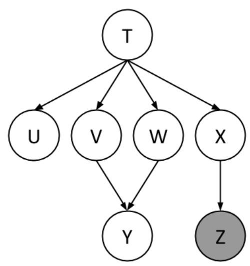
CS 188
Summer 2022

Regular Discussion 4A Solutions

1 Variable Elimination

Using the same Bayes Net (shown below), we want to compute $P(Y \mid +z)$. All variables have binary domains. Assume we run variable elimination to compute the answer to this query, with the following variable elimination ordering: X, T, U, V, W.



Complete the following description of the factors generated in this process:

After inserting evidence, we have the following factors to start out with:

$$P(T), P(U|T), P(V|T), P(W|T), P(X|T), P(Y|V,W), P(+z|X)$$

(a) When eliminating X we generate a new factor f_1 as follows, which leaves us with the factors:

$$f_1(+z|T) = \sum_x P(x|T)P(+z|x) \qquad P(T), P(U|T), P(V|T), P(W|T), P(Y|V,W), f_1(+z|T)$$

(b) When eliminating T we generate a new factor f_2 as follows, which leaves us with the factors:

$$f_2(U,V,W,+z) = \sum_t P(t)P(U|t)P(V|t)P(W|t)f_1(+z|t) \qquad P(Y|V,W), f_2(U,V,W,+z)$$

(c) When eliminating U we generate a new factor f_3 as follows, which leaves us with the factors:

$$f_3(V,W,+z) = \sum_u f_2(u,V,W,+z) \qquad P(Y|V,W), f_3(V,W,+z)$$

Note that U could have just been deleted from the original graph, because $\sum_u P(U|t) = 1$. We can see this in the graph: we can remove any leaf node that is not a query variable or an evidence variable.

(d) When eliminating V we generate a new factor f_4 as follows, which leaves us with the factors:

$$f_4(W, Y, +z) = \sum_v f_3(v, W, +z) P(Y|v, W) \qquad f_4(W, Y, +z)$$

(e) When eliminating W we generate a new factor f_5 as follows, which leaves us with the factors:

$$f_5(Y, +z) = \sum_w f_4(w, Y, +z) \qquad f_5(Y, +z)$$

(f) How would you obtain $P(Y \mid +z)$ from the factors left above:
Simply renormalize $f_5(Y, +z)$ to obtain $P(Y \mid +z)$. Concretely,

$$P(y \mid +z) = \frac{f_5(y, +z)}{\sum_{y'} f_5(y', +z)}$$

(g) What is the size of the largest factor that gets generated during the above process?
$f_2(U, V, W, +z)$. This contains 3 unconditioned variables, so it will have $2^3 = 8$ probability entries (U, V, W are binary variables, and we only need to store the probability for $+z$ for each possible setting of these variables).

(m) Does there exist a better elimination ordering (one which generates smaller largest factors)?
Yes. One such ordering is X, U, T, V, W. All factors generated with this ordering contain at most 2 unconditioned variables, so the tables will have at most $2^2 = 4$ probability entries (as all variables are binary).