Announcements

- HW1 is due Wednesday, February 5, 11:59 PM PT
- Project 1 is due Friday, February 7, 11:59 PM PT
- Oliver's office hours in 329 Soda TuTh 2:30-4:00 except Thursday 2/6; remote office hours on Friday 2/7 from 2:30-4:00 on the class Zoom meeting channel (Meeting ID: 995 0435 8998 Passcode: 852823)
- All concurrent enrollment students will be added. Resubmit your request if not.
- Please attend discussion sections.

CS 188: Artificial Intelligence

Constraint Satisfaction Problems II

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University of California, Berkeley

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]

Today

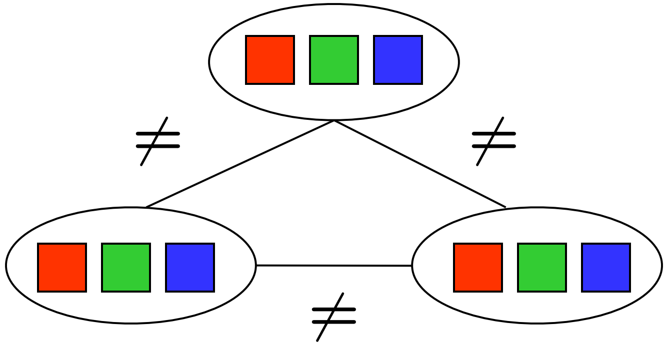
Efficient Solution of CSPs

Local Search

Reminder: CSPs

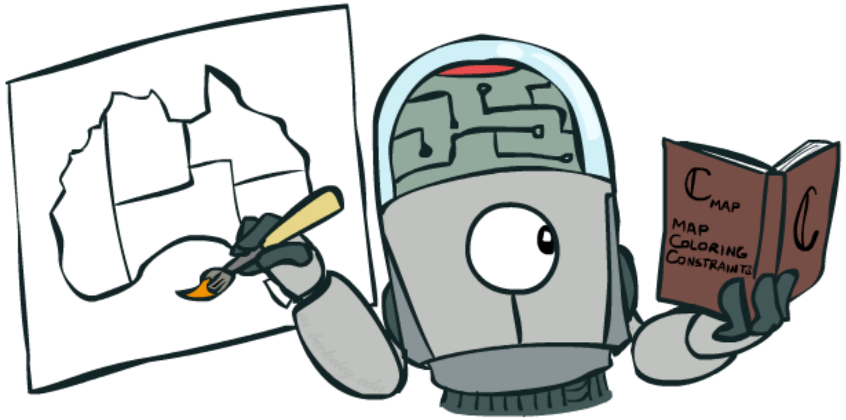
CSPs:

- Variables
- Domains
- Constraints
  - Implicit (provide code to compute)
  - Explicit (provide a list of the legal tuples)
  - Unary / Binary / N-ary

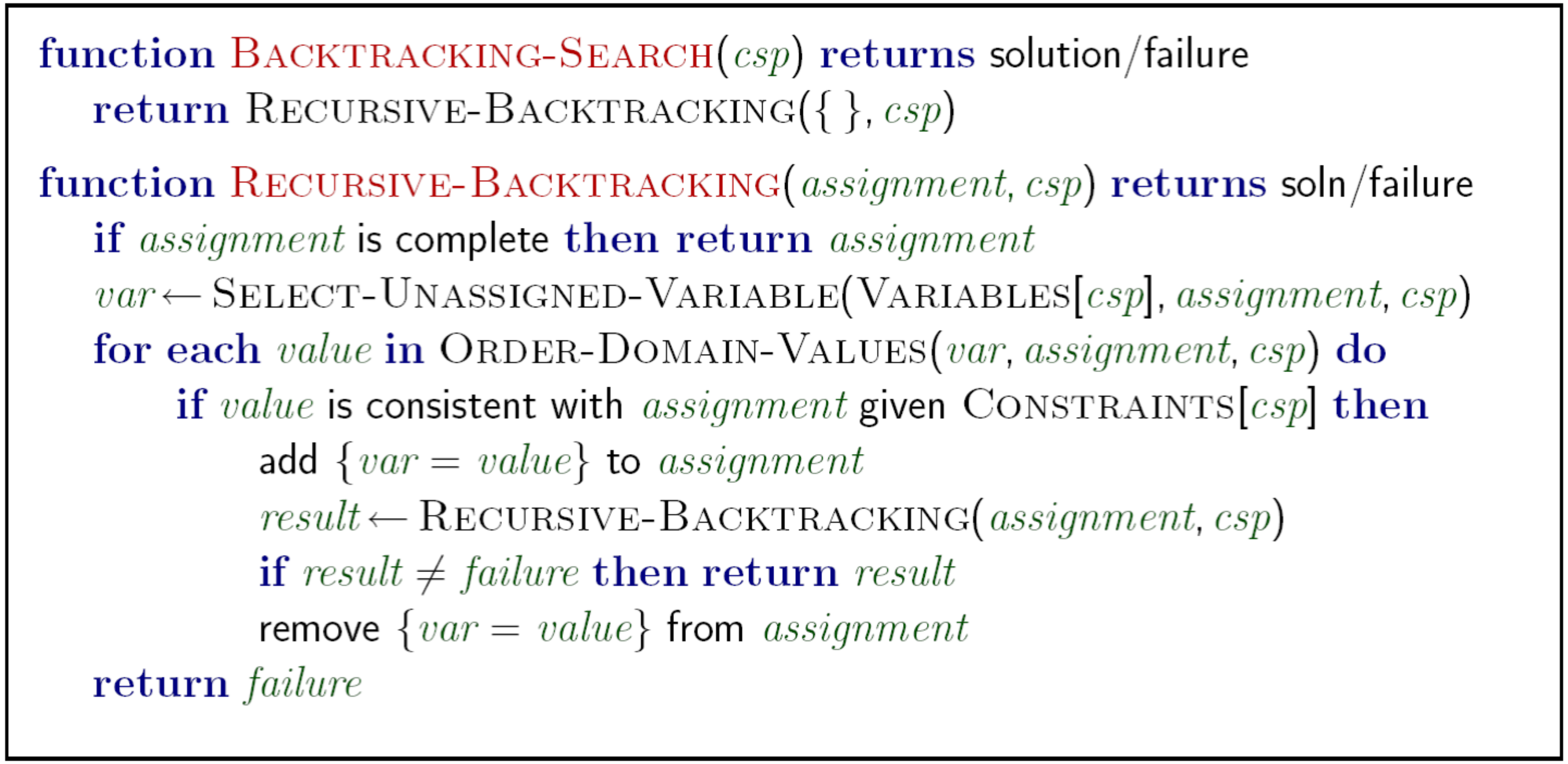


Goals:

- Here: find any solution
- Also: find all, find best, etc.



Backtracking Search



```
function BACKTRACKING-SEARCH(csp) returns solution/failure
    return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
    if assignment is complete then return assignment
    var $\leftarrow$ SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment given CONSTRAINTS[csp] then
            add {var = value} to assignment
            result $\leftarrow$ RECURSIVE-BACKTRACKING(assignment, csp)
            if result $\neq$ failure then return result
            remove {var = value} from assignment
    return failure
```

Improving Backtracking

General-purpose ideas give huge gains in speed

… but it's all still NP-hard

Filtering: Can we detect inevitable failure early?

Ordering:

Which variable should be assigned next? (MRV)

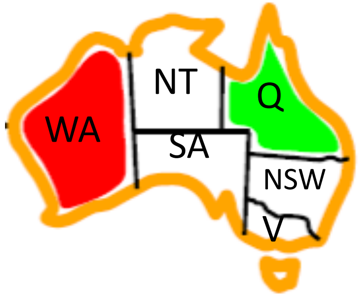
In what order should its values be tried? (LCV)

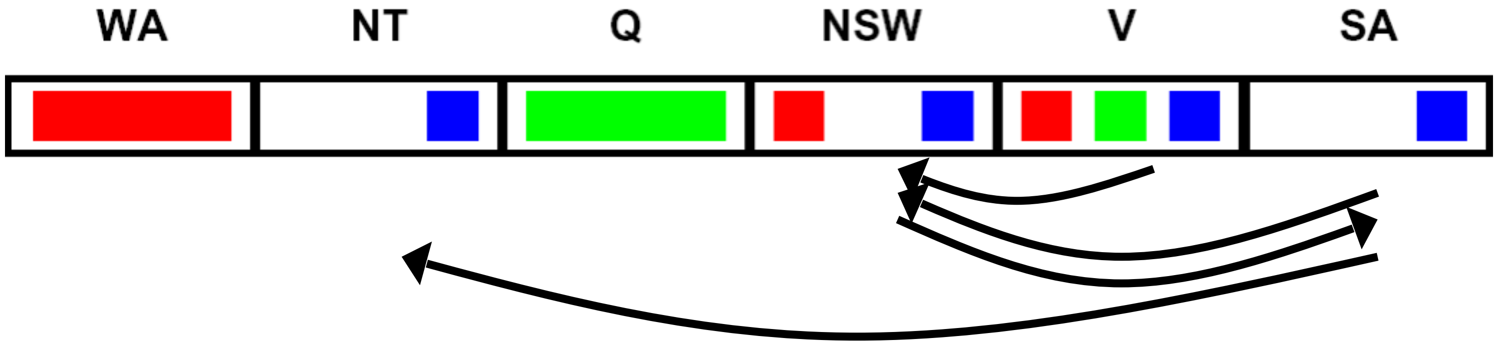
Structure: Can we exploit the problem structure?

Filtering

Arc Consistency of an Entire CSP

A simple form of propagation makes sure all arcs are consistent:





Important: If X loses a value, neighbors of X need to be rechecked!

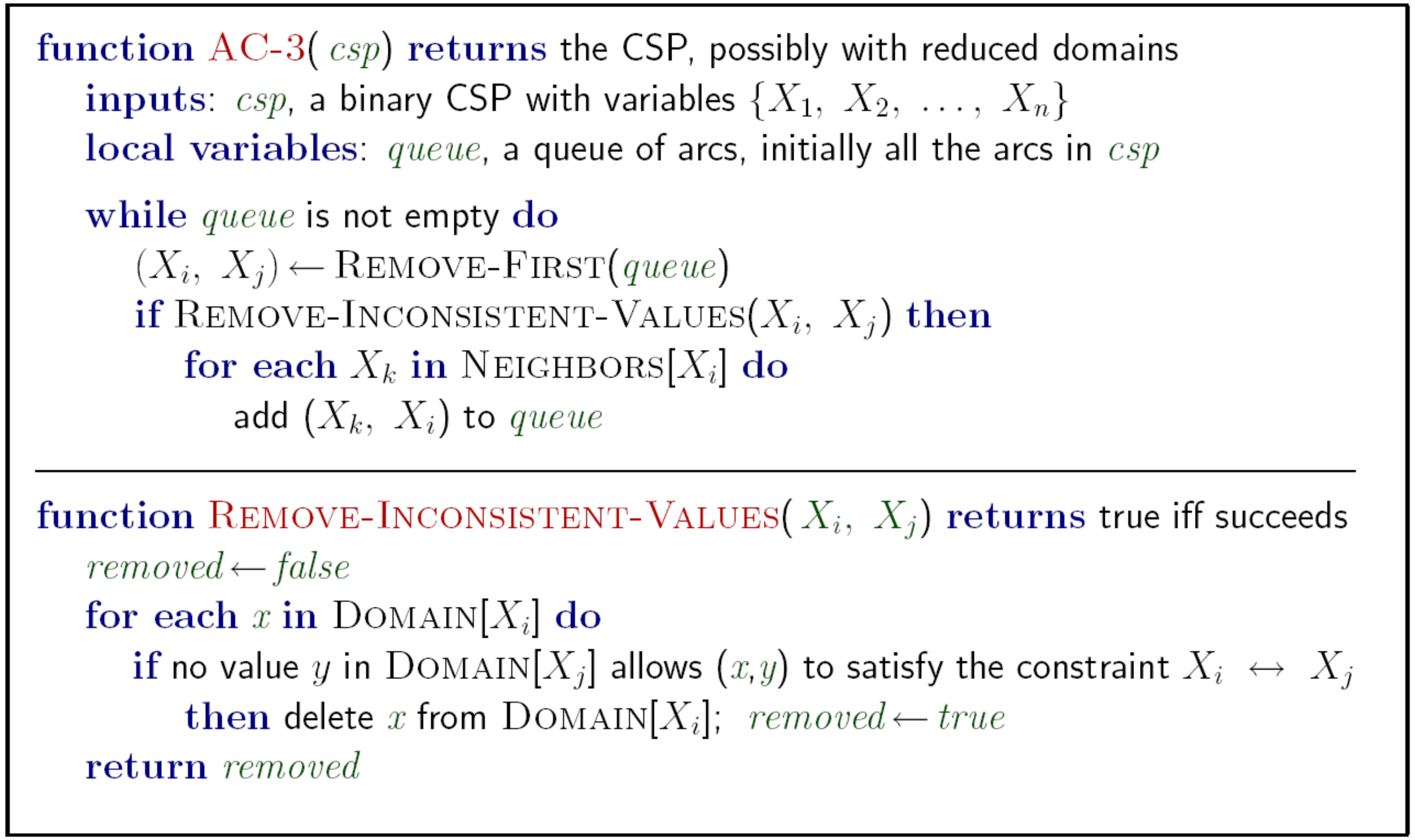
Arc consistency detects failure earlier than forward checking

Can be run as a preprocessor or after each assignment

What's the downside of enforcing arc consistency?

Remember: Delete from the tail!

