CS 188
Summer 2022

Regular Discussion 5B

Q1. VPI

You are the latest contestant on Monty Hall's game show. In the game, there are n closed doors: behind one door is a car ($U(car) = 1000$), while the other $n - 1$ doors each have a goat behind them ($U(goat) = 10$). You are permitted to open exactly one door and claim the prize behind it. You begin by choosing a door uniformly at random.

(a) What is your expected utility?

(b) After you choose a door but before you open it, Monty offers to open k other doors, each of which are guaranteed to have a goat behind it. If you accept this offer, should you keep your original choice of a door, or switch to a new door?

$EU(keep)$:

$EU(switch)$:

Action that achieves MEU:

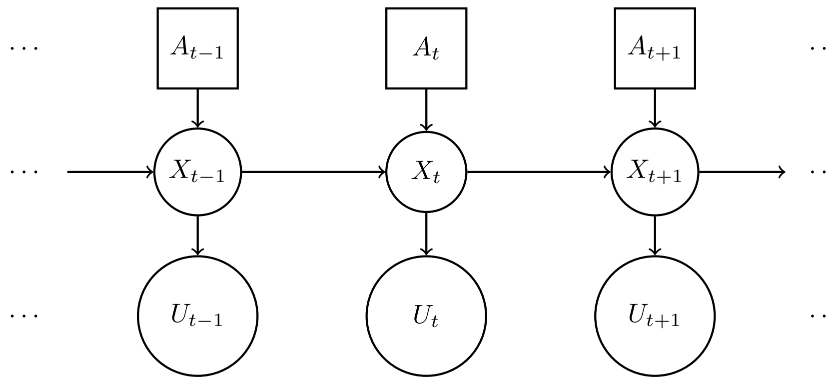
(c) What is the value of the information that Monty is offering you?

(d) Monty is changing his offer! After you choose your initial door, you are given the offer to choose any other door and open this second door. If you do, after you see what is inside the other door, you may switch your initial choice (to the newly opened door) or keep your initial choice. What is the value of this new offer?

(e) Monty is generalizing his offer: you can pay $\$d^3$ to open d doors as in the previous part. (Assume that $U(\$x) = x$.) You may now switch your choice to any of the open doors (or keep your initial choice). What is the largest value of d for which it would be rational to accept the offer?

Q2. Planning ahead with HMMs

Pacman is tired of using HMMs to estimate the location of ghosts. He wants to use HMMs to plan what actions to take in order to maximize his utility. Pacman uses the HMM (drawn to the right) of length T to model the planning problem. In the HMM, $X_{1:T}$ is the sequence of hidden states of Pacman's world, $A_{1:T}$ are actions Pacman can take, and U_t is the utility Pacman receives at the particular hidden state X_t. Notice that there are no evidence variables, and utilities are not discounted.



(a) The belief at time t is defined as $B_t(X_t) = p(X_t|a_{1:t})$. The forward algorithm update has the following form:

$$B_t(X_t) = \underline{\quad\textbf{(i)}\quad}\ \underline{\quad\textbf{(ii)}\quad}\ B_{t-1}(x_{t-1}).$$

Complete the expression by choosing the option that fills in each blank.

(i)	○ $\max_{x_{t-1}}$	○ $\sum_{x_{t-1}}$	○ $\max_{x_t}$	○ $\sum_{x_t}$	○ 1
(ii)	○ $p(X_t\|x_{t-1})$	○ $p(X_t\|x_{t-1})p(X_t\|a_t)$	○ $p(X_t)$	○ $p(X_t\|x_{t-1}, a_t)$	○ 1

○ None of the above combinations is correct

(b) Pacman would like to take actions $A_{1:T}$ that maximizes the expected sum of utilities, which has the following form:

$$\text{MEU}_{1:T} = \underline{\ \textbf{(i)}\ }\ \underline{\ \textbf{(ii)}\ }\ \underline{\ \textbf{(iii)}\ }\ \underline{\ \textbf{(iv)}\ }\ \underline{\ \textbf{(v)}\ }$$

Complete the expression by choosing the option that fills in each blank.

(i)	○ $\max_{a_{1:T}}$	○ $\max_{a_T}$	○ $\sum_{a_{1:T}}$	○ $\sum_{a_T}$	○ 1
(ii)	○ $\max_t$	○ $\prod_{t=1}^{T}$	○ $\sum_{t=1}^{T}$	○ $\min_t$	○ 1
(iii)	○ $\sum_{x_t, a_t}$	○ $\sum_{x_t}$	○ $\sum_{a_t}$	○ $\sum_{x_T}$	○ 1
(iv)	○ $p(x_t\|x_{t-1}, a_t)$	○ $p(x_t)$	○ $B_t(x_t)$	○ $B_T(x_T)$	○ 1
(v)	○ U_T	○ $\frac{1}{U_t}$	○ $\frac{1}{U_T}$	○ U_t	○ 1

○ None of the above combinations is correct

(c) A greedy ghost now offers to tell Pacman the values of some of the hidden states. Pacman needs your help to figure out if the ghost's information is useful. Assume that the transition function $p(x_t|x_{t-1}, a_t)$ is not deterministic. **With respect to the utility** U_t, mark all that can be True:

☐ $\text{VPI}(X_{t-1}|X_{t-2}) > 0$
☐ $\text{VPI}(X_{t-2}|X_{t-1}) > 0$
☐ $\text{VPI}(X_{t-1}|X_{t-2}) = 0$
☐ $\text{VPI}(X_{t-2}|X_{t-1}) = 0$
☐ None of the above

(d) Pacman notices that calculating the beliefs under this model is very slow using exact inference. He therefore decides to try out various particle filter methods to speed up inference. Order the following methods by how accurate their estimate of $B_T(X_T)$ is? If different methods give an equivalently accurate estimate, mark them as the same number.

	Most accurate			Least accurate
Exact inference	◯ 1	◯ 2	◯ 3	◯ 4
Particle filtering with no resampling	◯ 1	◯ 2	◯ 3	◯ 4
Particle filtering with resampling before every time elapse	◯ 1	◯ 2	◯ 3	◯ 4
Particle filtering with resampling before every other time elapse	◯ 1	◯ 2	◯ 3	◯ 4