Q1. Search

Each True/False question is worth 1 points. Leaving a question blank is worth 0 points. **Answering incorrectly is worth −1 points.**

(a) Consider a graph search problem where for every action, the cost is at least ϵ, with $\epsilon > 0$. Assume the used heuristic is consistent.

(i) [*true* or *false*] Depth-first graph search is guaranteed to return an optimal solution.
False. Depth first search has no guarantees of optimality. Further, it measures paths in length and not cost.

(ii) [*true* or *false*] Breadth-first graph search is guaranteed to return an optimal solution.
False. Breadth first search has no guarantees of optimality unless the actions all have the same cost, which is not the case here.

(iii) [*true* or *false*] Uniform-cost graph search is guaranteed to return an optimal solution.
True. UCS expands paths in order of least total cost so that the optimal solution is found.

(iv) [*true* or *false*] Greedy graph search is guaranteed to return an optimal solution.
False. Greedy search makes no guarantees of optimality. It relies solely on the heuristic and not the true cost.

(v) [*true* or *false*] A* graph search is guaranteed to return an optimal solution.
True, since the heuristic is consistent in this case.

(vi) [*true* or *false*] A* graph search is guaranteed to expand no more nodes than depth-first graph search.

False. Depth-first graph search could, for example, go directly to a sub-optimal solution.

(vii) [*true* or *false*] A* graph search is guaranteed to expand no more nodes than uniform-cost graph search.
True. The heuristic could help to guide the search and reduce the number of nodes expanded. In the extreme case where the heuristic function returns zero for every state, A* and UCS will expand the same number of nodes. In any case, A* with a consistent heuristic will never expand more nodes than UCS.

(b) Let $h_1(s)$ be an admissible A* heuristic. Let $h_2(s) = 2h_1(s)$. Then:

(i) [*true* or *false*] The solution found by A* tree search with h_2 is guaranteed to be an optimal solution.

False. h_2 is not guaranteed to be admissible since only one side of the admissibility inequality is doubled.

(ii) [*true* or *false*] The solution found by A* tree search with h_2 is guaranteed to have a cost at most twice as much as the optimal path.
True. In A* tree search we always have that as long as the optimal path to the goal has not been found, a prefix of this optimal path has to be on the fringe. Hence, if a non-optimal solution is found, then at time of popping the non-optimal path from the fringe, a path that is a prefix of the optimal path to the goal is sitting on the fringe. The cost $\bar{g}$ of a non-optimal solution when popped is its f-cost. The prefix of the optimal path to the goal has an f-cost of $g + h_0 = g + 2h_1 \leq 2(g + h_1) \leq 2C^*$, with C^* the optimal cost to the goal. Hence we have that $\bar{g} \leq 2C^*$ and the found path is at most twice as long as the optimal path.

(iii) [*true* or *false*] The solution found by A* graph search with h_2 is guaranteed to be an optimal solution.

False. h_2 is not guaranteed to be admissible and graph search further requires consistency for optimality.

(c) The heuristic values for the graph below are not correct. For which single state (S, A, B, C, D, or G) could you change the heuristic value to make everything admissible and consistent? What range of values are possible to make this correction?

State: | B | Range: | [2,3] |
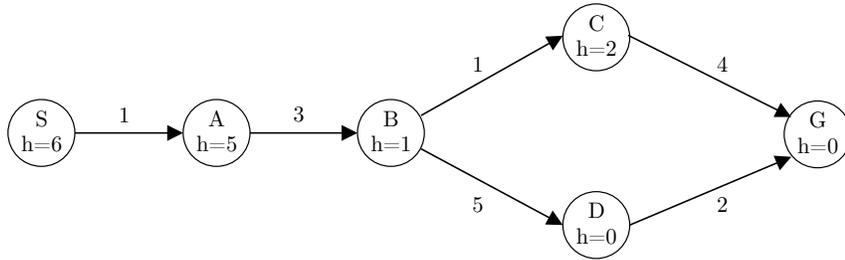
Q2. Search Algorithms Potpourri

(a) We will investigate various search algorithms for the following graph. Edges are labeled with their costs, and heuristic values h for states are labeled next to the states. S is the start state, and G is the goal state. In all search algorithms, assume ties are broken in alphabetical order.
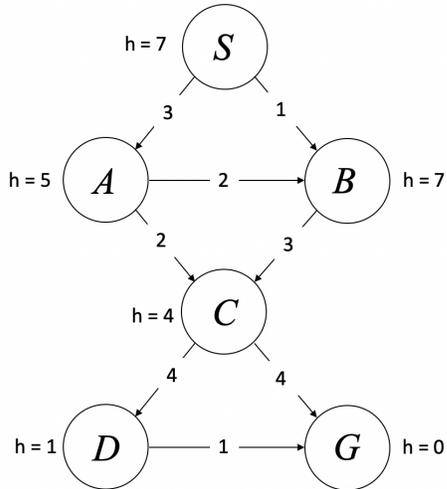
(i) Select all boxes that describe the given heuristic values.

