CS 188
Summer 2020 MT1 Review

# RL Solutions

## Q1. Reinforcement Learning

**(a)** Answer True/False, and try to explan your answer.

**(i)** [*true* or *false*] Temporal difference learning is an online learning method.
Temporal difference learning is used when we don't have the full MDP model and must collect online samples.

**(ii)** [*true* or *false*] Q-learning: Using an optimal exploration function leads to no regret while learning the optimal policy.

In order to learn the optimal policy, you must explore, and exploring in general has a non-zero chance of regret.

**(iii)** [*true* or *false*] In a deterministic MDP (i.e. one in which each state / action leads to a single deterministic next state), the Q-learning update with a learning rate of $\alpha = 1$ will correctly learn the optimal q-values (assume that all state/action pairs are visited sufficiently often). Remember that the learning rate is only there because we are trying to approximate a summation with a single sample. In a deterministic MDP where $s'$ is the single state that always follows when we take action a in state s, we have $Q(s, a) = R(s, a, s') + \max_{a'} Q(s', a')$, which is exactly the update we make.

**(iv)** [*true* or *false*] A small discount (close to 0) encourages greedy behavior.
A discount close to zero will place extremely small values on rewards more than one step away, leading to greedy behavior that looks for immediate rewards.

**(v)** [*true* or *false*] A large, negative living reward ($\ll 0$) encourages greedy behavior.
A negative living reward adds a penalty for every step taken. If that penalty is large, the agent will prefer to find an exit as soon as possible despite potential rewards on longer paths.

**(vi)** [*true* or *false*] A negative living reward can always be expressed using a discount $< 1$.
While both negative living rewards and discounts can encourage similar behavior, they are mathematically different. A discount has a multiplicative effect at each step, whereas a living reward only has an additive effect.

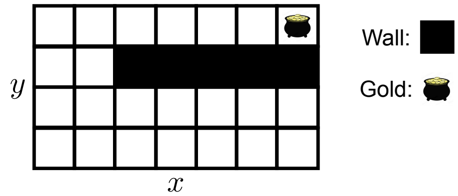
**(vii)** [*true* or *false*] A discount $< 1$ can always be expressed as a negative living reward.
While both negative living rewards and discounts can encourage similar behavior, they are mathematically different. A discount has a multiplicative effect at each step, whereas a living reward only has an additive effect.

**(b)** Given the following table of $Q$-values for the state $A$ and the set of actions $\{Forward, Reverse, Stop\}$, what is the probability that we will take each action on our next move when we following an $\epsilon$-greedy exploration policy (assuming any random movements are chosen uniformly from all actions)?

$Q(A, Forward) = 0.75$
$Q(A, Reverse) = 0.25$
$Q(A, Stop) = 0.5$

| Action | Probability (in terms of $\epsilon$) |
|---|---|
| $Forward$ | $(1-\epsilon) + \frac{\epsilon}{3} = 1 - \frac{2\epsilon}{3}$ |
| $Reverse$ | $\frac{\epsilon}{3}$ |
| $Stop$ | $\frac{\epsilon}{3}$ |

