CS 188 Summer 2020 Introduction to Artificial Intelligence

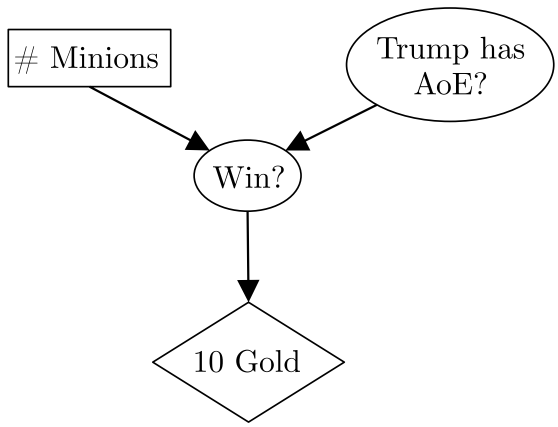
EP 8 Solutions

Q1. Hearthstone Decisions

You are playing the game Hearthstone. You are up against the famous player Trump.

On your turn, you can choose between playing 0, 1, or 2 minions. You realize Trump might be holding up an Area of Effect (AoE) card, which is more devastating the more minions you play.

- If Trump has the AoE, then your chances of winning are:
  - 60% if you play 0 minions
  - 50% if you play 1 minion
  - 20% if you play 2 minions
- If Trump does NOT have the AoE, then your chances of winning are:
  - 20% if you play 0 minions
  - 60% if you play 1 minion
  - 90% if you play 2 minions



You know that there is a 50% chance that Trump has an AoE.

Winning this game is worth 10 gold and losing is worth 0.

Solution notation: A: Trump has AoE?, W: Win?, M: Number of minions

(a) How much gold would you expect to win choosing 0 minions?

$\sum_w \sum_a (P(w|Minion = 0, a)P(a)R(w) = 10 \sum_a (P(w|Minion = 0, a)P(a) = 10(.6 \cdot .5 + .2 \cdot .5) = 4$

(b) How much gold would you expect to win choosing 1 minion?

$\sum_w \sum_a (P(w|Minion = 1, a)P(a)R(w) = 10 \sum_a (P(w|Minion = 1, a)P(a) = 10(.5 \cdot .5 + .6 \cdot .5) = 5.5$

(c) How much gold would you expect to win choosing 2 minions?

$\sum_w \sum_a (P(w|Minion = 2, a)P(a)R(w) = 10 \sum_a (P(w|Minion = 2, a)P(a) = 10(.2 \cdot .5 + .9 \cdot .5) = 5.5$

(d) How much gold would you expect to win if you know the AoE is in Trump's hand?

$\max_m \sum_w P(w|m, +a)R(w) = 10 \max_m P(w|m, +a) = 10 \max\{.6, .5, .2\} = 6$

(e) How much gold would you expect to win if you know the AoE is NOT in Trump's hand?

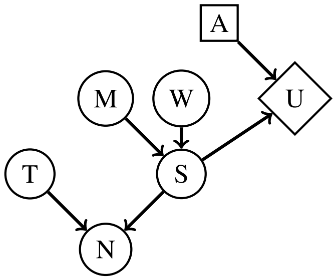
$\max_m \sum_w P(w|m, -a)R(w) = 10 \max_m P(w|m, -a) = 10 \max\{.2, .6, .9\} = 9$

(f) How much gold would you be willing to pay for to know whether or not the AoE is in Trump's hand? (Assume your utility of gold is the same as the amount of gold.)

Two. The difference between $MEU(\{\}) = 5.5$ and $MEU(\{A\}) = .5 * 6 + .5 * 9 = 7.5$ is 2.

Q2. Decision Networks and VPI

(a) Consider the decision network structure given below:



Mark all of the following statements that **could possibly be true**, for some probability distributions for $P(M)$, $P(W)$, $P(T)$, $P(S|M,W)$, and $P(N|T,S)$ and some utility function $U(S,A)$:

(i) [1.5 pts]

□ VPI(T) < 0 ■ VPI(T) $= 0$ □ VPI(T) > 0 ■ VPI(T) = VPI(N)

VPI can never be negative. VPI(T) = 0 must be true since T is independent of S. VPI(N) could also be zero if N and S are independent.