Enforcing Arc Consistency in a CSP



```
function AC-3( csp) returns the CSP, possibly with reduced domains
    inputs: csp, a binary CSP with variables $\{X_1,\ X_2,\ \ldots,\ X_n\}$
    local variables: queue, a queue of arcs, initially all the arcs in csp

    while queue is not empty do
        $(X_i,\ X_j) \leftarrow$ REMOVE-FIRST(queue)
        if REMOVE-INCONSISTENT-VALUES($X_i,\ X_j$) then
            for each $X_k$ in NEIGHBORS[$X_i$] do
                add $(X_k,\ X_i)$ to queue
```

```
function REMOVE-INCONSISTENT-VALUES( $X_i,\ X_j$) returns true iff succeeds
    removed $\leftarrow$ false
    for each $x$ in DOMAIN[$X_i$] do
        if no value $y$ in DOMAIN[$X_j$] allows $(x,y)$ to satisfy the constraint $X_i \leftrightarrow X_j$
            then delete $x$ from DOMAIN[$X_i$];  removed $\leftarrow$ true
    return removed
```

Runtime: $O(n^2d^3)$, can be reduced to $O(n^2d^2)$

… but detecting all possible future problems is NP-hard – why?

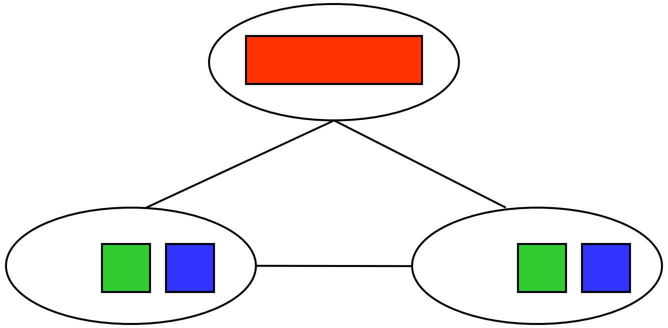
[Demo: CSP applet (made available by aispace.org) -- n-queens]

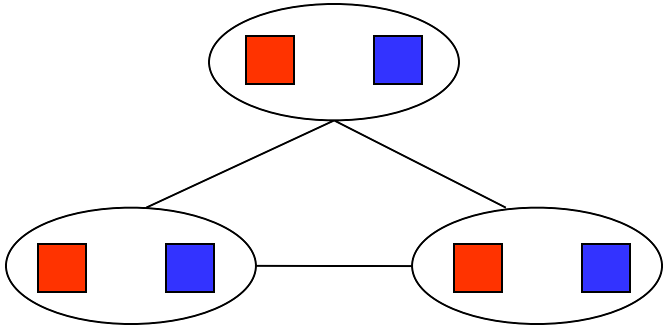
Limitations of Arc Consistency

After enforcing arc consistency:

- Can have one solution left
- Can have multiple solutions left
- Can have no solutions left (and not know it)

Arc consistency still runs inside a backtracking search!

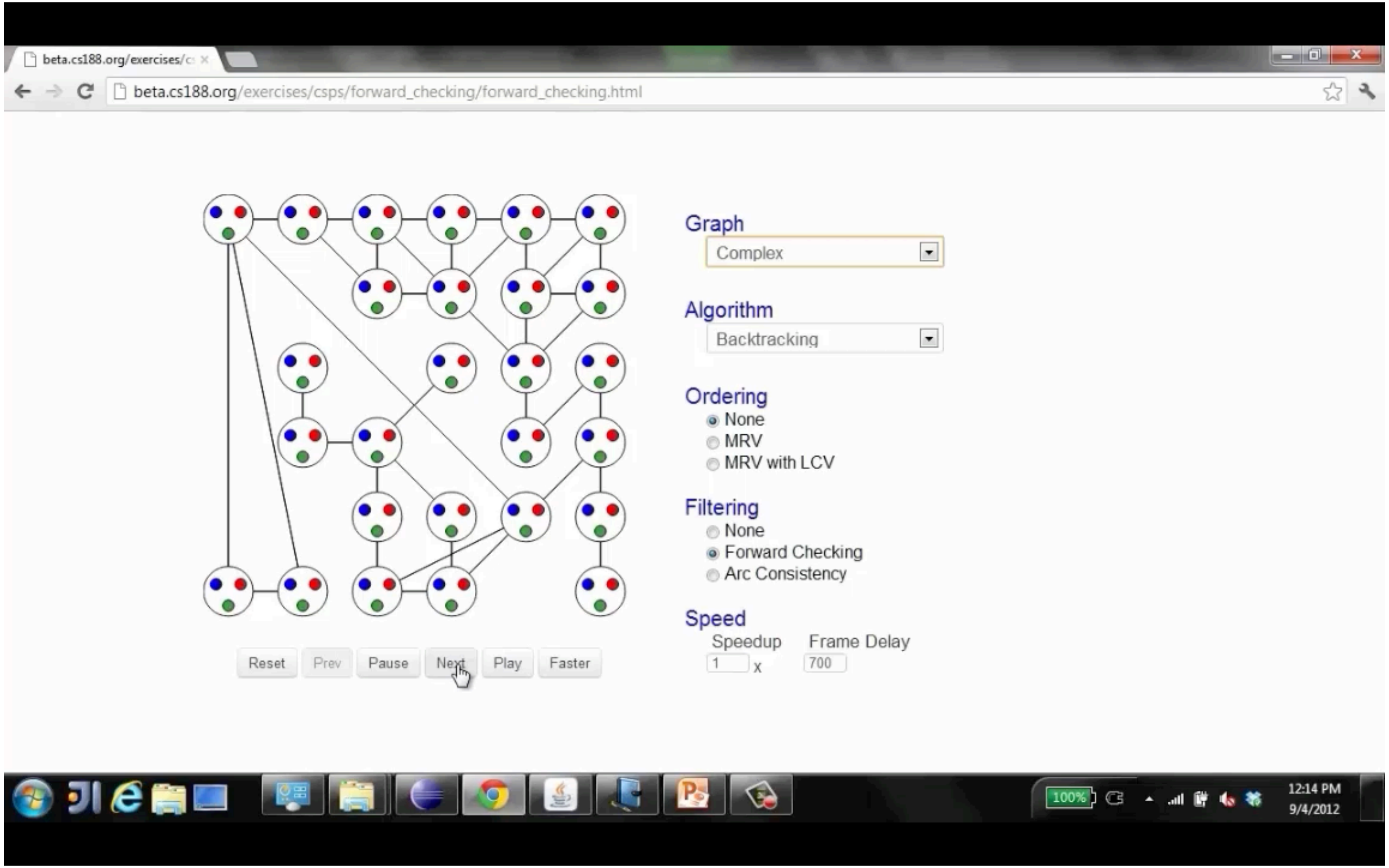




What went wrong here?

[Demo: coloring -- forward checking]
[Demo: coloring -- arc consistency]

Video of Demo Coloring – Backtracking with Forward Checking – Complex Graph



beta.cs188.org/exercises/csps/forward_checking/forward_checking.html

Graph: Complex

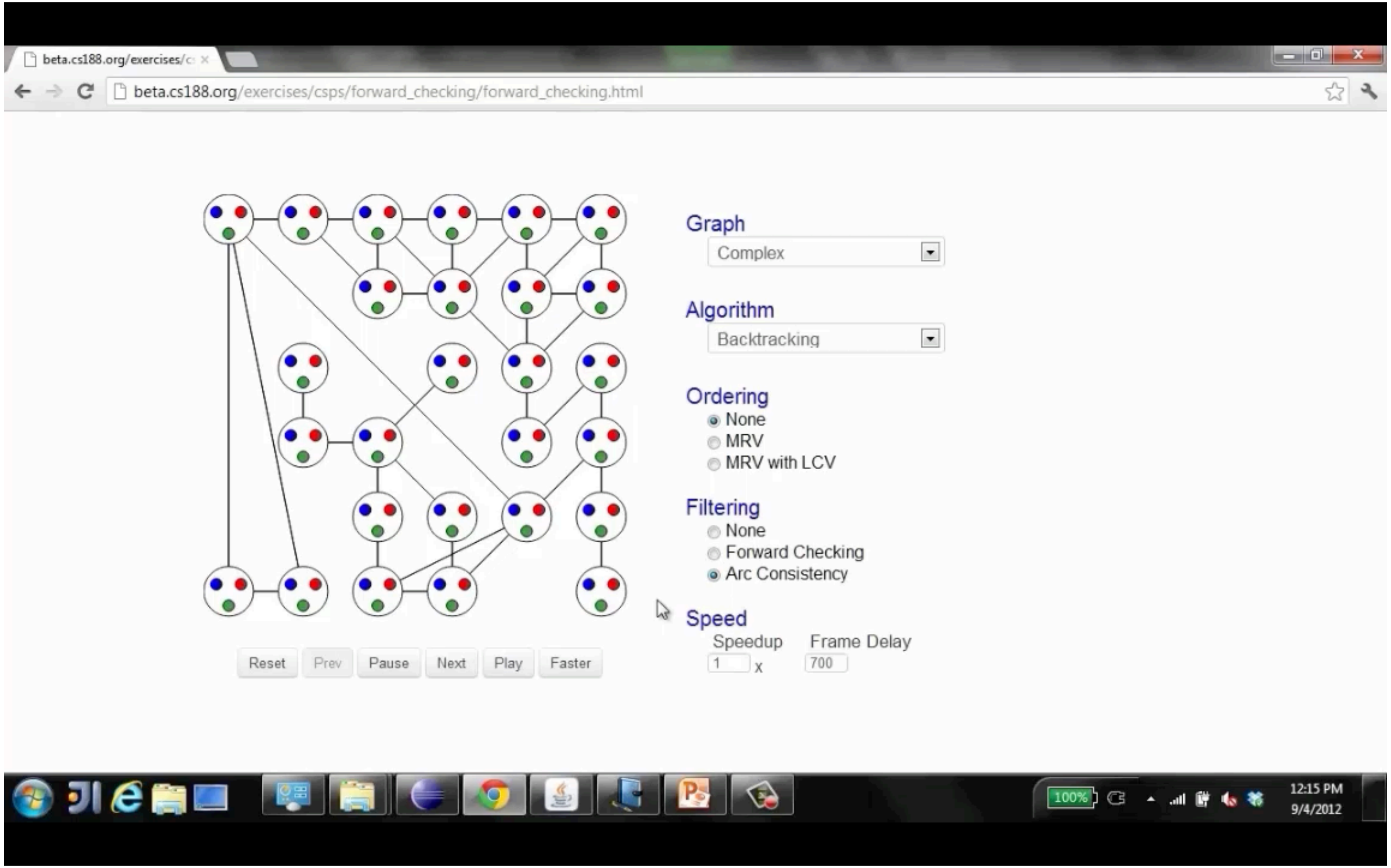
Algorithm: Backtracking

Ordering
● None
○ MRV
○ MRV with LCV

Filtering
○ None
● Forward Checking
○ Arc Consistency

Speed
Speedup: 1 x
Frame Delay: 700

Video of Demo Coloring – Backtracking with Arc Consistency – Complex Graph



Graph: Complex

Algorithm: Backtracking

Ordering
- ● None
- ○ MRV
- ○ MRV with LCV

Filtering
- ○ None
- ○ Forward Checking
- ● Arc Consistency

Speed

Speedup: 1 x

Frame Delay: 700

Ordering

Ordering: Minimum Remaining Values

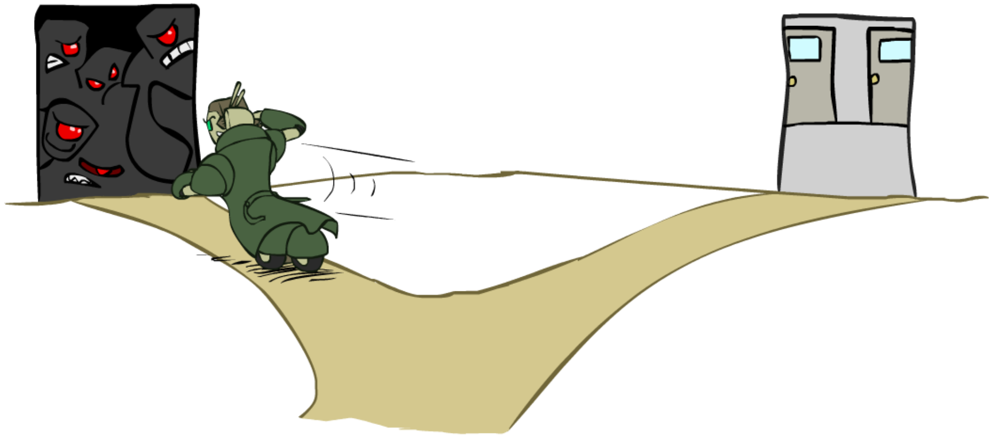
Variable Ordering: Minimum remaining values (MRV):

Choose the variable with the fewest legal left values in its domain

Why min rather than max?