■ admissible   ■ consistent   □ Neither

(ii) Given the above heuristics, what is the order that the states are going to be expanded in, assuming we run A* graph search with the heuristic values provided.

Index	1	2	3	4	5	Not Expanded
S	●	○	○	○	○	○
A	○	●	○	○	○	○
B	○	○	●	○	○	○
C	○	○	○	●	○	○
D	○	○	○	○	○	●
G	○	○	○	○	●	○

(iii) Assuming we run A* graph search with the heuristic values provided, what path is returned?
○ $S \to A \to B \to C \to D \to G$ ○ $S \to A \to C \to G$ ○ $S \to A \to C \to D \to G$
● $S \to B \to C \to G$ ○ $S \to A \to C \to D \to G$ ○ $S \to A \to C \to D \to G$
○ $S \to A \to B \to C \to G$ ○ None of the above

(iv) Given the above heuristics, what is the order that the states are going to be expanded in, assuming we run greedy graph search with the heuristic values provided.

Index	1	2	3	4	5	Not Expanded
S	●	○	○	○	○	○
A	○	●	○	○	○	○
B	○	○	○	○	○	●
C	○	○	●	○	○	○
D	○	○	○	○	○	●
G	○	○	○	●	○	○

(v) What path is returned by greedy graph search?
- ○ $S \to A \to B \to C \to D \to G$ ● $S \to A \to C \to G$ ○ $S \to A \to C \to D \to G$
- ○ $S \to A \to C \to D \to G$ ○ $S \to A \to C \to D \to G$ ○ None of the above

(b) Consider a complete graph, K_n, the undirected graph with n vertices where all n vertices are connected (there is an edge between every pair of vertices), resulting in $\binom{n}{2}$ edges. Please select the maximum possible depth of the resulting tree when the following **graph** search algorithms are run(assume any possible start and goal vertices).

	1	$\lceil \frac{n}{2} \rceil$	$n-1$	$\binom{n}{2}$	None of the above
BFS	●	○	○	○	○
DFS	○	○	●	○	○

(c) Given two admissible heuristics h_A and h_B.

(i) Which of the following are guaranteed to also be admissible heuristics?

- ☐ $h_A + h_B$ ■ $\frac{1}{2}(h_A)$ ■ $\frac{1}{2}(h_B)$ ■ $\frac{1}{2}(h_A + h_B)$ ☐ $h_A * h_B$ ■ $max(h_A, h_B)$
- ■ $min(h_A, h_B)$

(ii) Consider performing A* **tree** search. Which is generally best to use if we want to expand the fewest number of nodes? Note this was changed from graph to tree search during the exam

- ○ $h_A + h_B$ ○ $\frac{1}{2}(h_A)$ ○ $\frac{1}{2}(h_B)$ ○ $\frac{1}{2}(h_A + h_B)$ ○ $h_A * h_B$ ● $max(h_A, h_B)$
- ○ $min(h_A, h_B)$

(d) Consider performing tree search for some search graph. Let $depth(n)$ be the depth of search node n and $cost(n)$ be the total cost from the start state to node n. Let G_d be a goal node with minimum depth, and G_c be a goal node with minimum total cost.

(i) For iterative deepening (where we repeatedly run DFS and increase the maximum depth allowed by 1), mark all conditions that are guaranteed to be true for every node n that could be expanded during the search, or mark "None of the above" if none of the conditions are guaranteed.

- ☐ $cost(n) \leq cost(G_c)$
- ☐ $cost(n) \leq cost(G_d)$
- ■ $depth(n) \leq depth(G_c)$
- ■ $depth(n) \leq depth(G_d)$
- ○ None of the above

When running iterative deepening we will explore all nodes of depth k, before we explore any nodes of depth k + 1. As a result will never explore any nodes that have depth greater than G_d because we would stop exploring once we reached G_d. This also means we would never explore any nodes with depth greater than G_c because G_c has to have depth greater than or equal to the minimum depth goal, G_d

(ii) What is necessarily true regarding iterative deepening on any search tree?

