CS 188
Summer 2019

Section Handout 7

Q1. Independent Probability Tables

Suppose you have two random variables, C and N. C is the result of flipping a biased coin that lands on heads with probability 0.8. N is the number of heads that result from flipping a fair coin twice. Fill in the probability tables for $P(C)$, $P(N)$, and $P(C,N)$.

C	$P(C)$
h	
t	

N	$P(N)$
0	
1	
2	

C	N	$P(C,N)$
h	0	
h	1	
h	2	
t	0	
t	1	
t	2	

Note that in general, tables cannot be combined in this way. This works because of our simplifying assumption that the coin flips are independent.

Q2. Joining Probability Tables and Bayes's Rule

Consider two binary random variables, L and T. L takes on values $+l$ and l, corresponding to whether or not youre late for work. T takes on values $+t$ and t and corresponds to whether or not theres a traffic jam. For example, $+l,+t$ means youre late for work and theres a traffic jam. We are given the following probability tables:

T	$P(T)$
+t	0.4
–t	0.6

L	T	$P(L\|T)$
+l	+t	0.8
+l	–t	0.25
–l	+t	0.2
–l	–t	0.75

Use the above tables to fill in the following tables:

L	T	$P(L,T)$
+l	+t	
+l	–t	
–l	+t	
–l	–t	

L	T	$P(T\|L)$
+l	+t	
+l	–t	
–l	+t	
–l	–t	

Q3. Querying a Joint Distribution

A	B	C	$P(A,B,C)$
+a	+b	+c	1/16
+a	+b	−c	1/8
+a	−b	+c	1/16
+a	−b	−c	1/4
−a	+b	+c	1/4
−a	+b	−c	1/16
−a	−b	+c	1/16
−a	−b	−c	1/8

Compute the following values:

(a) $P(+c) =$

(b) $P(-b,+a) =$

(c) $P(-b|+a) =$

(d) $P(+c|-a,+b) =$

(e) $P(-a,+c|-b) =$

Compute the following tables:

(a) $P(B)$

(b) $P(+b|+a,C)$

(c) $P(A,C|+b)$

Q4. Independence

1. Here are two joint probability tables. In which are the variables independent?

A	B	$P(A,B)$
+a	+b	0.42
+a	–b	0.18
–a	+b	0.28
–a	–b	0.12

C	D	$P(C,D)$
+c	+d	0.4
+c	–d	0.15
–c	+d	0.4
–c	–d	0.05

2. (a) Given that $A \perp\!\!\!\perp B$, simplify $\sum_a P(a|B)P(C|a)$:

   (b) Given that $B \perp\!\!\!\perp C|A$, simplify $\frac{P(A)P(B|A)P(C|A)}{P(B|C)P(C)}$:

   (c) Given that $A \perp\!\!\!\perp B|C$, simplify $\frac{P(C,A|B)P(B)}{P(C)}$:

Q5. Expression Manipulation

Simplify the following expressions, as much as possible:

1. $\sum_x \sum_y f(x)h(x,y) =$

2. $\sum_a \sum_b \sum_c f(c)f(c,b)f(c,b,a) =$

3. $\sum_a \sum_b \sum_c f(b)f(c)f(a,b,c)f(a,j)f(a,m) =$

4. $\sum_d \sum_k \sum_c \sum_x f(c,k)f(x)f(c,k,x)f(k)f(c,d,k,x)f(k,x) =$

Q6. Expectation

Let's say we have a fair, 6-sided dice.

1. Let X_i be the result of the ith dice roll. What is $E[X_1]$?
2. What is $E[X_1^2]$?
3. What is $E[X_1+X_2]$?
4. Let K be an arbitrary random variable, and let Y_K be the sum of the first K dice rolls. What is $E[Y|K]$?
5. We play a game where we first flip a fair coin. If the coin lands heads, then we roll the dice twice and score points equal to the sum. If the coin lands tails, then we roll three dice instead. What is our expected score?