Also called "most constrained variable"

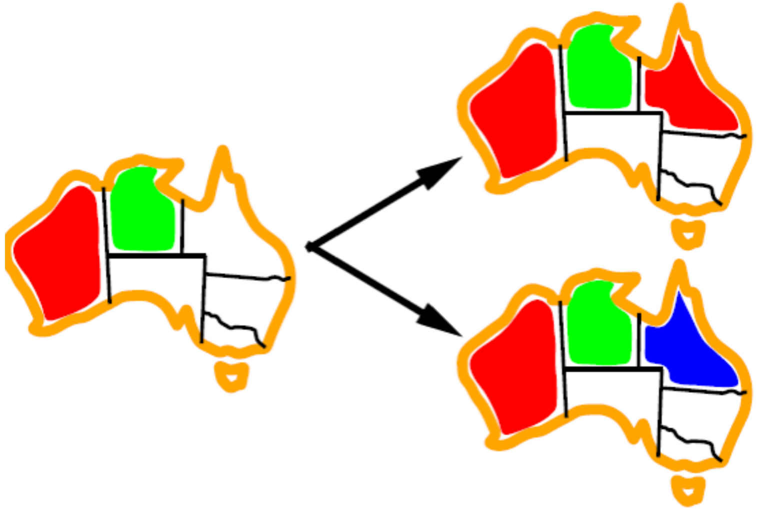
"Fail-fast" ordering



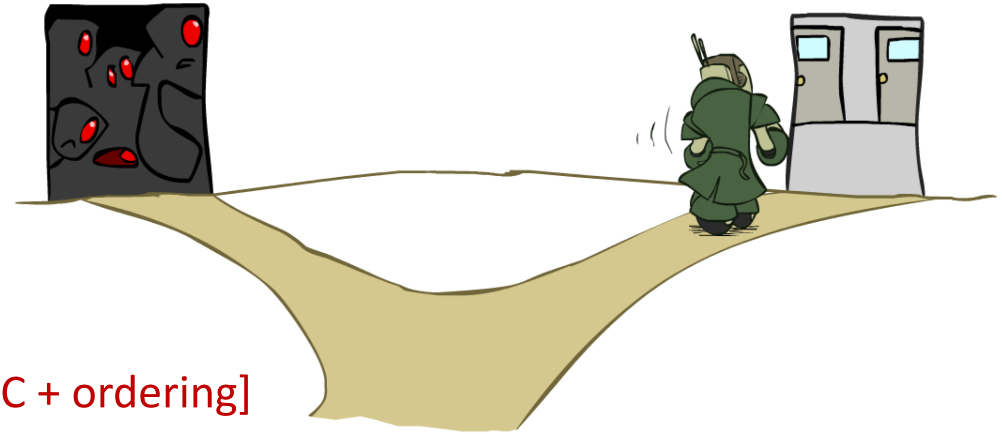
Ordering: Least Constraining Value

Value Ordering: Least Constraining Value

- Given a choice of variable, choose the *least constraining value*
- I.e., the one that rules out the fewest values in the remaining variables
- Note that it may take some computation to determine this! (E.g., rerunning filtering)



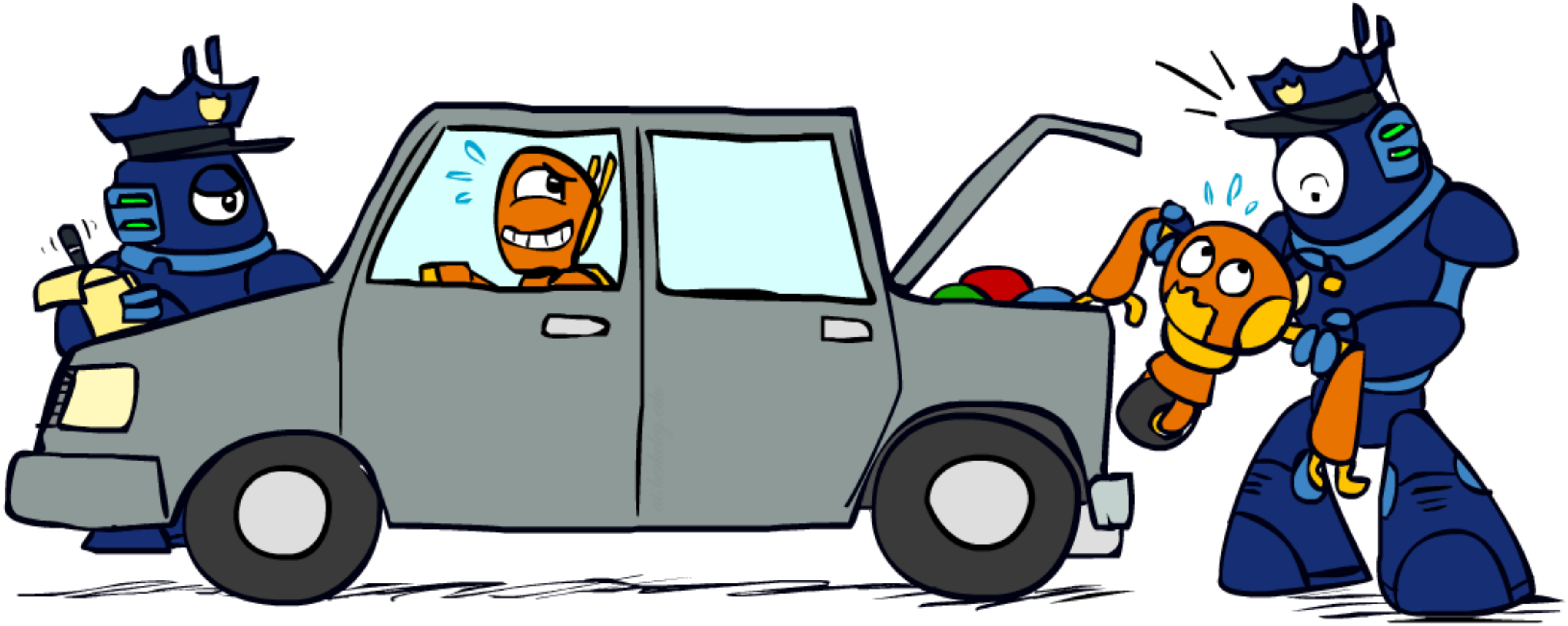
Why least rather than most?



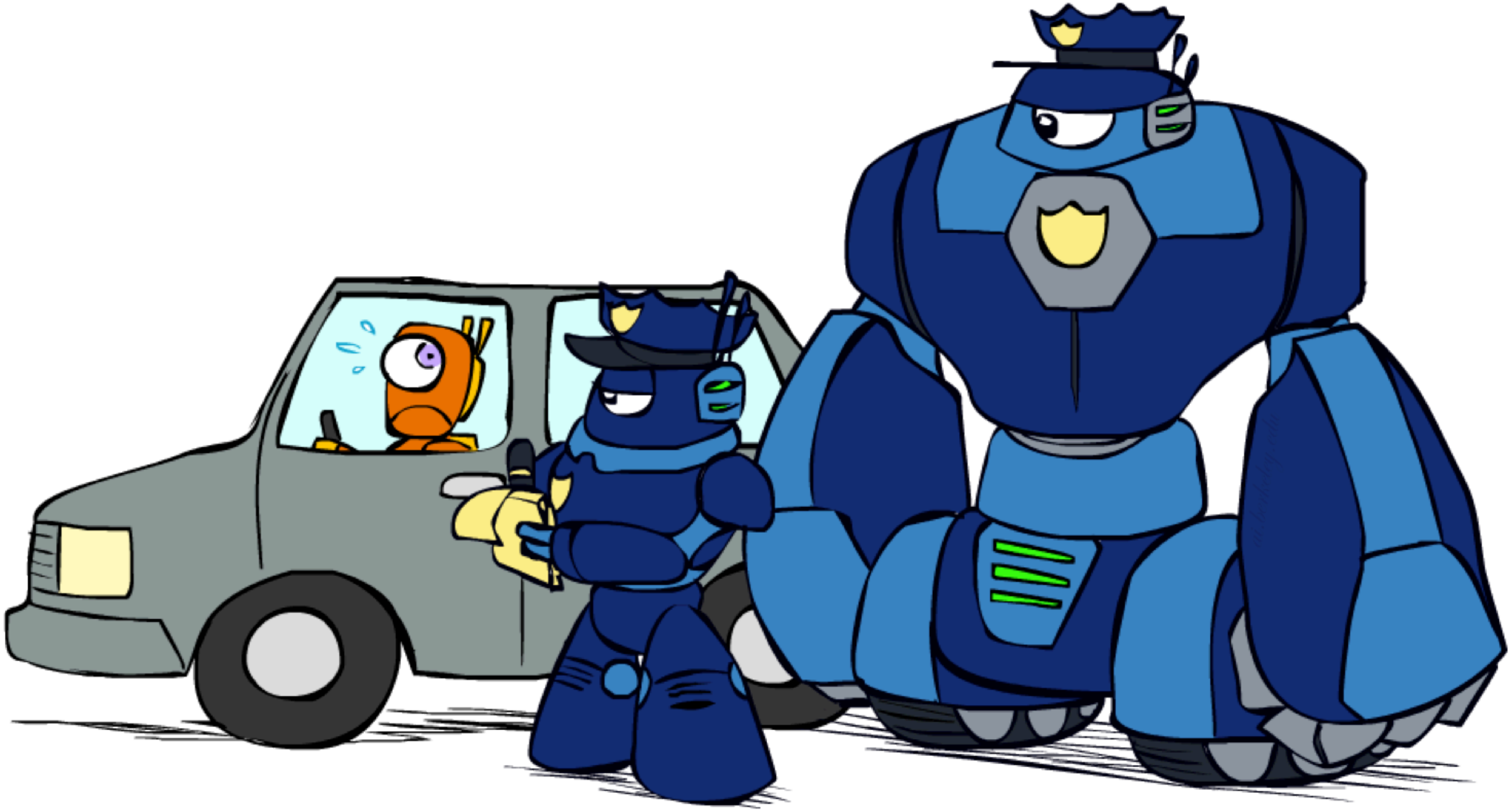
Combining these ordering ideas makes 1000 queens feasible

[Demo: coloring – backtracking + AC + ordering]

Arc Consistency and Beyond



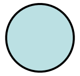
K-Consistency



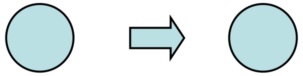
K-Consistency

Increasing degrees of consistency

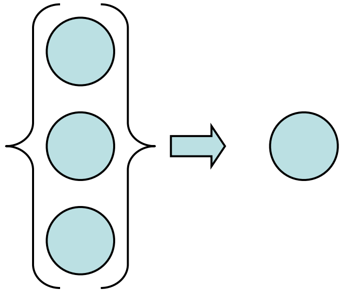
1-Consistency (Node Consistency): Each single node's domain has a value which meets that node's unary constraints



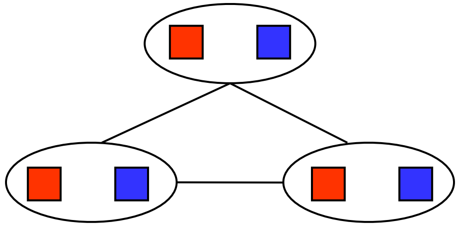
2-Consistency (Arc Consistency): For each pair of nodes, any consistent assignment to one can be extended to the other



K-Consistency: For each k nodes, any consistent assignment to k-1 can be extended to the kth node.



Higher k more expensive to compute



(You need to know the k=2 case: arc consistency)

Strong K-Consistency

Strong k-consistency: also k-1, k-2, … 1 consistent

Claim: strong n-consistency means we can solve without backtracking!

Why?

Choose any assignment to any variable

Choose a new variable

By 2-consistency, there is a choice consistent with the first

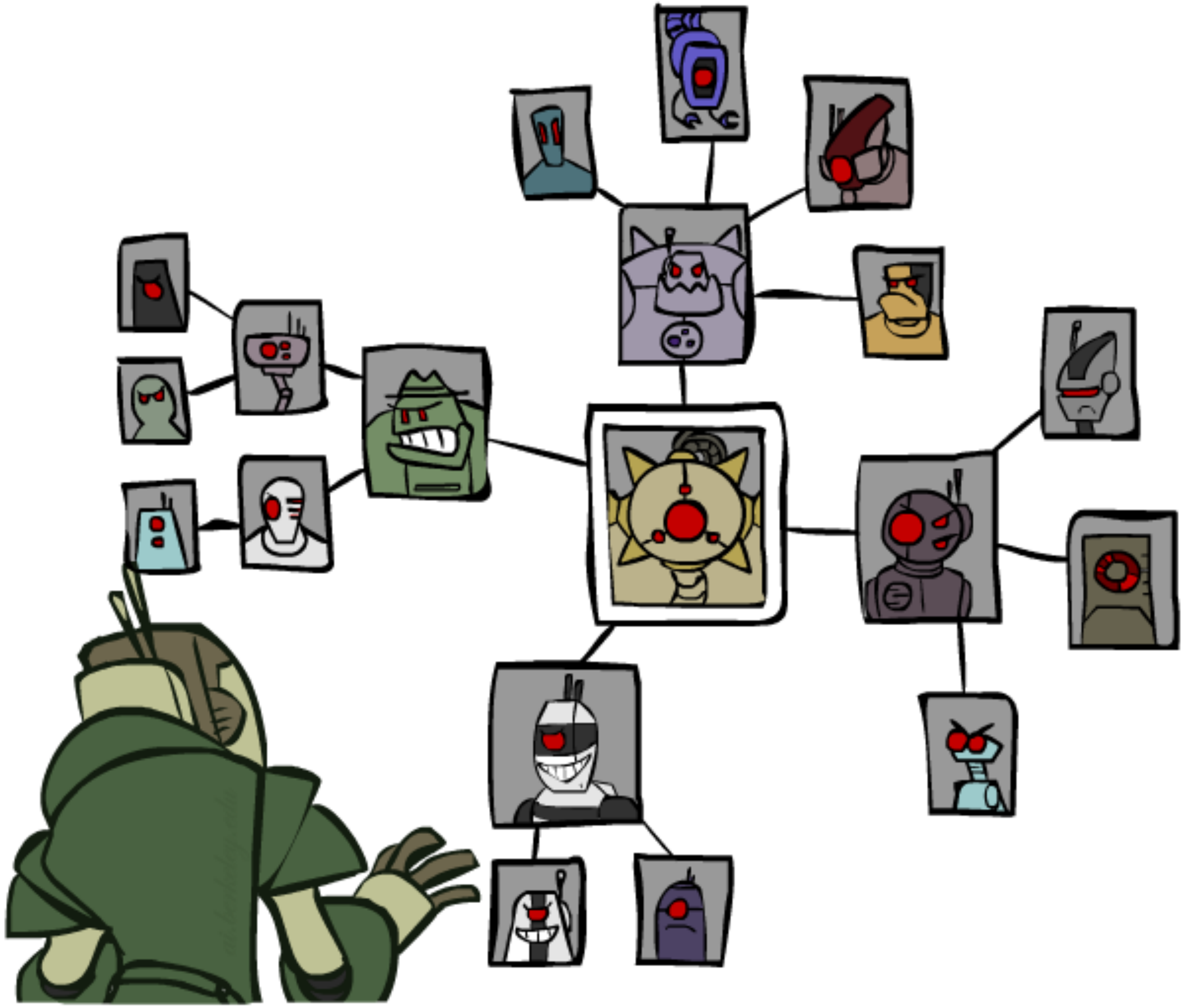
Choose a new variable

By 3-consistency, there is a choice consistent with the first 2

…

Lots of middle ground between arc consistency and n-consistency! (e.g. k=3, called path consistency)