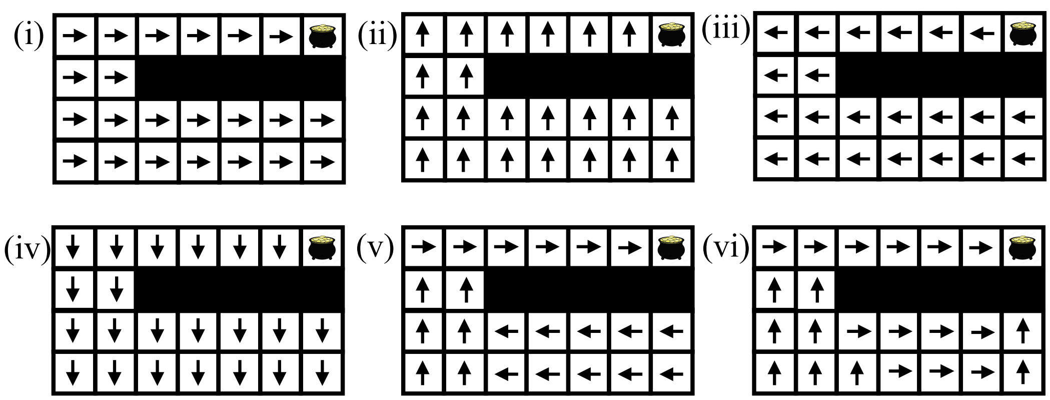
# Q2. MDPs & RL




Consider the grid-world MDP above. The goal of the game is to reach the pot of gold. As soon as you land on the pot of gold you receive a reward and the game ends. Your agent can move around the grid by taking the following actions: North, South, East, West. Moving into a square that is not a wall is always successful. If you attempt to move into a grid location occupied by a wall or attempt to move off the board, you remain in your current grid location.

Our goal is to build a value function that assigns values to each grid location, but instead of keeping track of a separate number for each location, we are going to use features. Specifically, suppose we represent the value of state $(x, y)$ (a grid location) as $V(x, y) = w^\top f(x, y)$. Here, $f(x, y)$ is a feature function that maps the grid location $(x, y)$ to a vector of features and $w$ is a weight vector that parameterizes our value function (note that entries in $w$ can be any real number, positive or negative).

In the next few questions, we will look at various possible feature functions $f(x, y)$. We will think about the value functions that are representable using each set of features, and, further, think about which policies could be extracted from those value functions. Assume that when a policy is extracted from a value function, ties can be broken arbitrarily. In our definition of feature functions we will make use of the location of the pot of gold. Let the gold's location be $(x^*, y^*)$. Keep in mind the policies $(i)$, $(ii)$, $(iii)$, $(iv)$, $(v)$, and $(vi)$ shown below.




**(a)** Suppose we use a single feature: the x-distance to the pot of gold. Specifically, suppose $f(x, y) = |x - x^*|$. Which of the policies could be extracted from a value function that is representable using this feature function? Assume the weights vector $w$ is not allowed to be 0. Fill in all that apply.

● (i) ○ (ii) ● (iii) ○ (iv) ○ (v) ○ (vi)

**(b)** Suppose we use a single feature: the y-distance to the pot of gold. Specifically, suppose $f(x, y) = |y - y^*|$. Which of the policies could be extracted from a value function that is representable using this feature function? Assume the weights vector $w$ is not allowed to be 0. Fill in all that apply.

○ (i) ● (ii) ○ (iii) ● (iv) ○ (v) ○ (vi)

**(c)** Suppose we use a single feature: the Manhattan distance to the pot of gold. Specifically, suppose $f(x, y) = |x - x^*| + |y - y^*|$. Which of the policies could be extracted from a value function that is representable using this feature function? Assume the weights vector $w$ is not allowed to be 0. Fill in all that apply.

○ (i) ○ (ii) ○ (iii) ○ (iv) ○ (v) ● (vi)

**(d)** Suppose we use a single feature: the length of the shortest path to the pot of gold. Which of the policies could be extracted from a value function that is representable using this feature function? Assume the weights vector $w$ is not allowed to be 0. Fill in all that apply.

○ (i) ○ (ii) ○ (iii) ○ (iv) ● (v) ○ (vi)

**(e)** Suppose we use two features: the x-distance to the pot of gold and the y-distance to the pot of gold. Specifically, suppose $f(x, y) = (|x - x^*|, |y - y^*|)$. Which of the policies could be extracted from a value function that is representable using this feature function? Assume the weights vector $w$ must have at least one non-zero entry. Fill in all that apply.

● (i) ● (ii) ● (iii) ● (iv) ○ (v) ● (vi)

# Q3. RL: Amusement Park

After the disastrous waterslide experience you decide to go to an amusement park instead. In the previous questions the MDP was based on a single ride (a water slide). Here our MDP is about choosing a ride from a set of many rides.

