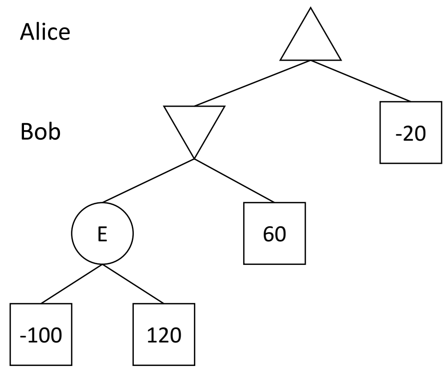
CS 188 Summer 2020 — Introduction to Artificial Intelligence

Final Review

VPI and Decision Nets Solutions

Q1. Value of Asymmetric Information

Alice and Bob are playing an adversarial game as shown in the game tree below. Alice (the MAX player) and Bob (the MIN player) are both rational and they both know that their opponent is also a rational player. The game tree has one chance node E whose outcome can be either $E = -100$ or $E = +120$ with equal 0.5 probability.



Each player's utility is equal to the amount of money he or she has. The value x of each leaf node in the game tree means that Bob will pay Alice x dollars after the game, so that Alice and Bob's utilities will be x and $-x$ respectively.

(a) Suppose neither Alice nor Bob knows the outcome of E before playing. What is Alice's expected utility?

Answer: 10 — E's expectation is 10. Using minimax, both Alice and Bob should go left.

(b) Carol, a good friend of Alice's, has access to E and can *secretly* tell Alice the outcome of E before the game starts (giving Alice the true outcome of E without lying). However, Bob is not aware of any communication between Alice and Carol, so he still assumes that Alice has no access to E.

(i) Suppose Carol secretly tells Alice that $E = -100$. What is Alice's expected utility in this case?

Answer: -20 — Here, Bob will still go left as before since he isn't aware of Alice's access to E. Given this, Alice should now choose to go right when $E = -100$.

(ii) Suppose Carol secretly tells Alice that $E = +120$. What is Alice's expected utility in this case?

Answer: 120 — Here, Bob will still go left as before since he isn't aware of Alice's access to E. Given this, Alice should now choose to go left when $E = +120$.

(iii) What is Alice's expected utility if Carol secretly tells Alice the outcome of E before playing?

Answer: 50 — E is equally likely to be -100 or $+120$. Averaging the two cases above, $-20 * 0.5 + 120 * 0.5 = 50$.

We define the *value of private information* $V_A^{\text{pri}}(X)$ of a random variable X to a player A as the difference in player A's expected utility after the outcome of X becomes a private information to player A, such that A has access to the outcome of X, while other players have no access to X and are not aware of A's access to X.

(iv) In general, the value of private information $V_A^{\text{pri}}(X)$ of a variable X to a player A

- [ ] always satisfies $V_A^{pri}(X) > 0$ in all cases.
- [x] always satisfies $V_A^{pri}(X) \geq 0$ in all cases.
- [ ] always satisfies $V_A^{pri}(X) = 0$ in all cases.
- [ ] can possibly satisfy $V_A^{pri}(X) < 0$ in certain cases.

Since player A can always choose to ignore this information and act in the same way as if he/she doesn't know this information, player A is guaranteed to obtain at least the same utility as before, so $V_A^{pri}(X) \geq 0$.

(v) What is $V_{Alice}^{pri}(E)$, the value of private information of E to Alice in the specific game tree above?

Answer: 40 Subtracting the answer of (a) from the answer of (b, iii), $50 - 10 = 40$.

(c) David also has access to E, and can make a *public* announcement of E (announcing the true outcome of E without lying), so that both Alice and Bob will know the outcome of E and are both aware that their opponent also knows the outcome of E. Also, Alice cannot obtain any information from Carol now.

(i) Suppose David publicly announces that $E = -100$. What is Alice's expected utility in this case?

Answer: -20 Using minimax with $E = -100$, Bob will go left and Alice will go right.

(ii) Suppose David publicly announces that $E = +120$. What is Alice's expected utility in this case?

Answer: 60 Using minimax with $E = +120$, Bob will go right and Alice will go left.

(iii) What is Alice's expected utility if David makes a public announcement of E before the game starts?

Answer: 20 E is equally likely to be -100 or $+120$. Averaging the two cases above, $-20 * 0.5 + 60 * 0.5 = 20$.

We define the *value of public information* $V_A^{pub}(X)$ of a random variable X to a player A as the difference in player A's expected utility after the outcome of X becomes a public information, such that everyone has access to the outcome of X and is aware that all other players also have access to X.

(iv) In general, the value of public information $V_A^{pub}(X)$ of a variable X to a player A

- [ ] always satisfies $V_A^{pub}(X) > 0$ in all cases.
- [ ] always satisfies $V_A^{pub}(X) \geq 0$ in all cases.
- [ ] always satisfies $V_A^{pub}(X) = 0$ in all cases.
- [x] can possibly satisfy $V_A^{pub}(X) < 0$ in certain cases.