Structure



Problem Structure

Extreme case: independent subproblems

Example: Tasmania and mainland do not interact

Independent subproblems are identifiable as connected components of constraint graph

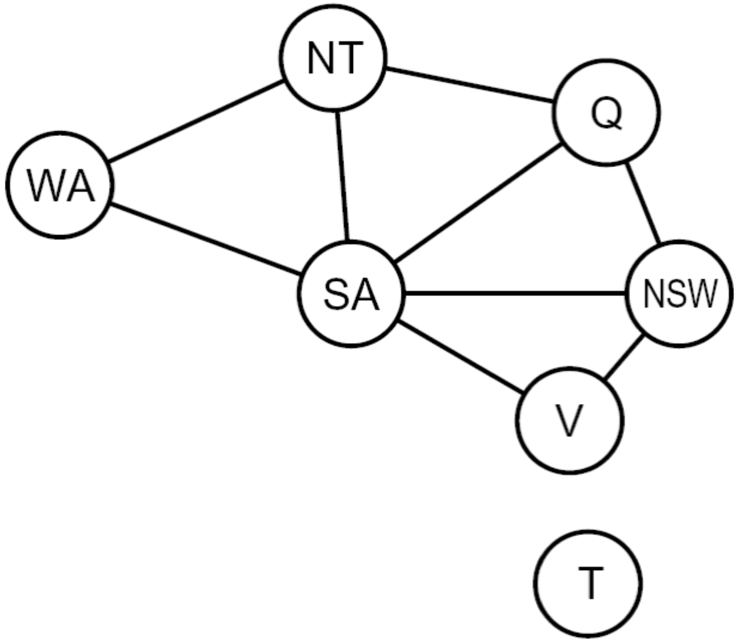
Suppose a graph of n variables can be broken into subproblems of only c variables:

Worst-case solution cost is $O((n/c)(d^c))$, linear in n

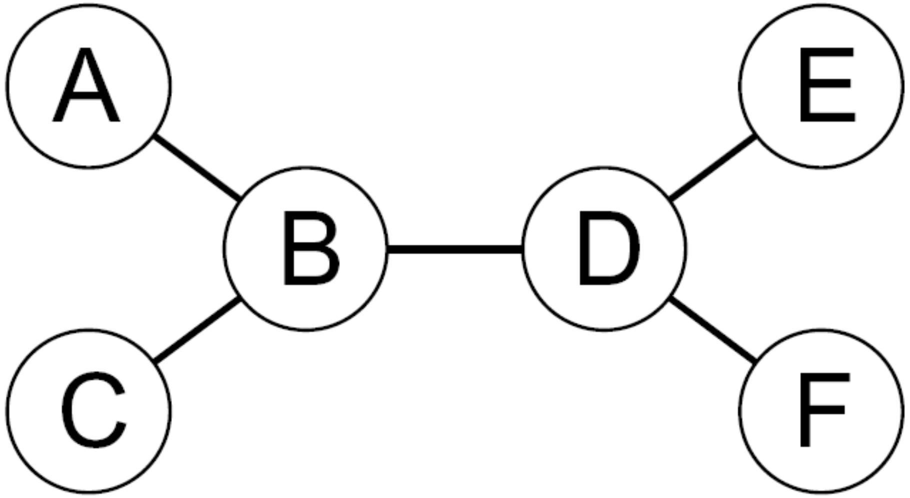
E.g., n = 80, d = 2, c =20

2^{80} = 4 billion years at 10 million nodes/sec

$(4)(2^{20})$ = 0.4 seconds at 10 million nodes/sec



Tree-Structured CSPs



Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\ d^2)$ time

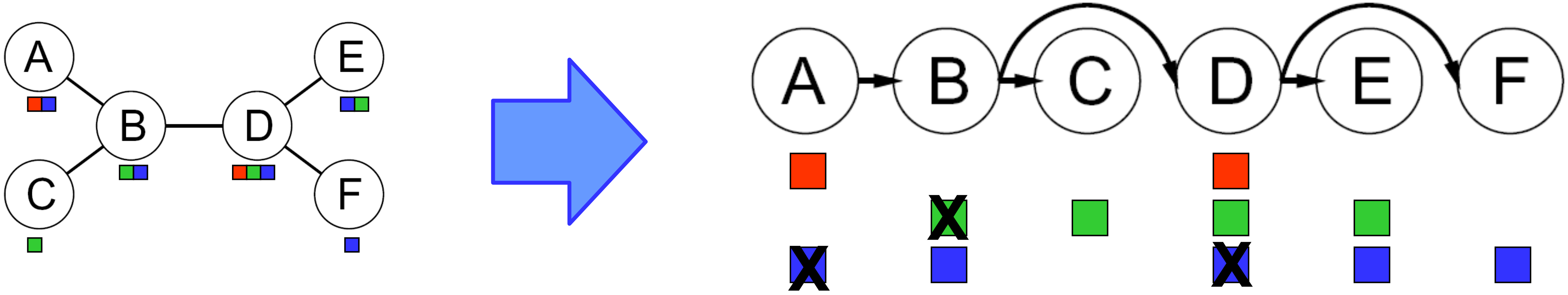
Compare to general CSPs, where worst-case time is $O(d^n)$

This property also applies to probabilistic reasoning (later): an example of the relation between syntactic restrictions and the complexity of reasoning

Tree-Structured CSPs

Algorithm for tree-structured CSPs:

Order: Choose a root variable, order variables so that parents precede children



Remove backward: For i = n : 2, apply RemoveInconsistent(Parent(X_i),X_i)

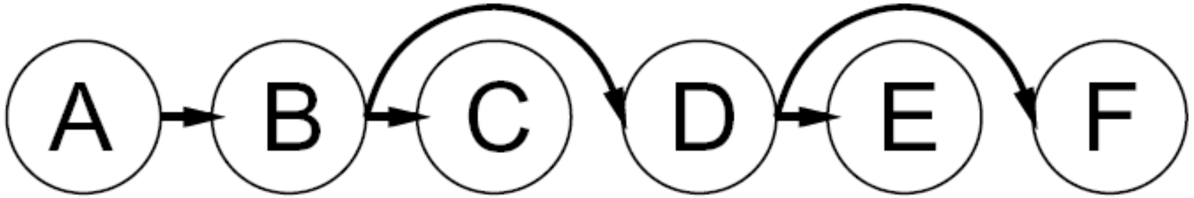
Assign forward: For i = 1 : n, assign X_i consistently with Parent(X_i)

Runtime: $O(n\ d^2)$ (why?)

Tree-Structured CSPs

Claim 1: After backward pass, all root-to-leaf arcs are consistent

Proof: Each X→Y was made consistent at one point and Y's domain could not have been reduced thereafter (because Y's children were processed before Y)



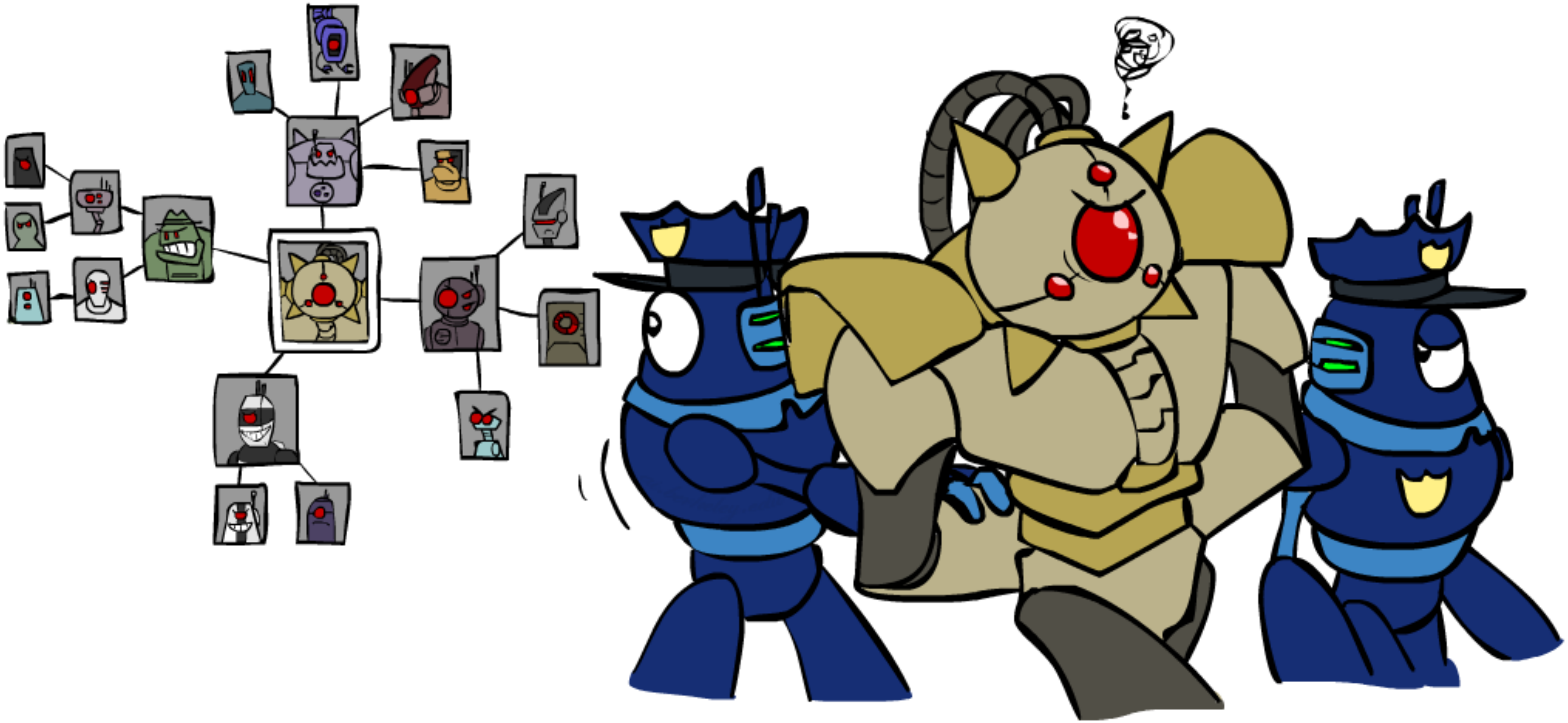
Claim 2: If root-to-leaf arcs are consistent, forward assignment will not backtrack

Proof: Induction on position

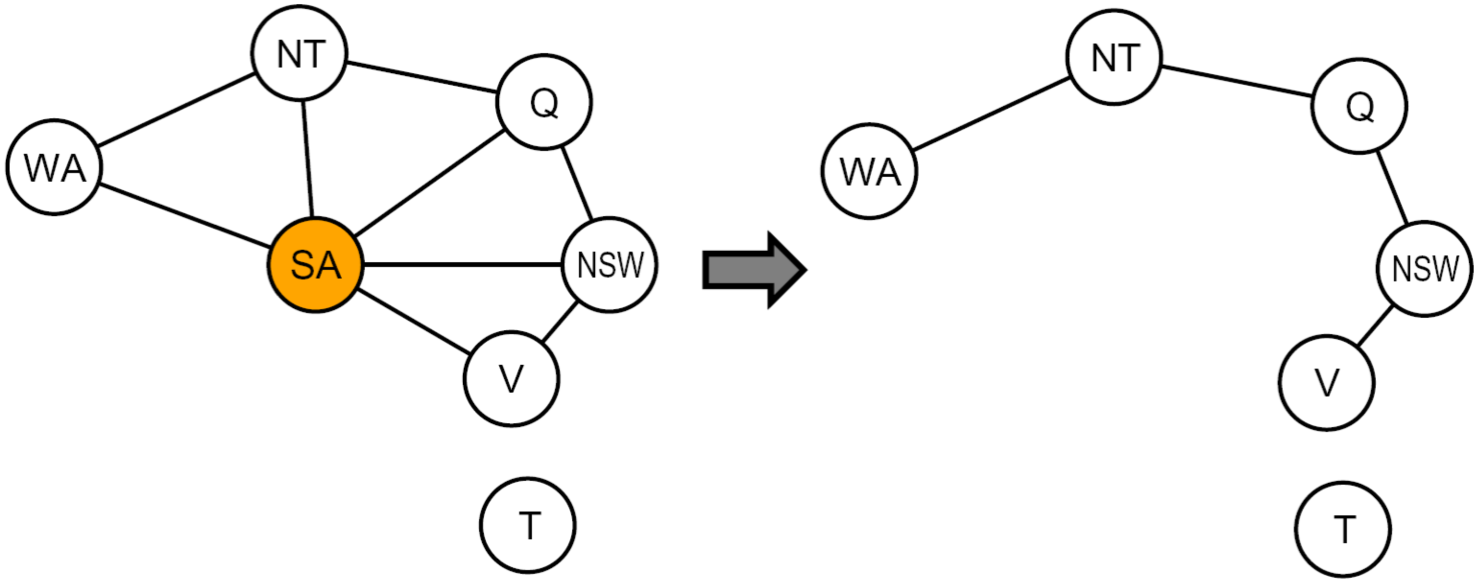
Why doesn't this algorithm work with cycles in the constraint graph?

