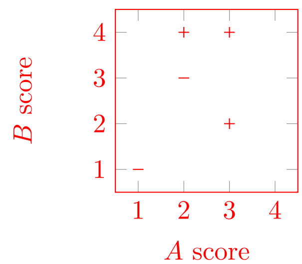
CS 188
Spring 2026

Regular Discussion 11 Solutions

1 Perceptron

You want to predict if movies will be profitable based on their screenplays. You hire two critics A and B to read a script you have and rate it on a scale of 1 to 4. The critics are not perfect; here are five data points including the critics' scores and the performance of the movie:

#	Movie Name	A	B	Profit?
1	Pellet Power	1	1	-
2	Ghosts!	3	2	+
3	Pac is Bac	2	4	+
4	Not a Pizza	3	4	+
5	Endless Maze	2	3	-



1. First, you would like to examine the linear separability of the data. Plot the data on the 2D plane above; label profitable movies with + and non-profitable movies with − and determine if the data are linearly separable.
   The data are linearly separable.

2. Now you decide to use a perceptron to classify your data. Suppose you directly use the scores given above as features, together with a bias feature. That is $f_0 = 1$, $f_1 =$ score given by A and $f_2 =$ score given by B.

   Run one pass through the data with the perceptron algorithm, filling out the table below. Go through the data points in order, e.g. using data point #1 at step 1.

step	Weights	Score	Correct?
1	[-1, 0, 0]	$-1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 = -1$	yes
2	[-1, 0, 0]	$-1 \cdot 1 + 0 \cdot 3 + 0 \cdot 2 = -1$	no
3	[0, 3, 2]	$0 \cdot 1 + 3 \cdot 2 + 2 \cdot 4 = 14$	yes
4	[0, 3, 2]	$0 \cdot 1 + 3 \cdot 3 + 2 \cdot 4 = 17$	yes
5	[0, 3, 2]	$0 \cdot 1 + 3 \cdot 2 + 2 \cdot 3 = 12$	no

   Final weights: [-1, 1, -1]

3. Have weights been learned that separate the data? With the current weights, points will be classified as positive if $-1 \cdot 1 + 1 \cdot A + -1 \cdot B \geq 0$, or $A - B \geq 1$. So we will have incorrect predictions for data points 3:

$$-1 \cdot 1 + 1 \cdot 2 + -1 \cdot 4 = -3 < 0$$

   and 4:

$$-1 \cdot 1 + 1 \cdot 3 + -1 \cdot 4 = -2 < 0$$

   Note that although point 2 has $w \cdot f = 0$, it will be classified as positive (since we classify as positive if $w \cdot f \geq 0$).

4. More generally, irrespective of the training data, you want to know if your features are powerful enough to allow you to handle a range of scenarios. Circle the scenarios for which a perceptron using the features above can indeed perfectly classify movies which are profitable according to the given rules:

(a) Your reviewers are awesome: if the total of their scores is more than 8, then the movie will definitely be profitable, and otherwise it won't be. Can classify (consider weights $[-8, 1, 1]$)

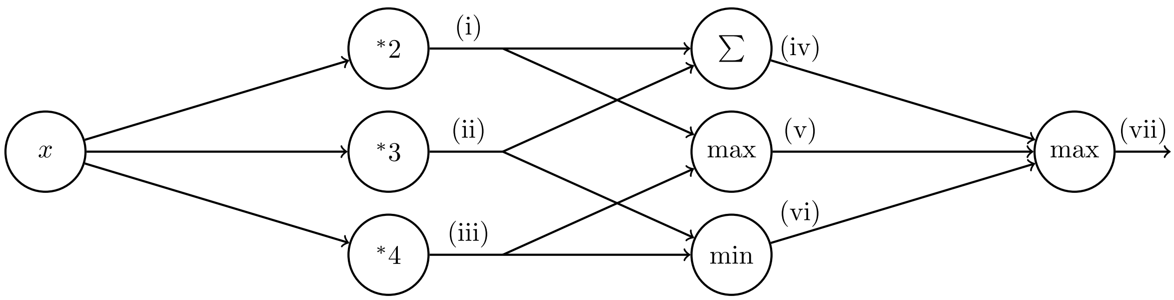
(b) Your reviewers are art critics. Your movie will be profitable if and only if each reviewer gives either a score of 2 or a score of 3. Cannot classify

(c) Your reviewers have weird but different tastes. Your movie will be profitable if and only if both reviewers agree. Cannot classify

Q2. Backpropagation

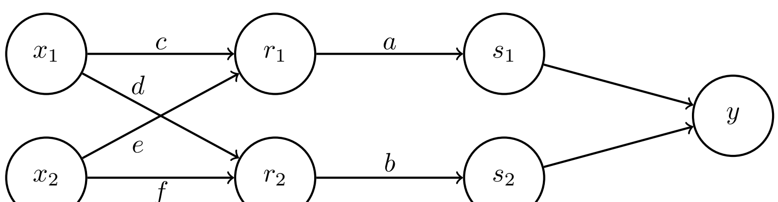
(a) Perform forward propagation on the neural network below for $x = 1$ by filling in the values in the table. Note that (i), ..., (vii) are outputs after performing the appropriate operation as indicated in the node.