Player A's utility may decrease if the outcome of X becomes a public information, since other players can now exploit this information to better play against player A, especially in an adversarial setting.

(v) What is $V_{Alice}^{pub}(E)$, the value of public information of E to Alice in the specific game tree above?

Answer: 10 Subtracting the answer of (a) from the answer of (c, iii), $20 - 10 = 10$.

(vi) Let $a = V_{Alice}^{pub}(E)$ be the value of public information of E to Alice. Suppose David will publicly announce the outcome of E if anyone (either Alice or Bob) pays him b dollars ($b > 0$), and will make no announcement otherwise. Which of the following statements are True?

- [x] The value of public information of E to Bob is $V_{Bob}^{pub}(E) = -a$.
- [x] If $b < a$, then Alice should pay David b dollars.
- [ ] If $b > a$, then Bob should pay David b dollars.

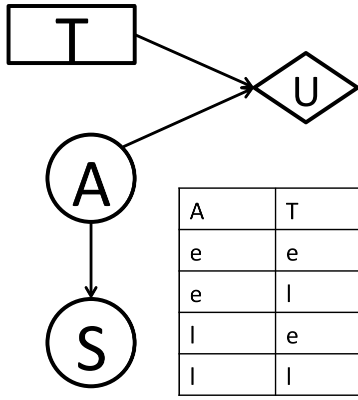
- [x] If $b < -a$, then Bob should pay David b dollars.
- [ ] If $b > -a$, then Alice should pay David b dollars.
- [ ] There exists some value $b > 0$ such that both Alice and Bob should pay David b dollars.
- [x] There exists some value $b > 0$ such that neither Alice nor Bob should pay David b dollars.

Since Alice and Bob's utilities always sum up to zero, if Alice's utility increases by a after the outcome of E becomes a public information, then Bob's utility will certainly decrease by a, so $V_{\text{Bob}}^{\text{pub}}(E) = -a$.

Alice should pay when $b < V_{\text{Alice}}^{\text{pub}}(E) = a$, and Bob should pay when $b < V_{\text{Bob}}^{\text{pub}}(E) = -a$, which cannot happen simultaneously since $b > 0$. When b is large enough ($b > |a|$), then neither Alice nor Bob should pay for the announcement.

Q2. Probability and Decision Networks

A	P(A)
e	0.5
l	0.5



S	P(S)
e	0.6
l	0.4

A	S	P(S|A)
e	e	0.8
e	l	0.2
l	e	0.4
l	l	0.6

A	T	U(A,T)
e	e	600
e	l	0
l	e	300
l	l	600

S	A	P(A|S)
e	e	2/3
e	l	1/3
l	e	1/4
l	l	3/4

Your parents are visiting you for graduation. You are in charge of picking them up at the airport. Their arrival time (A) might be early (e) or late (l). You decide on a time (T) to go to the airport, also either early (e) or late (l). Your sister (S) is a noisy source of information about their arrival time. The probability values and utilities are shown in the tables above.

(a) Compute $P(S)$, $P(A|S)$ and compute the quantities below.

$EU(T = e) = P(A = e)U(A = e, T = e) + P(A = l)U(A = l, T = e) = 0.5 * 600 + 0.5 * 300 = 450$

$EU(T = l) = P(A = e)U(A = e, T = l) + P(A = l)U(A = l, T = l) = 0.5 * 0 + 0.5 * 600 = 300$

$MEU(\{\}) = 450$

Optimal action with no observations is $T = e$

(b) Now we consider the case where you decide to ask your sister for input.

$EU(T = e|S = e) = P(A = e|S = e)U(A = e, T = e) + P(A = l|S = e)U(A = l, T = e) = \frac{2}{3}600 + \frac{1}{3}300 = 500$

$EU(T = l|S = e) = P(A = E|S = e)U(A = e, T = l) + P(A = l|S = e)U(A = l, T = l) = \frac{2}{3}0 + \frac{1}{3}600 = 200$

$MEU(\{S = e\}) = 500$

Optimal action with observation $\{S = e\}$ is $T = e$

(c) $EU(T = e|S = l) = P(A = e|S = l)U(A = e, T = e) + P(A = l|S = e)U(A = l, T = e) = \frac{1}{4}600 + \frac{3}{4}300 = 375$

$EU(T = l|S = l) = P(A = e|S = l)U(A = e, T = l) + P(A = l|S = l)U(A = l, T = l) = \frac{1}{4}0 + \frac{3}{4} * 600 = 450$

$MEU(\{S = l\}) = 450$

Optimal action with observation $S = l$ is $T = l$

$VPI(S) = P(S = e)MEU(\{S = e\}) + P(S = l)MEU(\{S = l\}) - MEU(\{\}) = 0.6 * 500 + 0.4 * 450 - 450 = 30$