Note: we'll see this basic idea again with Bayes' nets

Improving Structure



Nearly Tree-Structured CSPs



Conditioning: instantiate a variable, prune its neighbors' domains

Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree

Cutset size c gives runtime $O((d^c) (n-c) d^2)$, very fast for small c

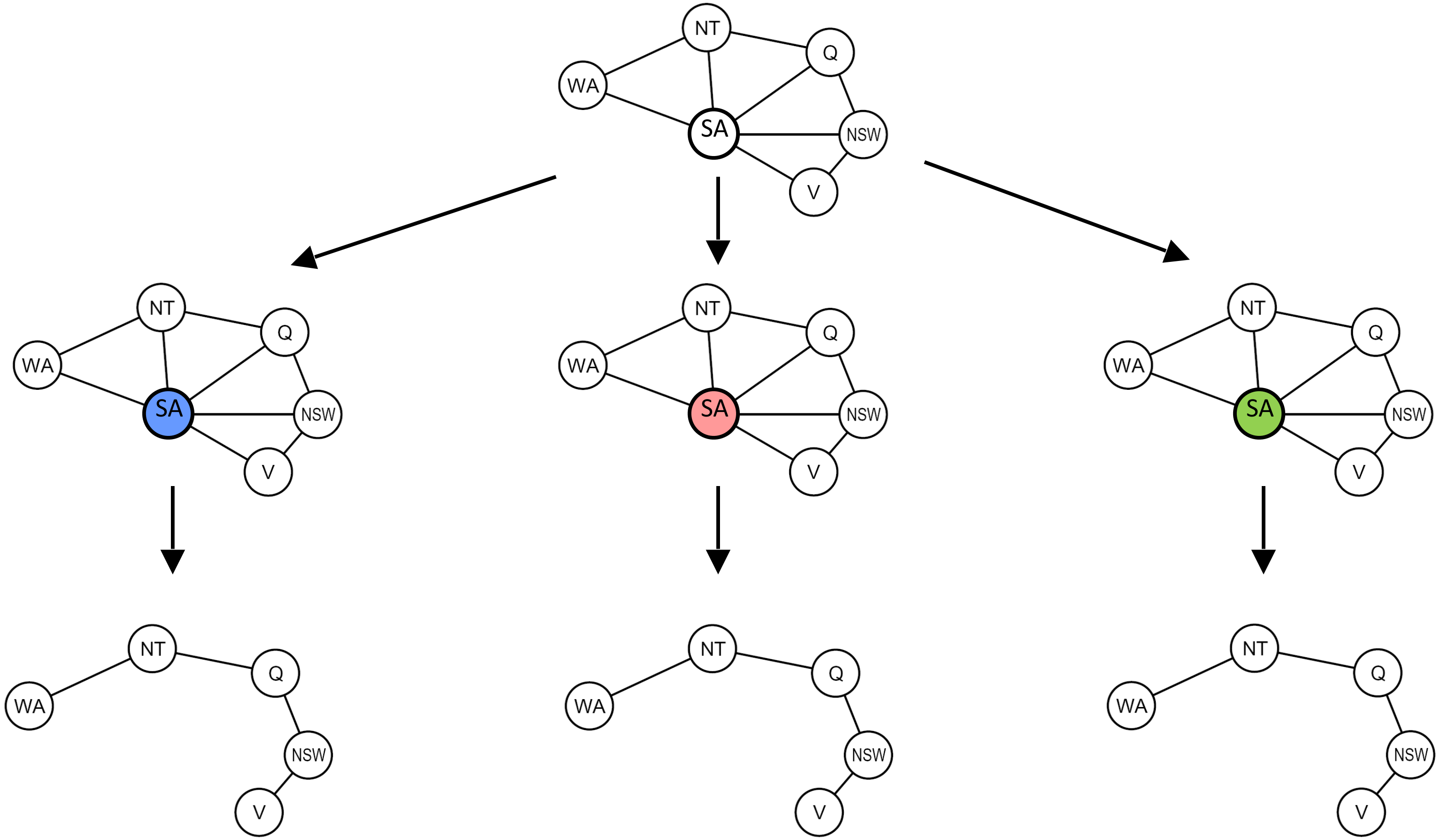
Cutset Conditioning

Choose a cutset

Instantiate the cutset (all possible ways)

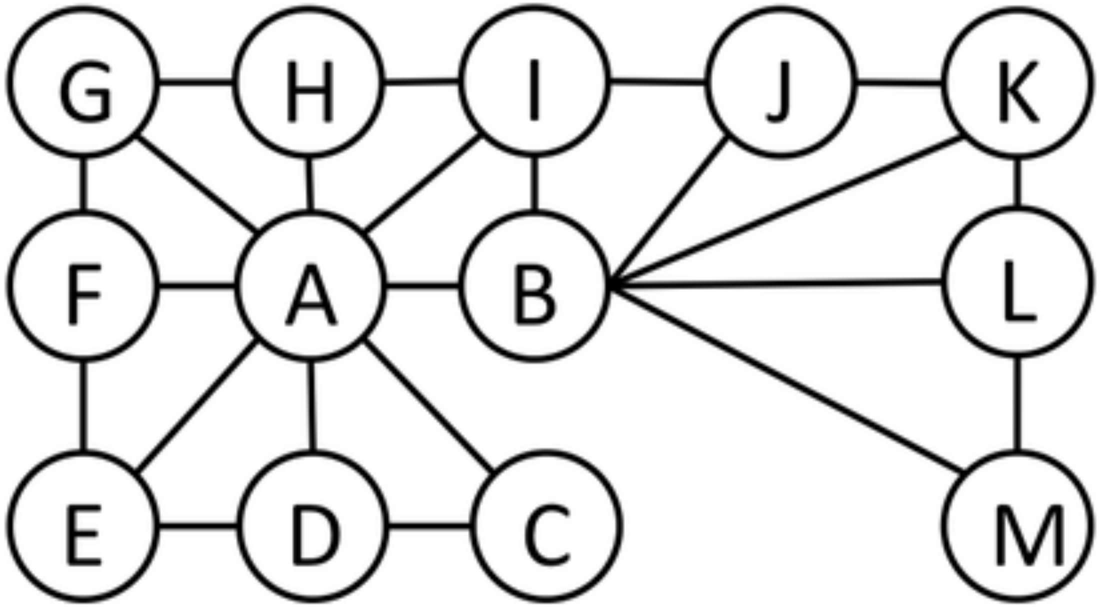
Compute residual CSP for each assignment

Solve the residual CSPs (tree structured)



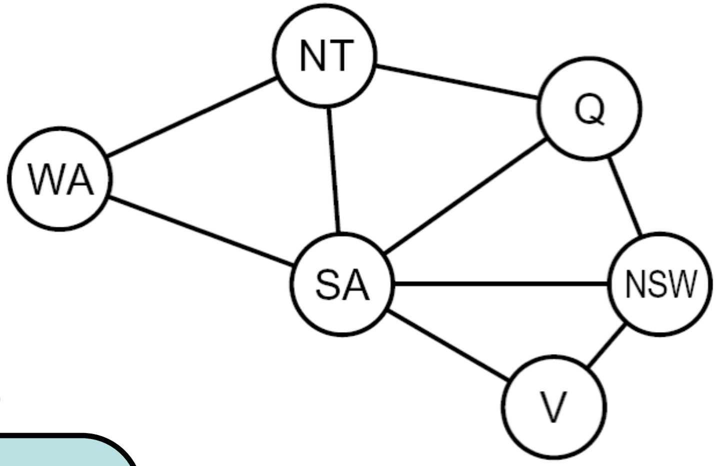
Cutset Quiz

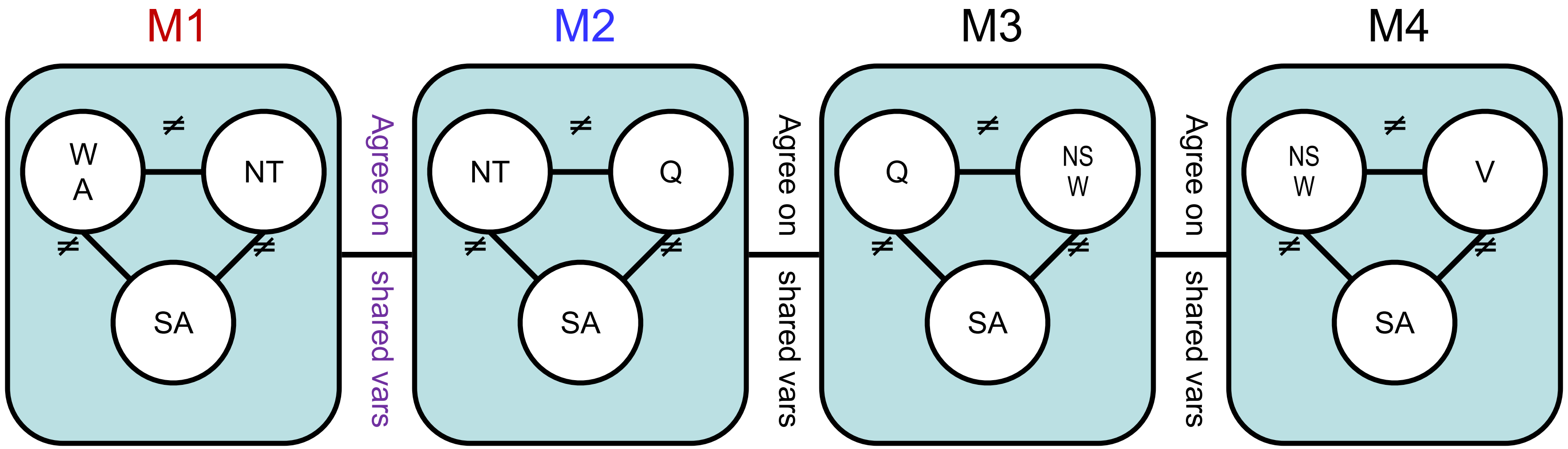
Find the smallest cutset for the graph below.



Tree Decomposition*

- Idea: create a tree-structured graph of mega-variables
- Each mega-variable encodes part of the original CSP
- Subproblems overlap to ensure consistent solutions



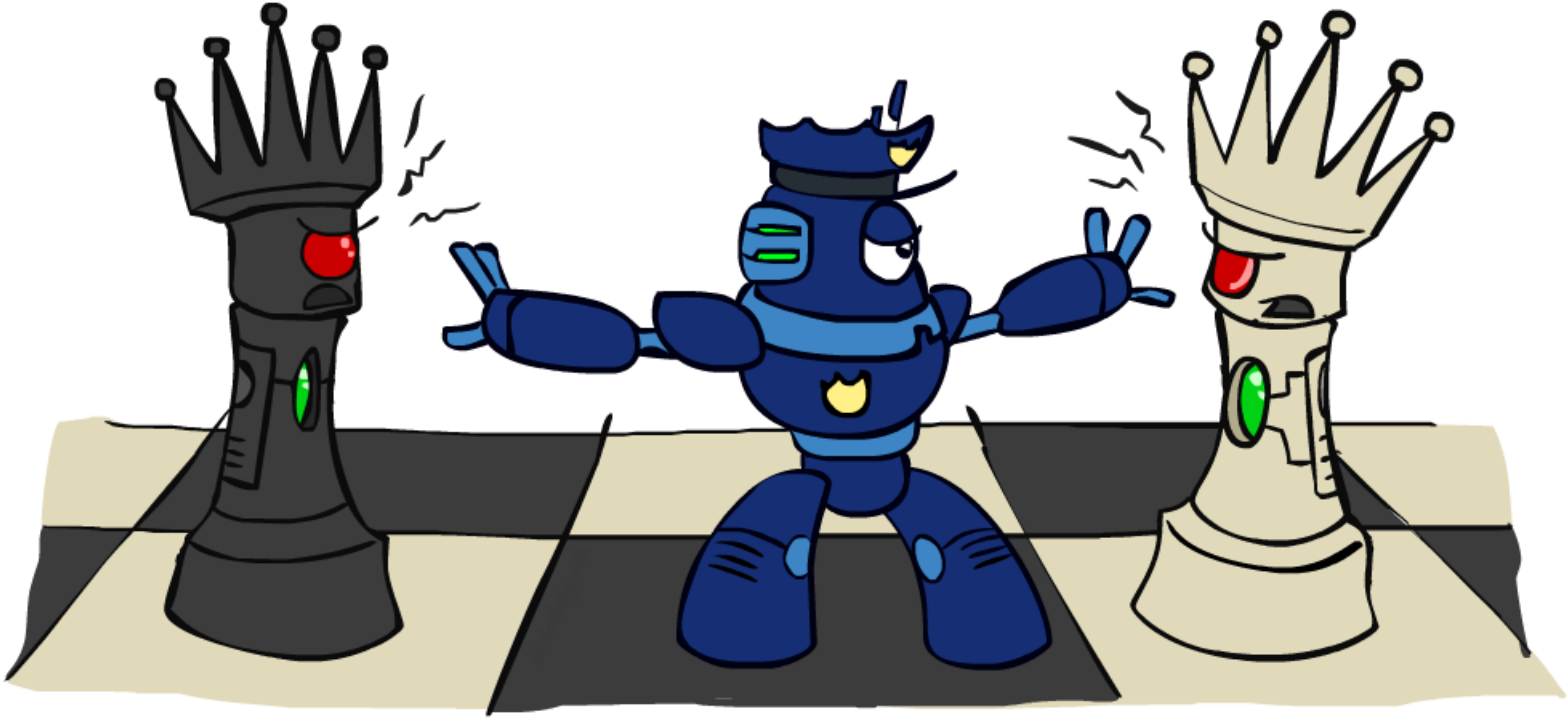


{(WA=r,SA=g,NT=b),
(WA=b,SA=r,NT=g),
…}

{(NT=r,SA=g,Q=b),
(NT=b,SA=g,Q=r),
…}

Agree: (M1,M2) ∈
{((WA=g,SA=g,NT=g), (NT=g,SA=g,Q=g)), …}

Iterative Improvement

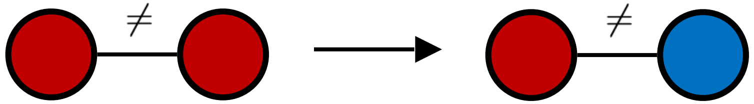


Iterative Algorithms for CSPs

Local search methods typically work with "complete" states, i.e., all variables assigned

