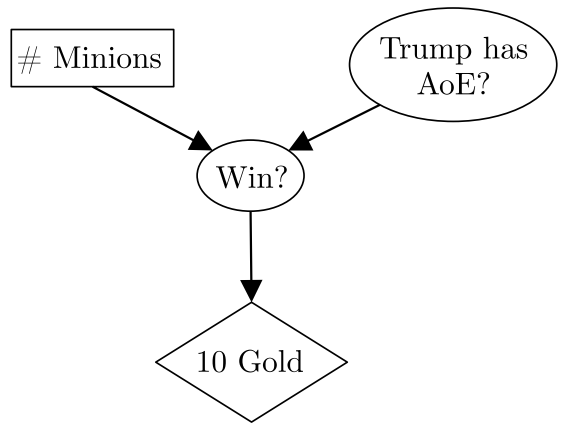
Q1. Hearthstone Decisions

You are playing the game Hearthstone. You are up against the famous player Trump.

On your turn, you can choose between playing 0, 1, or 2 minions. You realize Trump might be holding up an Area of Effect (AoE) card, which is more devastating the more minions you play.

- If Trump has the AoE, then your chances of winning are:
  - 60% if you play 0 minions
  - 50% if you play 1 minion
  - 20% if you play 2 minions
- If Trump does NOT have the AoE, then your chances of winning are:
  - 20% if you play 0 minions
  - 60% if you play 1 minion
  - 90% if you play 2 minions



You know that there is a 50% chance that Trump has an AoE.

Winning this game is worth 10 gold and losing is worth 0.

(a) How much gold would you expect to win choosing 0 minions?

(b) How much gold would you expect to win choosing 1 minion?

(c) How much gold would you expect to win choosing 2 minions?

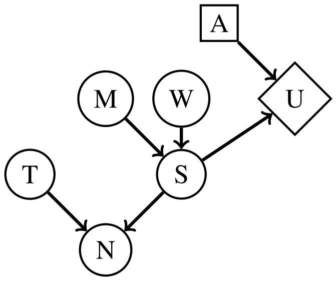
(d) How much gold would you expect to win if you know the AoE is in Trump's hand?

(e) How much gold would you expect to win if you know the AoE is NOT in Trump's hand?

(f) How much gold would you be willing to pay for to know whether or not the AoE is in Trump's hand? (Assume your utility of gold is the same as the amount of gold.)

Q2. Decision Networks and VPI

(a) Consider the decision network structure given below:



Mark all of the following statements that **could possibly be true**, for some probability distributions for $P(M)$, $P(W)$, $P(T)$, $P(S|M, W)$, and $P(N|T, S)$ and some utility function $U(S, A)$:

(i) [1.5 pts]

☐ $\text{VPI}(T) < 0$ ☐ $\text{VPI}(T) = 0$ ☐ $\text{VPI}(T) > 0$ ☐ $\text{VPI}(T) = \text{VPI}(N)$

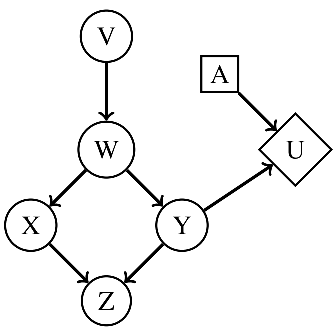
(ii) [1.5 pts]

☐ $\text{VPI}(T|N) < 0$ ☐ $\text{VPI}(T|N) = 0$ ☐ $\text{VPI}(T|N) > 0$ ☐ $\text{VPI}(T|N) = \text{VPI}(T|S)$

(iii) [1.5 pts]

☐ $\text{VPI}(M) > \text{VPI}(W)$ ☐ $\text{VPI}(M) > \text{VPI}(S)$ ☐ $\text{VPI}(M) < \text{VPI}(S)$ ☐ $\text{VPI}(M|S) > \text{VPI}(S)$

(b) Consider the decision network structure given below.



Mark all of the following statements that are **guaranteed to be true**, regardless of the probability distributions for any of the chance nodes and regardless of the utility function.

(i) [1.5 pts]

☐ $\text{VPI}(Y) = 0$

☐ $\text{VPI}(X) = 0$

☐ $\text{VPI}(Z) = \text{VPI}(W, Z)$

☐ $\text{VPI}(Y) = \text{VPI}(Y, X)$

(ii) [1.5 pts]

☐ $\text{VPI}(X) \leq \text{VPI}(W)$

☐ $\text{VPI}(V) \leq \text{VPI}(W)$

☐ $\text{VPI}(V \mid W) = \text{VPI}(V)$

☐ $\text{VPI}(W \mid V) = \text{VPI}(W)$

(iii) [1.5 pts]

☐ $\text{VPI}(X \mid W) = 0$

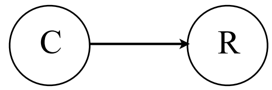
☐ $\text{VPI}(Z \mid W) = 0$

☐ $\text{VPI}(X, W) = \text{VPI}(V, W)$

☐ $\text{VPI}(W, Y) = \text{VPI}(W) + \text{VPI}(Y)$

Q3. Value of Perfect Information with an Incorrect Model

You would like to predict whether it will rain tomorrow. Your model of the situation is a very simple one. The is one variable C indicating the presence of clouds, and other variable indicating whether it will rain today. You model the weather with a simple Bayes net:



Let $P(C = +c) = 0.5$, $P(R = +r|C = +c) = 0.9$ and $P(R = +r|C = -c) = 0.2$. Assume that your utility is 1 if you correctly predict rain (or not rain) and 0 if you incorrectly predict rain (or not rain).

1. Calculate the Value of Perfect Information of observing C.

2. Now suppose that the true distribution P^* conditional distribution of rain given cloud is $P^*(R = +r|C = +c) = 0.3$ and $P^*(R = +r|C = -c) = 0.6$, and we still have $P^*(C = +c) = 0.5$. If you act so as to maximize your expected utility under P, what is your Value of Perfect Information (under $P*$)?

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