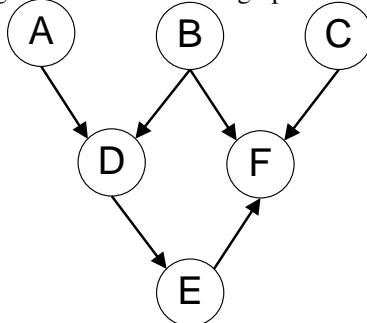


## Q1. Bayes Nets

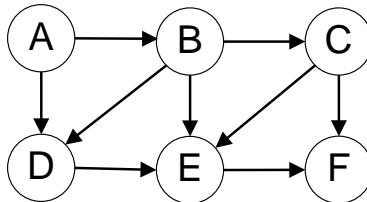
- (a) For the following graphs, explicitly state the minimum size set of edges that must be removed such that the corresponding independence relations are guaranteed to be true.

Marked the removed edges with an 'X' on the graphs.



$$\begin{aligned} A \perp\!\!\!\perp B | F \\ A \perp\!\!\!\perp F | D \\ B \perp\!\!\!\perp C \end{aligned}$$

(i)



$$\begin{aligned} A \perp\!\!\!\perp D | B \\ A \perp\!\!\!\perp F | C \\ C \perp\!\!\!\perp D | B \end{aligned}$$

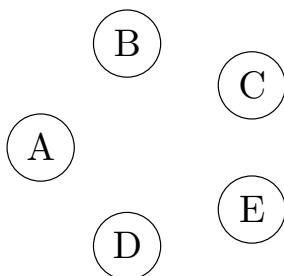
(ii)

- (b) You're performing variable elimination over a Bayes Net with variables  $A, B, C, D, E$ . So far, you've finished joining over (but not summing out)  $C$ , when you realize you've lost the original Bayes Net!

Your current factors are  $f(A), f(B), f(B, D), f(A, B, C, D, E)$ . Note: these are factors, NOT joint distributions. You don't know which variables are conditioned or unconditioned.

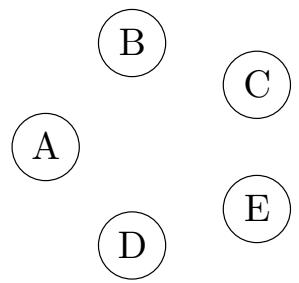
- (i) What's the smallest number of edges that could have been in the original Bayes Net? Draw out one such Bayes Net below.

Number of edges =



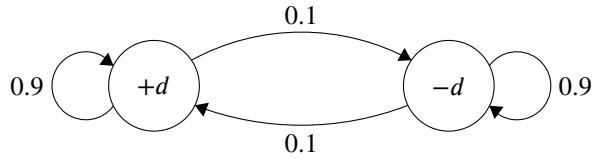
- (ii) What's the largest number of edges that could have been in the original Bayes Net? Draw out one such Bayes Net below.

Number of edges =

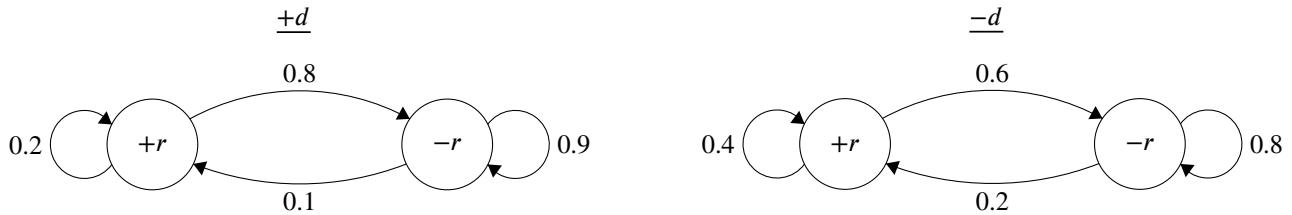


## Q2. I Heard You Like Markov Chains

In California, whether it rains or not from each day to the next forms a Markov chain (note: this is a terrible model for real weather). However, sometimes California is in a drought and sometimes it is not. Whether California is in a drought from each day to the next itself forms a Markov chain, and the state of this Markov chain affects the transition probabilities in the rain-or-shine Markov chain. This is the state diagram for droughts:



These are the state diagrams for rain given that California is and is not in a drought, respectively:



- (a) Draw a dynamic Bayes net which encodes this behavior. Use variables  $D_{t-1}$ ,  $D_t$ ,  $D_{t+1}$ ,  $R_{t-1}$ ,  $R_t$ , and  $R_{t+1}$ . Assume that on a given day, it is determined whether or not there is a drought before it is determined whether or not it rains that day.