To apply to CSPs:

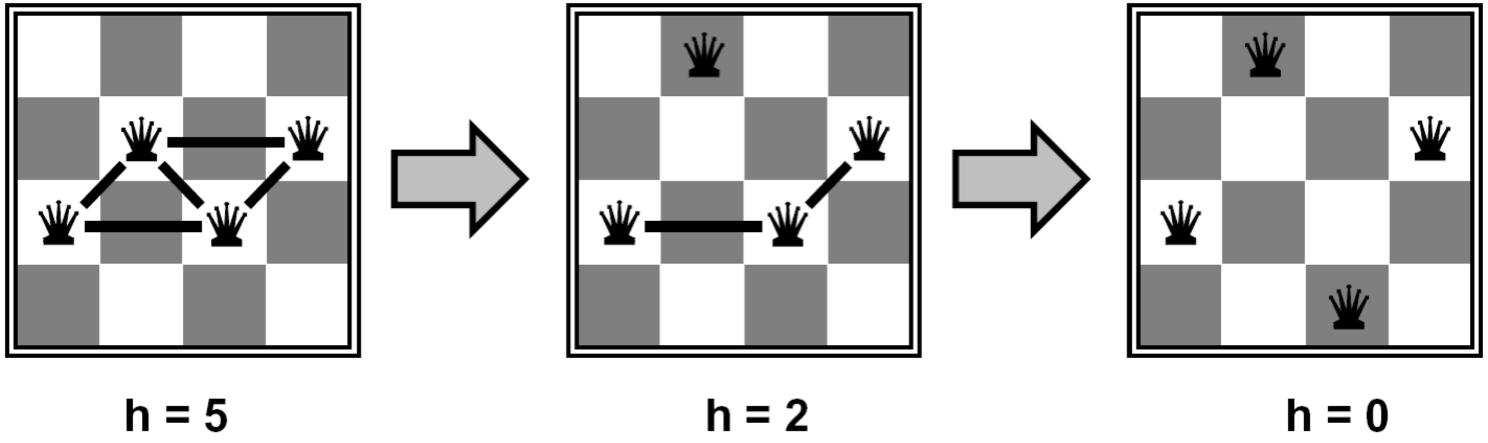
- Take an assignment with unsatisfied constraints
- Operators *reassign* variable values
- No fringe! Live on the edge.



Algorithm: While not solved,

- Variable selection: randomly select any conflicted variable
- Value selection: min-conflicts heuristic:
  - Choose a value that violates the fewest constraints
  - I.e., hill climb with h(n) = total number of violated constraints

Example: 4-Queens



States: 4 queens in 4 columns (4^4 = 256 states)

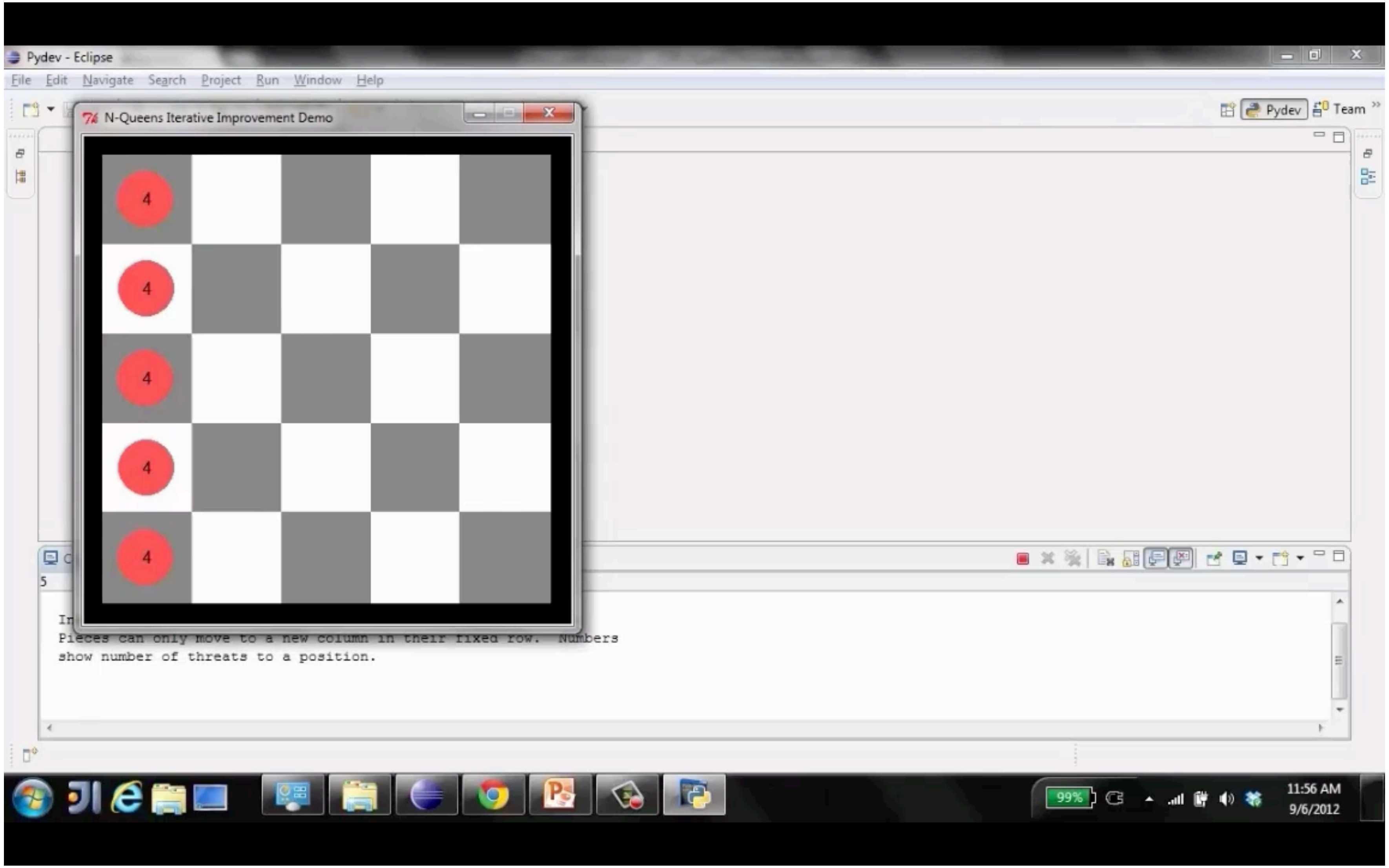
Operators: move queen in column

Goal test: no attacks

Evaluation: c(n) = number of attacks

[Demo: n-queens – iterative improvement (L5D1)]
[Demo: coloring – iterative improvement]

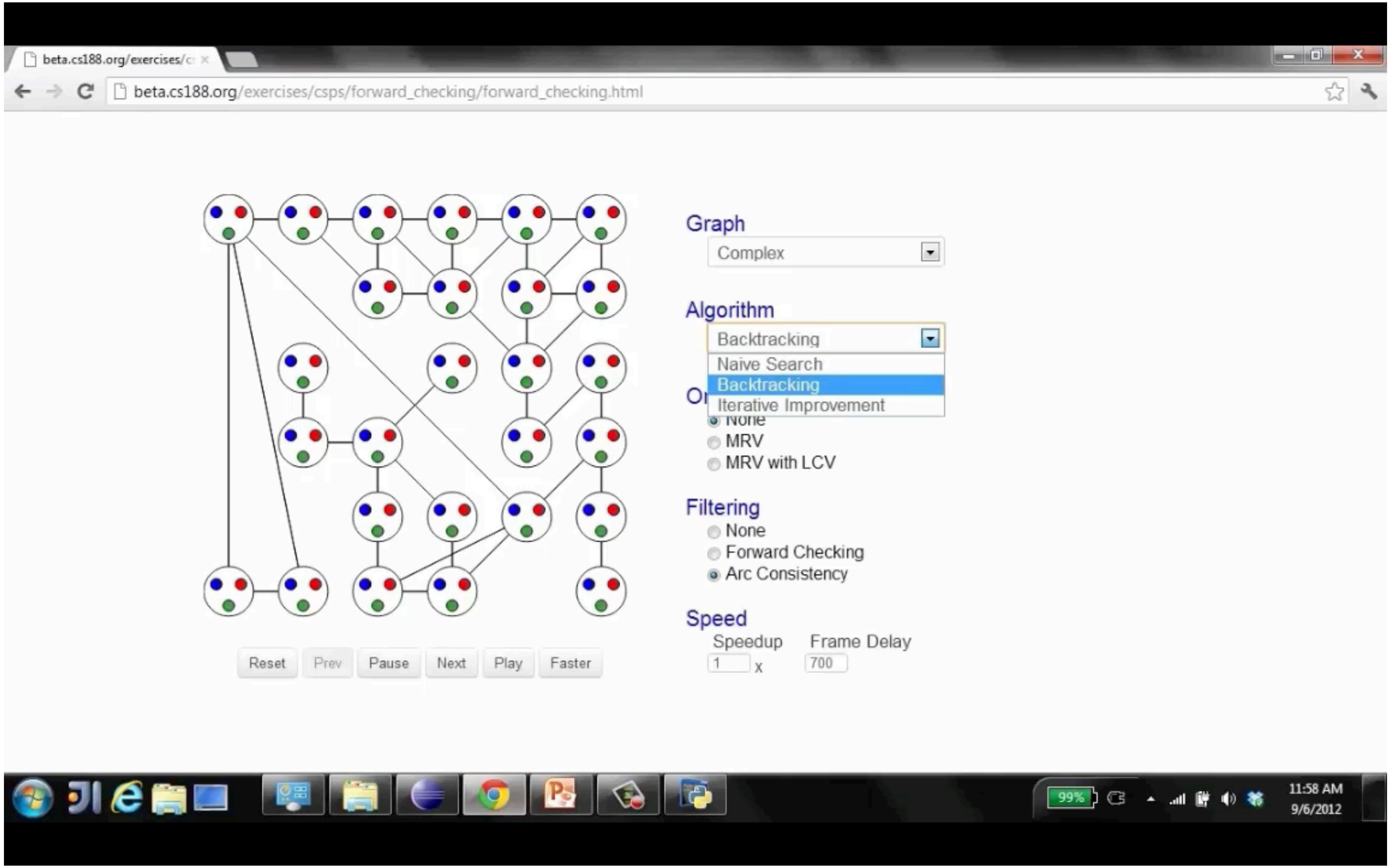
Video of Demo Iterative Improvement – n Queens



```
5

In
Pieces can only move to a new column in their fixed row.  Numbers
show number of threats to a position.
```

