what is the probability of the $X_1 - X_2$ edge pointing left?

○ $\frac{1}{2}$

○ $\frac{2^{n-2}-1}{2^{n-1}}$

○ $\frac{2^{n-2}-(n-1)}{2^{n-1}}$

○ $\frac{1}{4}$

○ $\frac{2^{n-2}-1}{2^{n-1}-n}$

○ $\frac{2^{n-2}}{2^{n-1}-n}$

● $\frac{2^{n-2}-(n-1)}{2^{n-1}-n}$

○ None of the above

We can create such a table, where we already have the information to fill in everything except the 2nd column for rows Right and Left (those squares can be filled in with simple subtraction).

$X_1 - X_2$ direction	$X_1 \perp\!\!\!\perp X_4$ not guaranteed	$X_1 \perp\!\!\!\perp X_4$ guaranteed	Total
Right	1	$2^{n-2} - 1$	2^{n-2}
Left	$n-1$	$2^{n-2} - (n-1)$	2^{n-2}
TOTAL	n	$2^{n-1} - n$	2^{n-1}

From this table, since all bayes net configurations are equally likely, we can read off $\frac{2^{n-2}-(n-1)}{2^{n-1}-n}$ as the probability.