- (b) Draw the CPT for  $D_t$  in the above DBN. Fill in the actual numerical probabilities.

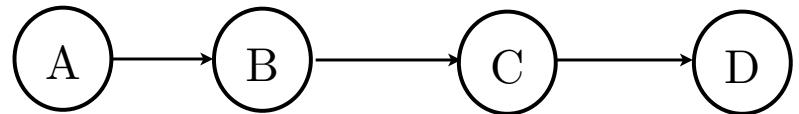
- (c) Draw the CPT for  $R_t$  in the above DBN. Fill in the actual numerical probabilities.

Suppose we are observing the weather on a day-to-day basis, but we cannot directly observe whether California is in a drought or not. We want to predict whether or not it will rain on day  $t + 1$  given observations of whether or not it rained on days 1 through  $t$ .

- (d) First, we need to determine whether California will be in a drought on day  $t + 1$ . Derive a formula for  $P(D_{t+1}|r_{1:t})$  in terms of the given probabilities (the transition probabilities on the above state diagrams) and  $P(D_t|r_{1:t})$  (that is, you can assume we've already computed the probability there is a drought today given the weather over time).
- (e) Now derive a formula for  $P(R_{t+1}|r_{1:t})$  in terms of  $P(D_{t+1}|r_{1:t})$  and the given probabilities.

## Q3. Bayes' Nets Sampling

Assume the following Bayes' net, and the corresponding distributions over the variables in the Bayes' net:



$P(A)$	
$-a$	$3/4$
$+a$	$1/4$

$P(B A)$		
$-a$	$-b$	$2/3$
$-a$	$+b$	$1/3$
$+a$	$-b$	$4/5$
$+a$	$+b$	$1/5$

$P(C B)$		
$-b$	$-c$	$1/4$
$-b$	$+c$	$3/4$
$+b$	$-c$	$1/2$
$+b$	$+c$	$1/2$

$P(D C)$		
$-c$	$-d$	$1/8$
$-c$	$+d$	$7/8$
$+c$	$-d$	$5/6$
$+c$	$+d$	$1/6$

- (a) You are given the following samples:

$$\begin{array}{cccc} +a & +b & -c & -d \\ +a & -b & +c & -d \\ -a & +b & +c & -d \\ -a & -b & +c & -d \end{array} \quad \begin{array}{cccc} +a & -b & -c & +d \\ +a & +b & +c & -d \\ -a & +b & -c & +d \\ -a & -b & +c & -d \end{array}$$

- (i) Assume that these samples came from performing Prior Sampling, and calculate the sample estimate of  $P(+c)$ .

- (ii) Now we will estimate  $P(+c | +a, -d)$ . Above, clearly cross out the samples that would **not** be used when doing Rejection Sampling for this task, and write down the sample estimate of  $P(+c | +a, -d)$  below.

- (b) Using Likelihood Weighting Sampling to estimate  $P(-a | +b, -d)$ , the following samples were obtained. Fill in the weight of each sample in the corresponding row.

Sample	Weight
$-a$ $+b$ $+c$ $-d$	_____
$+a$ $+b$ $+c$ $-d$	_____
$+a$ $+b$ $-c$ $-d$	_____
$-a$ $+b$ $-c$ $-d$	_____

- (c) From the weighted samples in the previous question, estimate  $P(-a | +b, -d)$ .

- (d) Which query is better suited for likelihood weighting,  $P(D | A)$  or  $P(A | D)$ ? Justify your answer in one sentence.

(e) Recall that during Gibbs Sampling, samples are generated through an iterative process.

Assume that the only evidence that is available is  $A = +a$ . Clearly fill in the circle(s) of the sequence(s) below that could have been generated by Gibbs Sampling.

Sequence 1

1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$+d$
3 :	$+a$	$-b$	$+c$	$+d$

Sequence 2

1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$-d$
3 :	$-a$	$-b$	$-c$	$+d$

Sequence 3

1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$-d$
3 :	$+a$	$+b$	$-c$	$-d$

Sequence 4

1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$-d$
3 :	$+a$	$+b$	$-c$	$+d$