(ii) [1.5 pts]

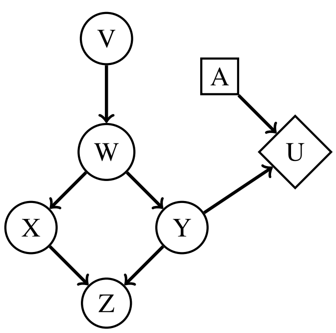
□ VPI($T|N$) < 0 ■ VPI($T|N$) $= 0$ ■ VPI($T|N$) > 0 ■ VPI($T|N$) = VPI($T|S$)

VPI can never be negative. $VPI(T|N) = 0$ if N is conditionally independent of S given N, but will usually be positive. $VPI(T|S) = 0$, and as we've seen $VPI(T|N)$ could also be zero.

(iii) [1.5 pts]

■ VPI(M) > VPI(W) □ VPI(M) > VPI(S) ■ VPI(M) < VPI(S) □ VPI($M|S$) > VPI(S)

(b) Consider the decision network structure given below.



Mark all of the following statements that are **guaranteed to be true**, regardless of the probability distributions for any of the chance nodes and regardless of the utility function.

(i) [1.5 pts]

□ VPI(Y) = 0 Observing Y could increase MEU

□ VPI(X) = 0 Y can depend on X because of the path through W

□ VPI(Z) = VPI(W,Z) Consider a case where Y is independent of Z but not independent of W. Then $VPI(Z) = 0 < VPI(W,Z)$

■ VPI(Y) = VPI(Y,X) After Y is revealed, X will add no more information about Y.

(ii) [1.5 pts]

■ VPI(X) $\leq$ VPI(W) $VPI(W \mid X) + VPI(X) = VPI(X,W) = VPI(X \mid W) + VPI(W)$. We know $VPI(X \mid W) = 0$, since X is conditionally independent of Y, given W. So $VPI(W \mid X) + VPI(X) = VPI(W)$. Since VPI is non-negative, $VPI(W \mid X) \geq 0$, so $VPI(X) \leq VPI(W)$.

■ VPI(V) $\leq$ VPI(W) Since the only path from V to Y is through W, revealing V cannot give more information about Y than revealing W.

□ VPI($V \mid W$) = VPI(V) $VPI(V \mid W) = 0$ by conditional independence, but $VPI(V)$ is not necessarily 0

☐ VPI($W \mid V$) = VPI(W) Consider a case where W is a deterministic function of V and Y is a deterministic function of W, then $VPI(W \mid V) = 0 \neq VPI(W)$

(iii) [1.5 pts]

■ VPI($X \mid W$) = 0 X is independent of Y given W

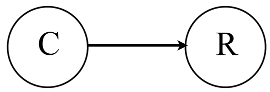
☐ VPI($Z \mid W$) = 0 Y could depend on Z, given W

■ VPI(X, W) = VPI(V, W) Both are equal to VPI(W), since both X and V are conditionally independent of Y given W.

☐ VPI(W, Y) = VPI(W) + VPI(Y) VPI(W,Y) = VPI(Y), and we can have $VPI(W) > 0$

Q3. Value of Perfect Information with an Incorrect Model

You would like to predict whether it will rain tomorrow. Your model of the situation is a very simple one. The is one variable C indicating the presence of clouds, and other variable indicating whether it will rain today. You model the weather with a simple Bayes net:



Let $P(C = +c) = 0.5$, $P(R = +r|C = +c) = 0.9$ and $P(R = +r|C = -c) = 0.2$. Assume that your utility is 1 if you correctly predict rain (or not rain) and 0 if you incorrectly predict rain (or not rain).

1. Calculate the Value of Perfect Information of observing C.

   Expected utility of "predict rain" is 0.5*0.9 + 0.5 * 0.2 = 0.55.

   Expected utility of "predict not rain" is 0.1*0.5 + 0.8*0.5 = 0.45.

   If $C = +c$, MEU is 0.9 for predict rain.

   If $C = -c$, MEU is 0.8 for predict not rain.

   So VPI is 0.85 - 0.55 = 0.3.

2. Now suppose that the true distribution P^* conditional distribution of rain given cloud is $P^*(R = +r|C = +c) = 0.3$ and $P^*(R = +r|C = -c) = 0.6$, and we still have $P^*(C = +c) = 0.5$. If you act so as to maximize your expected utility under P, what is your Value of Perfect Information (under P*)?

   Expected utility of "predict rain" is 0.45; this is because acting under the assumption of distribution P, "we" should always choose to "predict rain", which, under the actual distribution P*, gives an EU of $\frac{1}{2} \cdot 0.3 + \frac{1}{2} \cdot 0.6 = 0.45$ only.

   Expected utility of predicting rain given $C = +c$ is 0.3, and expected utility of predicting not rain for $C = -c$ is 0.4. So expected utility given C is 0.35, and difference is 0.35-0.45 = -0.1.