Video of Demo Iterative Improvement – Coloring



beta.cs188.org/exercises/csps/forward_checking/forward_checking.html

Graph: Complex

Algorithm: Backtracking

Or
● None
○ MRV
○ MRV with LCV

Filtering
○ None
○ Forward Checking
● Arc Consistency

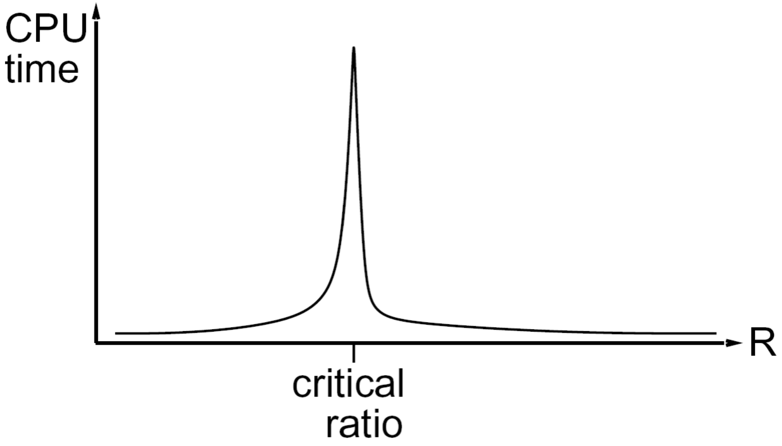
Speed
Speedup: 1 x
Frame Delay: 700

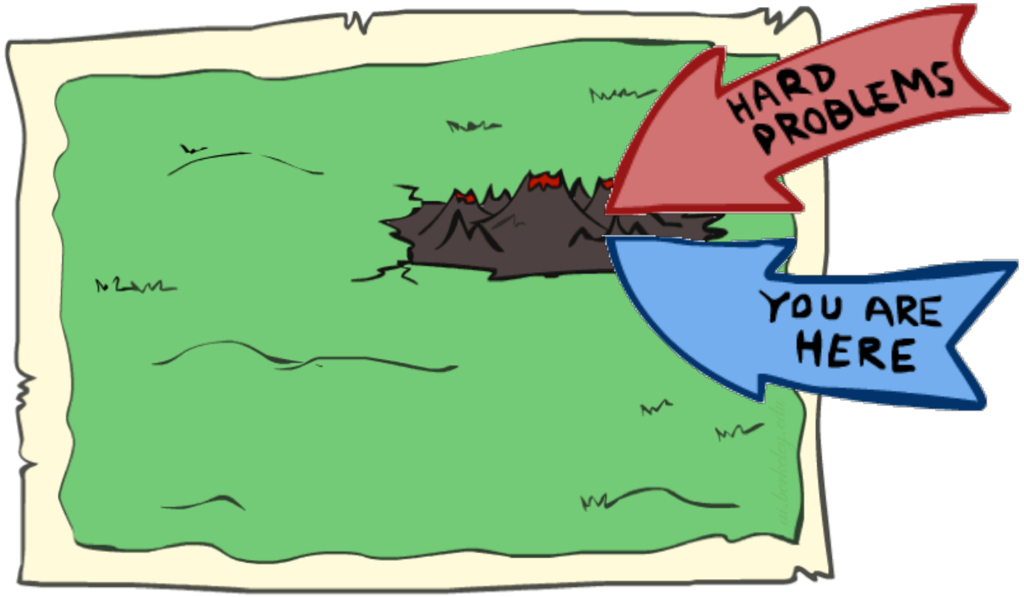
Performance of Min-Conflicts

Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000)!

The same appears to be true for any randomly-generated CSP *except* in a narrow range of the ratio

$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$





Summary: CSPs

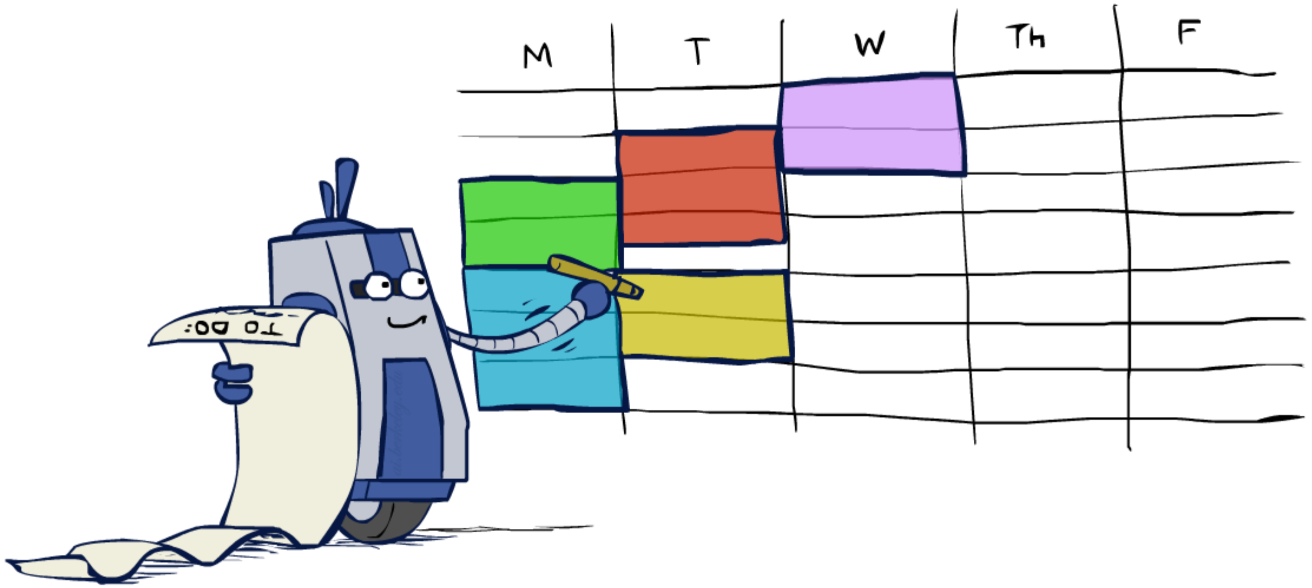
CSPs are a special kind of search problem:

- States are partial assignments
- Goal test defined by constraints

Basic solution: backtracking search

Speed-ups:

- Ordering
- Filtering
- Structure



Iterative min-conflicts is often effective in practice

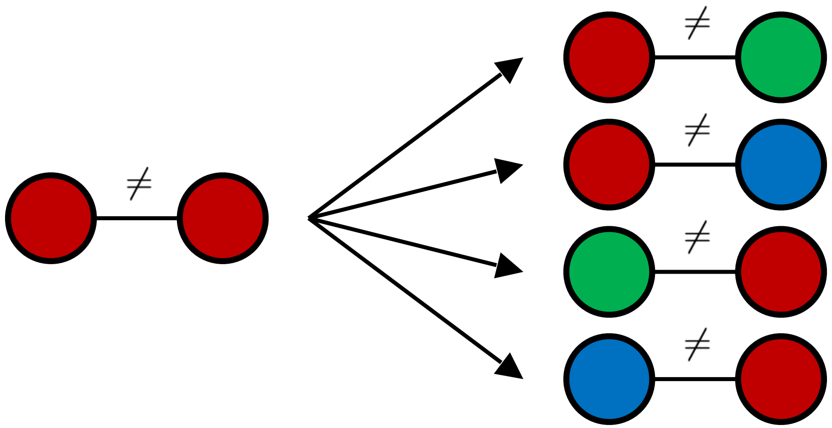
Local Search

Local Search

Tree search keeps unexplored alternatives on the fringe (ensures completeness)

Local search: improve a single option until you can't make it better (no fringe!)

New successor function: local changes



Generally much faster and more memory efficient (but incomplete and suboptimal)

Hill Climbing

Simple, general idea:

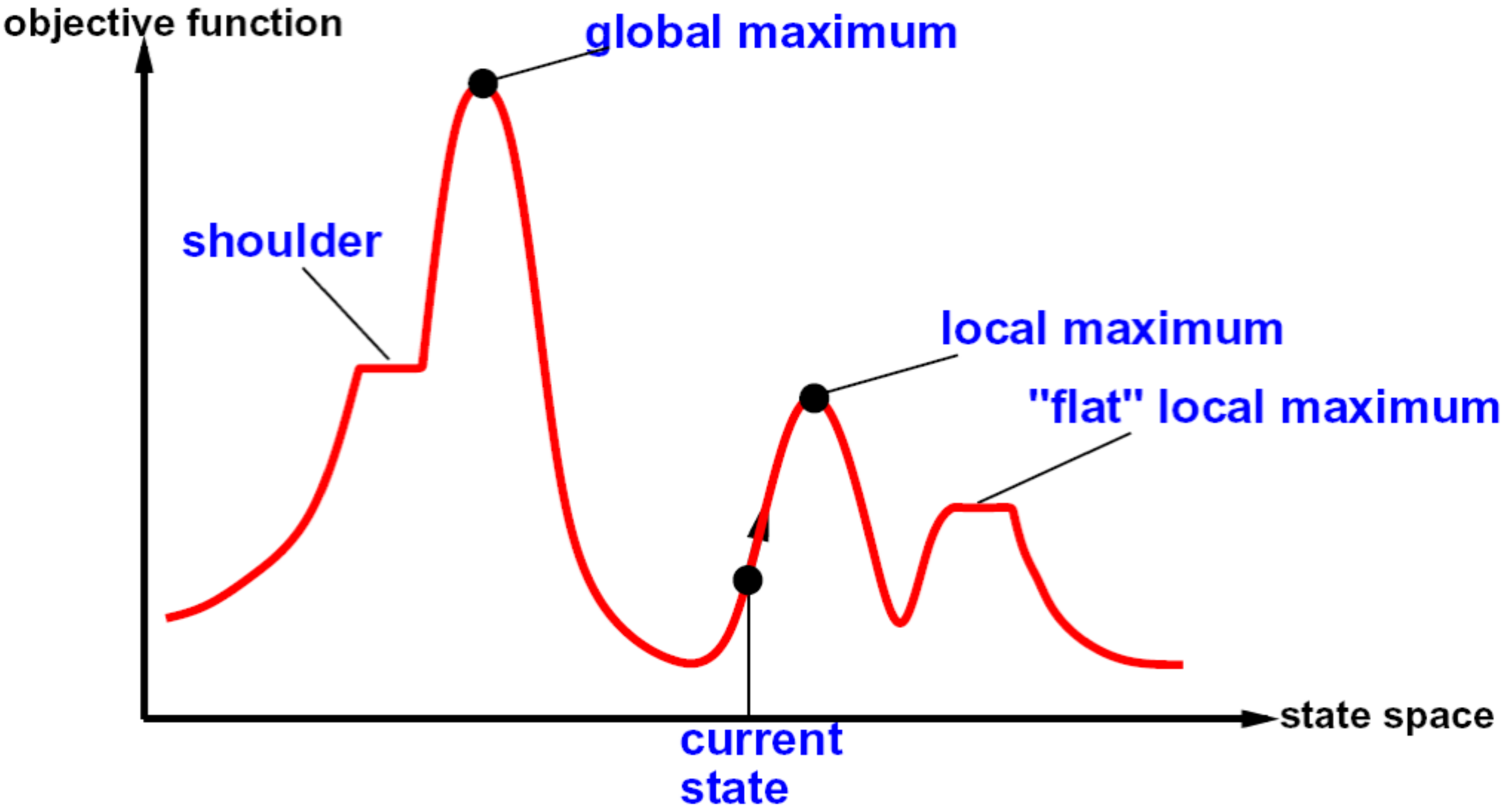
- Start wherever
- Repeat: move to the best neighboring state
- If no neighbors better than current, quit

What's bad about this approach?

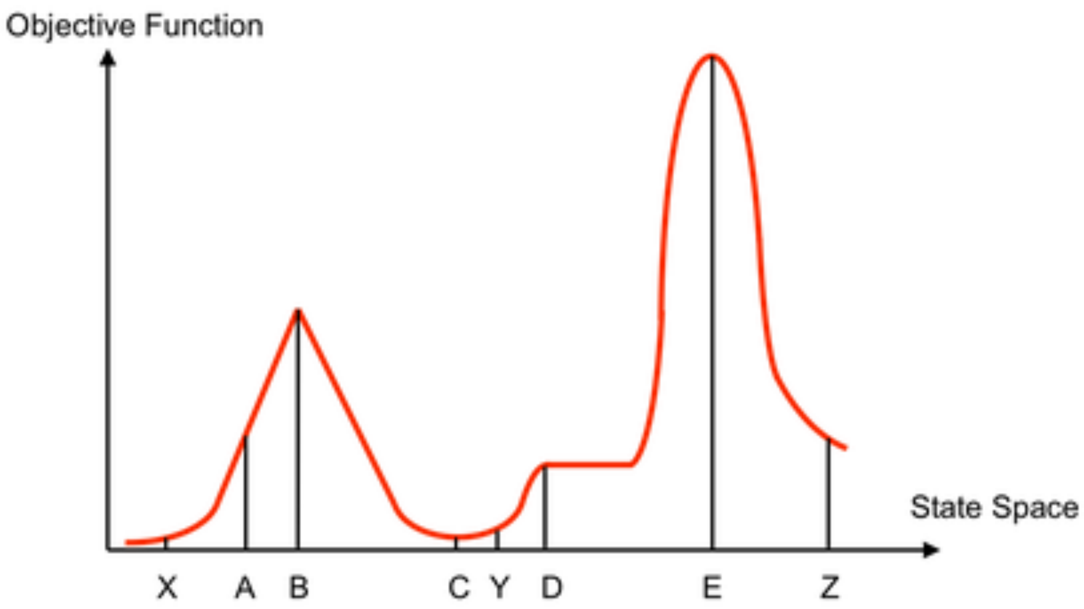
- Complete?
- Optimal?

What's good about it?

Hill Climbing Diagram



Hill Climbing Quiz



Starting from X, where do you end up ?

