CS 188 Summer 2022 Introduction to Artificial Intelligence — Written HW 3

Due: Wednesday 07/13/2022 at 11:59pm (submit via Gradescope).

Policy: Can be solved in groups (acknowledge collaborators) but must be written up individually

Submission: It is recommended that your submission be a PDF that matches this template. You may also fill out this template digitally (e.g. using a tablet). **However, if you do not use this template, you will still need to write down the below four fields on the first page of your submission.**

First name	
Last name	
SID	
Collaborators	

For staff use only:

Q1.	Probability Review	/30
Q2.	Quadcopter: Spectator	/30
	Total	/60

Q1. [30 pts] Probability Review

This question is meant to review part of the probability prerequisite.

Let A, B, C, D be four random variables.

(a) What is the smallest set of independence or conditional independence relationships we need to assume for the following scenarios?

(i) [1 pt] $P(A, B) = P(A|B)P(B)$

(ii) [1 pt] $P(A, B) = P(A)P(B)$

(iii) [2 pts] $P(A, B, C) = P(A|B)P(B|C)P(C)$

(iv) [3 pts] $P(A, B, C) = P(A)P(B|C)P(C)$

(v) [3 pts] $P(A, B, C) = P(A)P(B)P(C)$

(b) Simplify the following expressions to one probability expression. Please show your work.

(i) [3 pts] $\frac{P(A,B)}{\sum_a P(a,B)}$

(ii) [3 pts] $\frac{P(A,B,C,D)}{\sum_a \sum_b P(a,b,C,D)}$

(iii) [4 pts] $\frac{P(A,C,D|B)}{P(C,D|B)}$

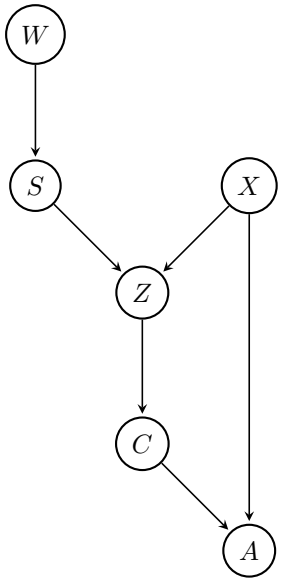
(iv) [4 pts] $\frac{P(A|B)}{\sum_c P(c|B)}$

(v) [6 pts]
$\frac{\sum_b P(A,b|C)P(D|A,b,C)}{P(A|B,C)}$, given $A \perp\!\!\!\perp B|C$

Q2. [30 pts] Quadcopter: Spectator

Flying a quadcopter can be modeled using a Bayes Net with the following variables:

- W (weather) $\in$ {clear, cloudy, rainy}
- S (signal strength) $\in$ {strong, medium, weak}
- X (true position) $= (x, y, z, \theta)$ where x, y, z **each** can take on values $\in \{0, 1, 2, 3, 4\}$ and θ can take on values $\in \{0°, 90°, 180°, 270°\}$
- Z (reading of the position) $= (x, y, z, \theta)$ where x, y, z **each** can take on values $\in \{0, 1, 2, 3, 4\}$ and θ can take on values $\in \{0°, 90°, 180°, 270°\}$
- C (control from the pilot) $\in$ {forward, backward, rotate left, rotate right, ascend, descend} (6 controls in total)
- A (smart alarm to warn pilot if that control could cause a collision) $\in$ {bad, good}



(a) Representation

(i) [3 pts] What is N_x, where N_x is the domain size of the variable X? Please explain your answer.

Answer: $N_x =$

Explanation:

(ii) [4 pts] Please list **all** of the Conditional Probability Tables that are needed in order to represent the Bayes Net above. Note that there are 6 of them.

(iii) [3 pts] What is the size of the Conditional Probability Table for Z? You may use N_x in your answer.

Now, assume that we look at this setup from the perspective of Spencer – a spectator who can observe A and W. Spencer observes A=bad and W=clear, and he now wants to infer the signal strength. In BN terminology, he wants to calculate $P(S|A = \text{bad}, W = \text{clear})$.

(b) [5 pts] Inference by Enumeration

If Spencer chooses to solve for this quantity using inference by enumeration, what are the different probability terms that need to be multiplied together in the summation?

(c) [15 pts] Inference by Variable Elimination

Spencer chooses to solve for this quantity by performing variable elimination in the order of $Z - X - C$. Answer the following prompts to work your way through this procedure.

(1a) First, we need to eliminate Z. Which factors (from the 6 CPTs above) are involved?

(1b) Describe how you eliminate the variable of interest by multiplication of those factors. What conditional probability **factor** results from this step?

(2a) Second, we need to eliminate X. Which factors are involved?

(2b) Describe how you eliminate the variable of interest by multiplication of those factors. What conditional probability **factor** results from this step?

(3a) Third, we need to eliminate C. Which factor/s are involved?

(3b) Describe how you eliminate the variable of interest by multiplication of those factors. What conditional probability **factor** results from this step?

(4) List the 3 conditional probability factors that you calculated as a result of the 3 elimination steps above, along with their domain sizes. You may use N_x in your answer. Which factor is the biggest? Is this bigger or smaller than the biggest factor from the "inference by enumeration" approach?

(5) List the **1** unused conditional probability factor from the 3 that you calculated above, and also list the **2** remaining conditional probability factors from the 6 original CPTs.

(6) Finally, let's solve for the original quantity of interest: $P(S|A = \text{bad}, W = \text{clear})$. After writing the equations to show how to use the factors from (5) in order to solve for $f(S|A = bad, W = clear)$, don't forget to write how to turn that into a probability $P(S|A = bad, W = clear)$.
Hint: use the definition of conditional probability, and use the 3 resulting factors that you listed in the previous question.