You start off feeling well, getting positive rewards from rides, some larger than others. However, there is some chance of each ride making you sick. If you continue going on rides while sick there is some chance of becoming well again, but you don't enjoy the rides as much, receiving lower rewards (possibly negative).

You have never been to an amusement park before, so you don't know how much reward you will get from each ride (while well or sick). You also don't know how likely you are to get sick on each ride, or how likely you are to become well again. What you do know about the rides is:

| Actions / Rides | Type | Wait | Speed |
|---|---|---|---|
| Big Dipper | Rollercoaster | Long | Fast |
| Wild Mouse | Rollercoaster | Short | Slow |
| Hair Raiser | Drop tower | Short | Fast |
| Moon Ranger | Pendulum | Short | Slow |
| Leave the Park | Leave | Short | Slow |

We will formulate this as an MDP with two states, well and sick. Each ride corresponds to an action. The 'Leave the Park' action ends the current run through the MDP. Taking a ride will lead back to the same state with some probability or take you to the other state. We will use a feature based approximation to the Q-values, defined by the following four features and associated weights:

| Features | Initial Weights |
|---|---|
| $f_0(state, action) = 1$ (this is a bias feature that is always 1) | $w_0 = 1$ |
| $f_1(state, action) = \begin{cases} 1 & \text{if } action \text{ type is Rollercoaster} \\ 0 & \text{otherwise} \end{cases}$ | $w_1 = 2$ |
| $f_2(state, action) = \begin{cases} 1 & \text{if } action \text{ wait is Short} \\ 0 & \text{otherwise} \end{cases}$ | $w_2 = 1$ |
| $f_3(state, action) = \begin{cases} 1 & \text{if } action \text{ speed is Fast} \\ 0 & \text{otherwise} \end{cases}$ | $w_3 = 0.5$ |

**(a)** Calculate Q(*'Well'*, *'Big Dipper'*):

$1 + 2 + 0 + 0.5 = 3.5$

**(b)** Apply a Q-learning update based on the sample (*'Well'*, *'Big Dipper'*, *'Sick'*, $-10.5$), using a learning rate of $\alpha = 0.5$ and discount of $\gamma = 0.5$. What are the new weights?

Difference $= -10.5 + 0.5 * max(4, 3.5, 2.5, 2.0, 2.0) - 3.5 = -12$
$w_0 = 1 - 6 * 1 = -5$
$w_1 = 2 - 6 * 1 = -4$
$w_2 = 1 - 6 * 0 = 1$
$w_3 = 0.5 - 6 * 1 = -5.5$

**(c)** Using our approximation, are the Q-values that involve the sick state the same or different from the corresponding Q-values that involve the well state? In other words, is Q(*'Well'*, action) = Q(*'Sick'*, action) for each possible action? Why / Why not? (in just one sentence)

Same
They are the same because we have no features that distinguish between the two states.

Now we will consider the exploration / exploitation tradeoff in this amusement park.

**(d)** Assume we have the original weights from the table on the previous page. What action will an $\epsilon$-greedy approach choose from the well state? If multiple actions could be chosen, give each action and its probability.

With probability $(1 - \epsilon\frac{4}{5})$ we will choose the Wild Mouse. Each other action will be chosen with probability $\frac{\epsilon}{5}$

**(e)** When running Q-learning another approach to dealing with this tradeoff is using an exploration function:

$$f(u, n) = u + \frac{k}{n}$$

**(i)** How is this function used in the Q-learning equations? (a single sentence)
The update replaces the max over Q values with a max over this function (with Q and N as arguments)

What are each of the following? (a single sentence each)

**(ii)** $u$ :
The utility, given by Q

**(iii)** $n$ :
The number of times this state has been visited

**(iv)** $k$ :
A constant, by adjusting it we can change how optimistic we are about states we haven't visited much.