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
2	3	4	5	4	3	5



(b) Below is a neural network with weights a, b, c, d, e, f. The inputs are x_1 and x_2.
The first hidden layer computes $r_1 = \max(c \cdot x_1 + e \cdot x_2, 0)$ and $r_2 = \max(d \cdot x_1 + f \cdot x_2, 0)$.
The second hidden layer computes $s_1 = \frac{1}{1+\exp(-a \cdot r_1)}$ and $s_2 = \frac{1}{1+\exp(-b \cdot r_2)}$.
The output layer computes $y = s_1 + s_2$. Note that the weights a, b, c, d, e, f are indicated along the edges of the neural network here.

Suppose the network has inputs $x_1 = 1, x_2 = -1$.
The weight values are $a = 1, b = 1, c = 4, d = 1, e = 2, f = 2$.
Forward propagation then computes $r_1 = 2, r_2 = 0, s_1 = 0.9, s_2 = 0.5, y = 1.4$. Note: some values are rounded.



Using the values computed from forward propagation, use backpropagation to numerically calculate the following partial derivatives. Write your answers as a single number (not an expression). You do not need a calculator. Use scratch paper if needed.
Hint: For $g(z) = \frac{1}{1+\exp(-z)}$, the derivative is $\frac{\partial g}{\partial z} = g(z)(1 - g(z))$.

$\frac{\partial y}{\partial a}$	$\frac{\partial y}{\partial b}$	$\frac{\partial y}{\partial c}$	$\frac{\partial y}{\partial d}$	$\frac{\partial y}{\partial e}$	$\frac{\partial y}{\partial f}$
0.18	0	0.09	0	−0.09	0

$$
\begin{aligned}
\frac{\partial y}{\partial a} &= \frac{\partial y}{\partial s_1}\frac{\partial s_1}{\partial a} \\
&= 1 \cdot \frac{\partial g(a \cdot r_1)}{\partial a} \\
&= r_1 \cdot g(a \cdot r_1)(1 - g(a \cdot r_1)) \\
&= r_1 \cdot s_1(1 - s_1) \\
&= 2 \cdot 0.9 \cdot (1 - 0.9) \\
&= 0.18 \\
\frac{\partial y}{\partial b} &= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial b} \\
&= 1 \cdot \frac{\partial g(b \cdot r_2)}{\partial b} \\
&= r_2 \cdot g(b \cdot r_2)(1 - g(b \cdot r_2)) \\
&= r_2 \cdot s_2(1 - s_2) \\
&= 0 \cdot 0.5(1 - 0.5) \\
&= 0 \\
\frac{\partial y}{\partial c} &= \frac{\partial y}{\partial s_1}\frac{\partial s_1}{\partial r_1}\frac{\partial r_1}{\partial c} \\
&= 1 \cdot [a \cdot g(a \cdot r_1)(1 - g(a \cdot r_1))] \cdot x_1 \\
&= [a \cdot s_1(1 - s_1)] \cdot x_1 \\
&= [1 \cdot 0.9(1 - 0.9)] \cdot 1 \\
&= 0.09 \\
\frac{\partial y}{\partial d} &= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial r_2}\frac{\partial r_2}{\partial d} \\
&= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial r_2} \cdot 0 \\
&= 0 \\
\frac{\partial y}{\partial e} &= \frac{\partial y}{\partial s_1}\frac{\partial s_1}{\partial r_1}\frac{\partial r_1}{\partial e} \\
&= 1 \cdot [a \cdot g(a \cdot r_1)(1 - g(a \cdot r_1))] \cdot x_2 \\
&= [a \cdot s_1(1 - s_1)] \cdot x_2 \\
&= [1 \cdot 0.9(1 - 0.9)] \cdot -1 \\
&= -0.09 \\
\frac{\partial y}{\partial f} &= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial r_2}\frac{\partial r_2}{\partial f} \\
&= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial r_2} \cdot 0 \\
&= 0
\end{aligned}
$$