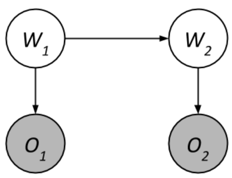
Regular Discussion 8

1 HMMs

Consider the following Hidden Markov Model. O_1 and O_2 are supposed to be shaded.



W_1	$P(W_1)$
0	0.3
1	0.7

W_t	W_{t+1}	$P(W_{t+1} \| W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t \| W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

Suppose that we observe $O_1 = a$ and $O_2 = b$.
Using the forward algorithm, compute the probability distribution $P(W_2|O_1 = a, O_2 = b)$ one step at a time.

(a) Compute $P(W_1, O_1 = a)$.

(b) Using the previous calculation, compute $P(W_2, O_1 = a)$.

(c) Using the previous calculation, compute $P(W_2, O_1 = a, O_2 = b)$.

(d) Finally, compute $P(W_2|O_1 = a, O_2 = b)$.

Dynamic Bayesian Networks

The Decaying Sensor

We are tracking a robot moving through a 1D grid. Let L_t be the robot's true location at time t. The robot is equipped with a distance sensor that gives a reading S_t at time t. However, the sensor runs on a separate battery, B_t. As the battery drains, the sensor readings become less reliable.

The dynamics of this world are modeled by a DBN with the following properties:

- The robot's current location L_t depends only on its previous location L_{t-1}.
- The sensor's battery level B_t depends only on its previous battery level B_{t-1}.
- The sensor reading S_t depends on both the robot's current location L_t and the current battery level B_t.

1. Write down the full joint probability distribution for the first two time steps, $t = 1$ and $t = 2$, in terms of the initial state distributions, transition models, and sensor models.

   0.5in

2. Are L_2 and B_2 guaranteed to be independent given no observations? Prove it algebraically by marginalizing the joint distribution.

   2.5in

3. Are L_2 and B_2 guaranteed to be independent given the sensor reading S_2? Briefly explain using the concept of common effects (explaining away).

   1.5in