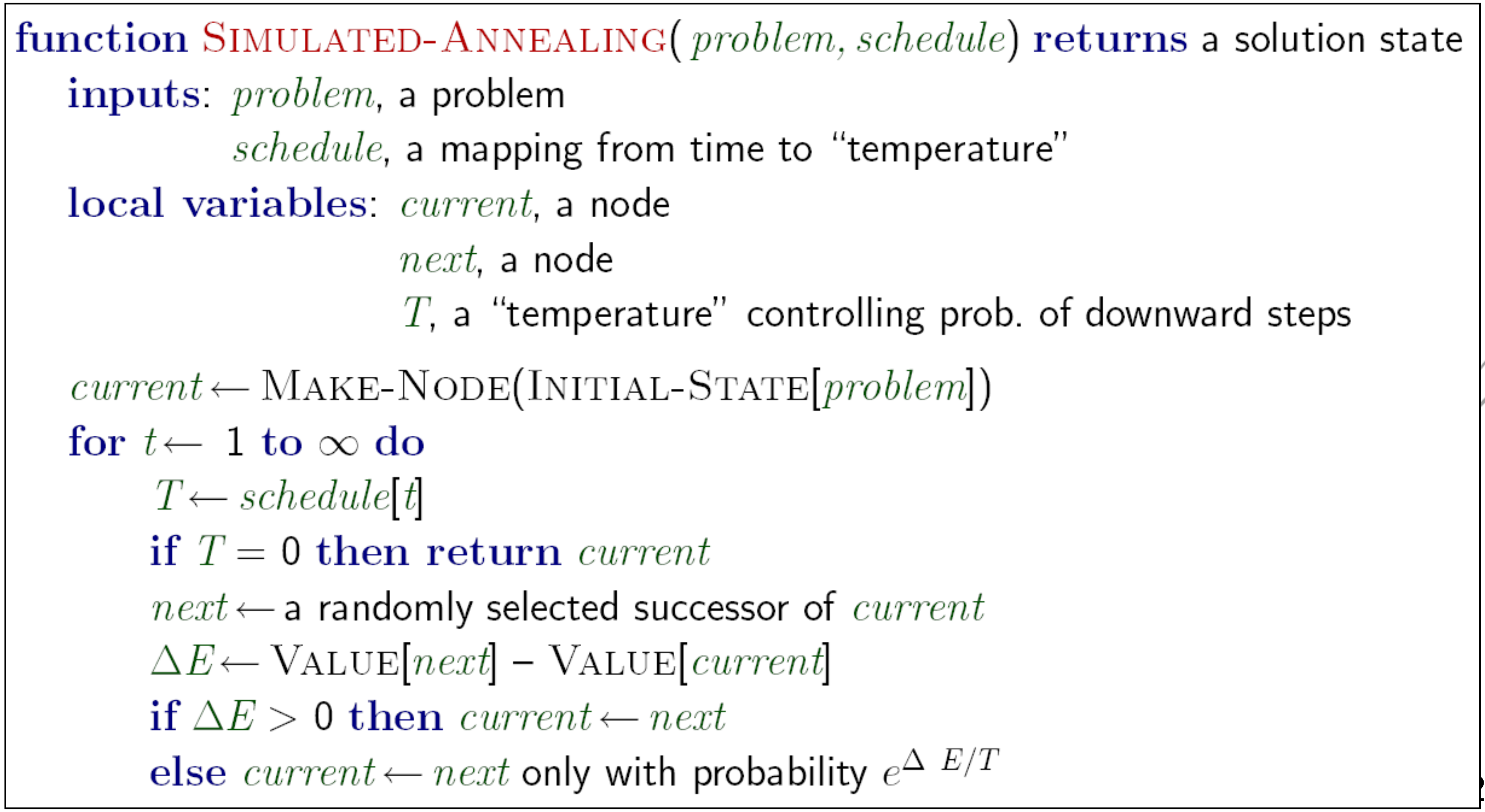
Starting from Y, where do you end up ?

Starting from Z, where do you end up ?

Simulated Annealing

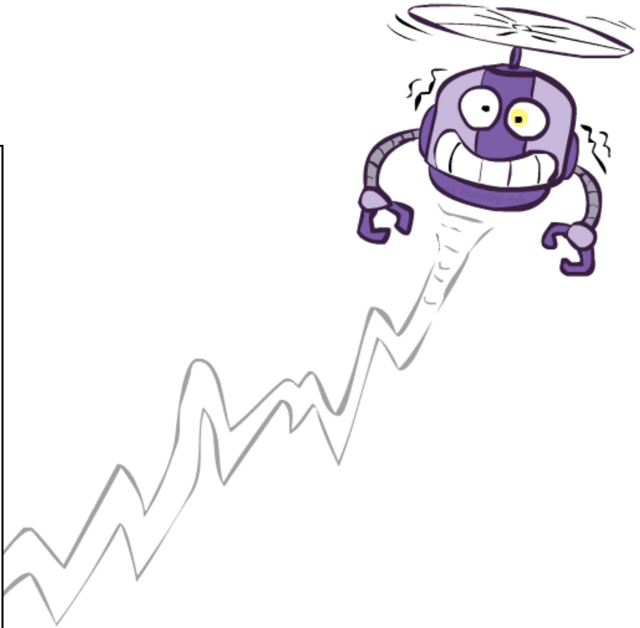
Idea: Escape local maxima by allowing downhill moves

But make them rarer as time goes on



```
function SIMULATED-ANNEALING( problem, schedule) returns a solution state
  inputs: problem, a problem
          schedule, a mapping from time to "temperature"
  local variables: current, a node
                   next, a node
                   T, a "temperature" controlling prob. of downward steps

  current ← MAKE-NODE(INITIAL-STATE[problem])
  for t ← 1 to ∞ do
      T ← schedule[t]
      if T = 0 then return current
      next ← a randomly selected successor of current
      $\Delta E$ ← VALUE[next] – VALUE[current]
      if $\Delta E > 0$ then current ← next
      else current ← next only with probability $e^{\Delta\ E/T}$
```

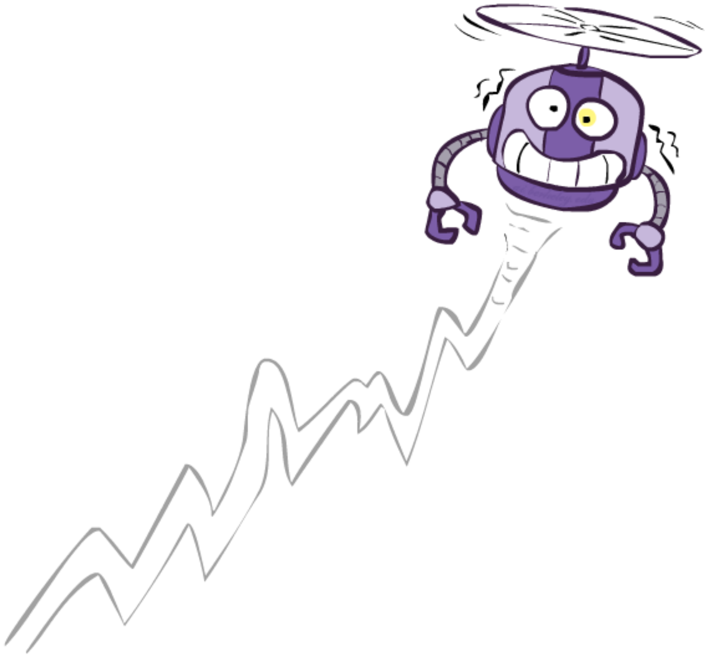


Simulated Annealing

Theoretical guarantee:

Stationary distribution: $p(x) \propto e^{\frac{E(x)}{kT}}$

If T decreased slowly enough,
will converge to optimal state!



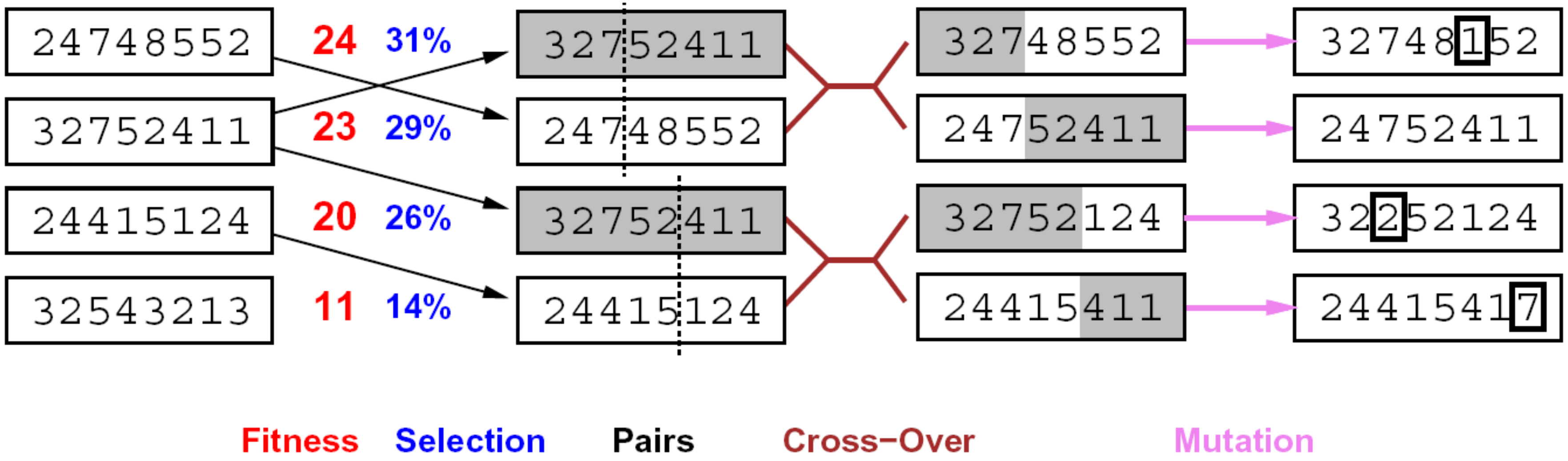
Is this an interesting guarantee?

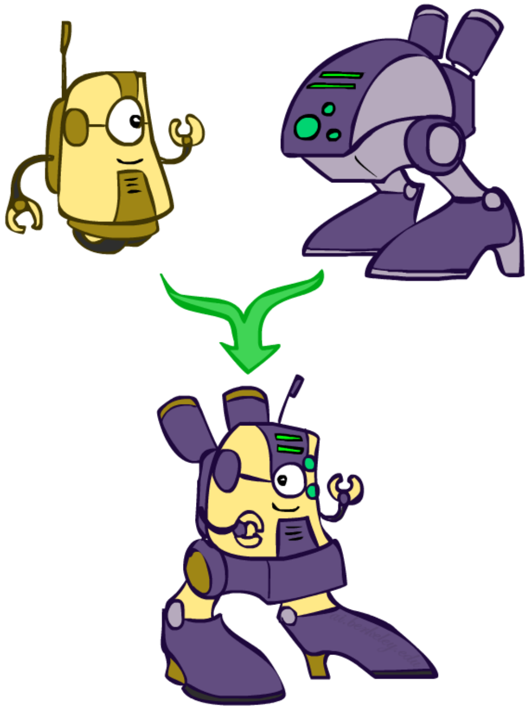
Sounds like magic, but reality is reality:

The more downhill steps you need to escape a local optimum, the less likely you are to ever make them all in a row

People think hard about *ridge operators* which let you jump around the space in better ways

Genetic Algorithms



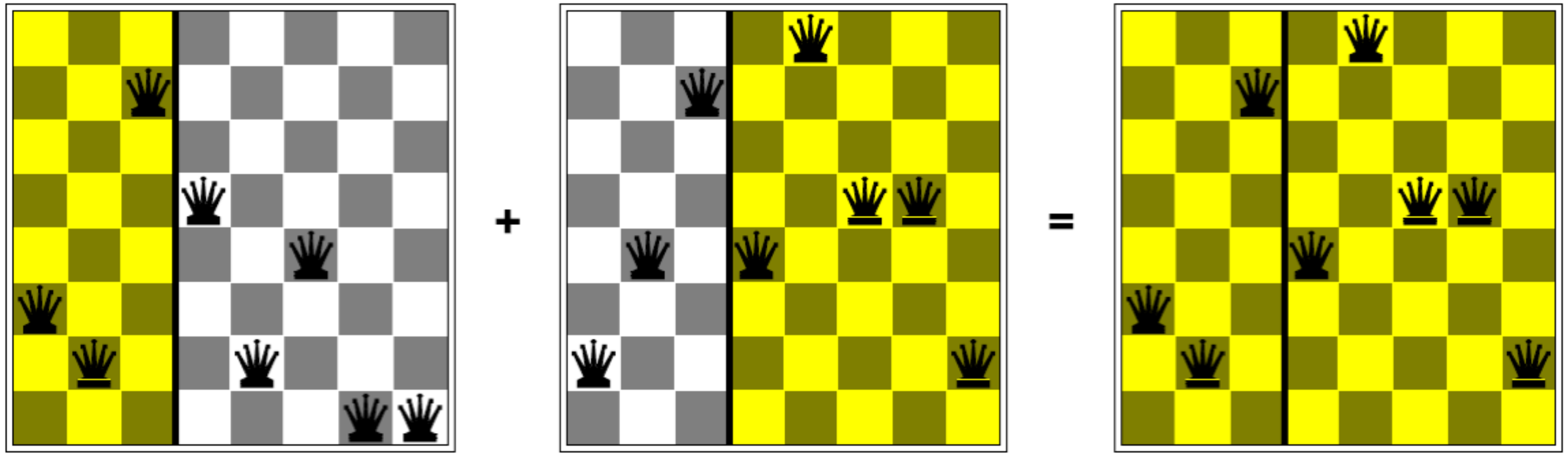


Genetic algorithms use a natural selection metaphor

- Keep best N hypotheses at each step (selection) based on a fitness function
- Also have pairwise crossover operators, with optional mutation to give variety

Possibly the most misunderstood, misapplied (and even maligned) technique around

Example: N-Queens



Why does crossover make sense here?

When wouldn’t it make sense?

What would mutation be?

What would a good fitness function be?

Next Time: Adversarial Search!