CS 188 Summer 2020 Introduction to Artificial Intelligence

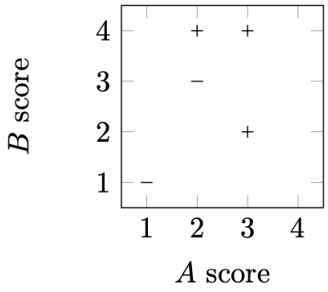
Disc 11/EP 10

Perceptron & Naive Bayes Solutions

1 Perceptron

You want to predict if movies will be profitable based on their screenplays. You hire two critics A and B to read a script you have and rate it on a scale of 1 to 4. The critics are not perfect; here are five data points including the critics' scores and the performance of the movie:

#	Movie Name	A	B	Profit?
1	Pellet Power	1	1	-
2	Ghosts!	3	2	+
3	Pac is Bac	2	4	+
4	Not a Pizza	3	4	+
5	Endless Maze	2	3	-



1. First, you would like to examine the linear separability of the data. Plot the data on the 2D plane above; label profitable movies with + and non-profitable movies with − and determine if the data are linearly separable. The data are linearly separable.

2. Now you decide to use a perceptron to classify your data. Suppose you directly use the scores given above as features, together with a bias feature. That is $f_0 = 1$, f_1 = score given by A and f_2 = score given by B.

   Run one pass through the data with the perceptron algorithm, filling out the table below. Go through the data points in order, e.g. using data point #1 at step 1.

step	Weights	Score	Correct?
1	[-1, 0, 0]	$-1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 = -1$	yes
2	[-1, 0, 0]	$-1 \cdot 1 + 0 \cdot 3 + 0 \cdot 2 = -1$	no
3	[0, 3, 2]	$0 \cdot 1 + 3 \cdot 2 + 2 \cdot 4 = 14$	yes
4	[0, 3, 2]	$0 \cdot 1 + 3 \cdot 3 + 2 \cdot 4 = 17$	yes
5	[0, 3, 2]	$0 \cdot 1 + 3 \cdot 2 + 2 \cdot 3 = 12$	no

   Final weights: [-1, 1, -1]

3. Have weights been learned that separate the data? With the current weights, points will be classified as positive if $-1 \cdot 1 + 1 \cdot A + -1 \cdot B \geq 0$, or $A - B \geq 1$. So we will have incorrect predictions for data points 3:

$$-1 \cdot 1 + 1 \cdot 2 + -1 \cdot 4 = -3 < 0$$

   and 4:

$$-1 \cdot 1 + 1 \cdot 3 + -1 \cdot 4 = -2 < 0$$

   Note that although point 2 has $w \cdot f = 0$, it will be classified as positive (since we classify as positive if $w \cdot f \geq 0$).

4. More generally, irrespective of the training data, you want to know if your features are powerful enough to allow you to handle a range of scenarios. Circle the scenarios for which a perceptron using the features above can indeed perfectly classify movies which are profitable according to the given rules:

   (a) Your reviewers are awesome: if the total of their scores is more than 8, then the movie will definitely be profitable, and otherwise it won't be. Can classify (consider weights $[-8, 1, 1]$)

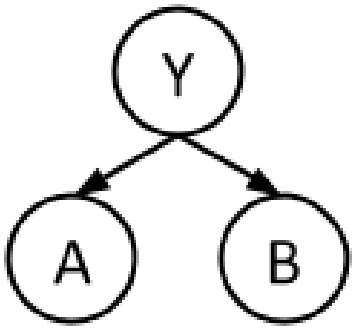
   (b) Your reviewers are art critics. Your movie will be profitable if and only if each reviewer gives either a score of 2 or a score of 3. Cannot classify

   (c) Your reviewers have weird but different tastes. Your movie will be profitable if and only if both reviewers agree. Cannot classify

Q2. Naive Bayes

In this question, we will train a Naive Bayes classifier to predict class labels Y as a function of input features A and B. Y, A, and B are all binary variables, with domains 0 and 1. We are given 10 training points from which we will estimate our distribution.

A	1	1	1	1	0	1	0	1	1	1
B	1	0	0	1	1	1	1	0	1	1
Y	1	1	0	0	0	1	1	0	0	0



1. What are the maximum likelihood estimates for the tables $P(Y)$, $P(A|Y)$, and $P(B|Y)$?

Y	$P(Y)$
0	3/5
1	2/5

A	Y	$P(A\|Y)$
0	0	1/6
1	0	5/6
0	1	1/4
1	1	3/4

B	Y	$P(B\|Y)$
0	0	1/3
1	0	2/3
0	1	1/4
1	1	3/4

2. Consider a new data point ($A = 1$, $B = 1$). What label would this classifier assign to this sample?

$$
\begin{aligned}
P(Y = 0, A = 1, B = 1) &= P(Y = 0)P(A = 1|Y = 0)P(B = 1|Y = 0) && (1)\\
&= (3/5)(5/6)(2/3) && (2)\\
&= 1/3 && (3)\\
P(Y = 1, A = 1, B = 1) &= P(Y = 1)P(A = 1|Y = 1)P(B = 1|Y = 1) && (4)\\
&= (2/5)(3/4)(3/4) && (5)\\
&= 9/40 && (6)\\
& && (7)
\end{aligned}
$$

Our classifier will predict label 0.

3. Let's use Laplace Smoothing to smooth out our distribution. Compute the new distribution for $P(A|Y)$ given Laplace Smoothing with $k = 2$.

A	Y	$P(A\|Y)$
0	0	3/10
1	0	7/10
0	1	3/8
1	1	5/8