- ■ Complete as opposed to DFS tree search
- ☐ Strictly faster than DFS tree search
- ☐ Strictly faster than BFS tree search
- ■ More memory efficient than BFS tree search

☐ A type of stochastic local search
◯ None of the above

Q3. They See Me Rolling (Search Problem)

Pacman buys a car to start Rolling in the Pac-City! But driving a car requires a new set of controls because he can now travel faster than 1 grid per turn (second). Instead of solely moving [North, South, East, West, Stop], Pacman's car has two distinct integers to control: throttle, and steering.

Throttle: $t_i \in \{1, 0, -1\}$, corresponding to {Gas, Coast, Brake}. This controls the **speed** of the car by determining its acceleration. The integer chosen here will be added to his velocity for the next state. For example, if Pacman is currently driving at 5 grid/s and chooses Gas he will be traveling at 6 grid/s in the next turn.
Steering: $s_i \in \{1, 0, -1\}$, corresponding to {Turn Left, Neutral, Turn Right}. This controls the **direction** of the car. For example, if he is facing North and chooses Turn Left he will be facing West in the next turn.

(a) Suppose Pac-city has dimension m by n, but only $k < mn$ squares are legal roads. The speed limit of Pac-city is 3 grid/s. For this sub-part only, suppose Pacman is a law-abiding citizen, so $0 \leq v \leq 3$ at all time, and he only drives on legal roads.

  (i) Without any additional information, what is the **tightest upper bound** on the size of state space, if he wants to search a route (not necessarily the shortest) from his current location to anywhere in the city. Please note that your state space representation must be able to represent **all** states in the search space.

    ○ $4mn$ ○ $4k$ ○ $12mn$ ○ $12k$ ○ $16mn$ ● $16k$ ○ $48mn$ ○ $48k$

    Only legal grids count, so there are k legal position. At each legal position, there are 4 possible speed (0, 1, 2, 3), so a factor of 4 is multiplied. In addition, since change of direction depends on orientation of the car, another factor of 4 is multiplied. The size of state space is bounded by $k * 4 * 4 = 16k$

  (ii) What is the maximum branching factor? The answer should be an integer.

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    3 possible throttle inputs, and 3 possible steering inputs.

  (iii) Which algorithm(s) is/are guaranteed to return a path between two points, if one exists?

    ☐ Depth First Tree Search   ■ Breadth First Tree Search
    ■ Depth First Graph Search   ■ Breadth First Graph Search

  (iv) Is Breadth First Graph Search guaranteed to return the path with the shortest grid distance?
    ○ Yes   ● No

    The Breadth First Graph Search is guranteed to return the path with the shortest amount of time, because each edge here represent moving for 1 unit of time.

(b) Now let's remove the constraint that Pacman follows the speed limit. Now Pacman's speed is limited by the mechanical constraints of the car, which is 6 grid/s, double the speed limit.

Pacman is now able to drive twice as fast on the route to his destination. How do the following properties of the search problem change as a result of being able to drive twice as fast?

  (i) Size of State Space:
    ● Increases   ○ Stays the same   ○ Decreases
    At each legal position, there are 7 possible speed (0,1, 2, 3, 4, 5, 6), so a factor of 7 is multiplied. size of state space is now k * 7 * 4 = 28k

  (ii) Maximum Branching Factor:
    ○ Increases   ● Stays the same   ○ Decreases
    Branching factor is independent of top speed

For the following part, **mark all choices that could happen on any graph**

(iii) The number of nodes expanded with **Depth** First Graph Search:

■ Increases   ■ Stays the same   ■ Decreases

Anything can happen with Depth First Graph Search. For example, too fast can jump past an intersection that would lead to a shorter path to the goal

(c) Now we need to consider that there are $p > 0$ police cars waiting at $p > 0$ distinct locations trying to catch Pacman riding dirty!! All police cars are stationary, but once Pacman takes an action which lands him in the same grid as one police car, Pacman will be arrested and the game ends.

Pacman wants to find a route to his destination, without being arrested. How do the following properties of the search problem change as a result of avoiding the police?

- **(i)** Size of State Space:
  ◯ Increases   ⬤ Stays the same   ◯ Decreases
  The size of statespace is still the same because all the factors in state space calculation is the same

- **(ii)** Maximum Branching Factor:
  ◯ Increases   ⬤ Stays the same   ◯ Decreases
  Branching factor is independent of police presense

For the following part, **mark all choices that could happen on any graph**

- **(iii)** Number of nodes expanded with **Breadth** First Graph Search:
  ◼ Increases   ◼ Stays the same   ◼ Decreases
  Again anything could happen with BFS. Policemen could block out misleading paths to decrease the number of expansion. They could also not affect anything. They could also block the closest goal causing you to explore more nodes.