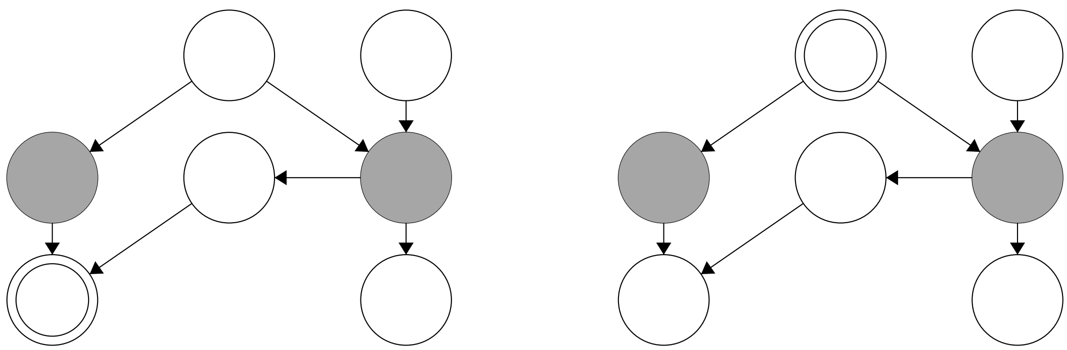
CS 188
Summer 2022

Midterm Review Bayes Nets

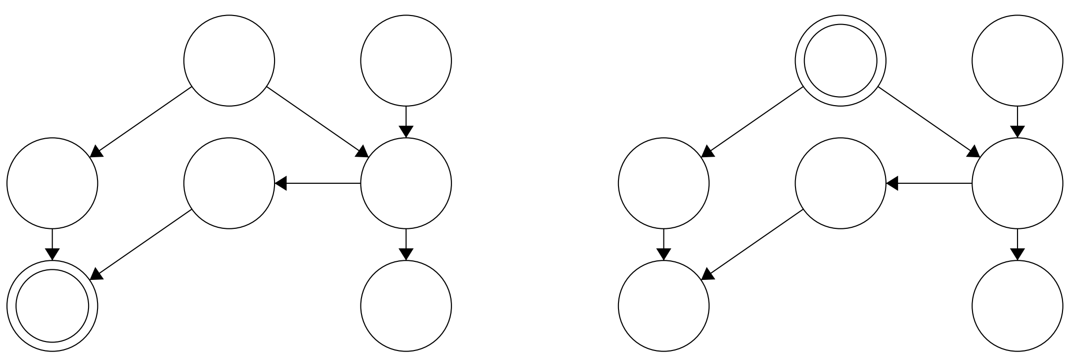
Q1. Moral Graphs

(a) For each of the following queries, we want to preprocess the Bayes net before performing variable elimination. Query variables are double-circled and evidence variables are shaded. Cross off all the variables that we can ignore in performing the query. If no variables can be ignored in one of the Bayes nets, write "None" under that Bayes net.



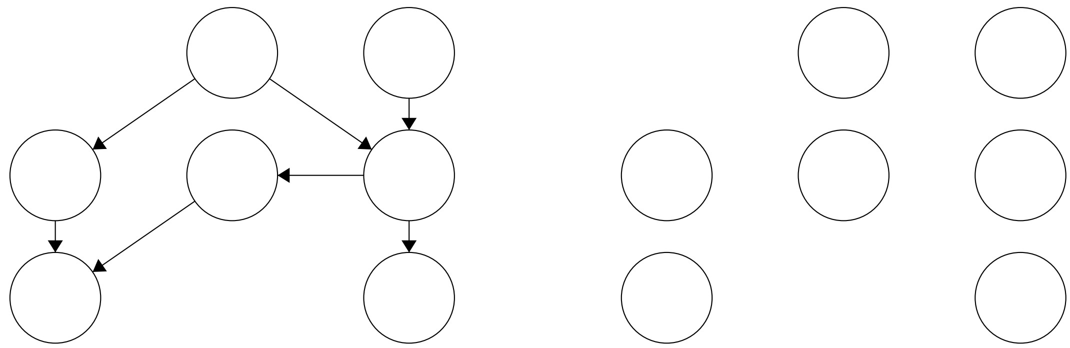
Let B be a Bayes net with a set of variables V. The **Markov blanket** of a variable $v \in V$ is the smallest set of variables $S \subset V$ such that for any variables $v' \in V$ such that $v \neq v'$ and $v' \notin S$, $v \perp\!\!\!\perp v' | S$. Less formally, v is independent from the entire Bayes net given all the variables in S.

(b) In each of the following Bayes nets, shade in the Markov blanket of the double-circled variable.

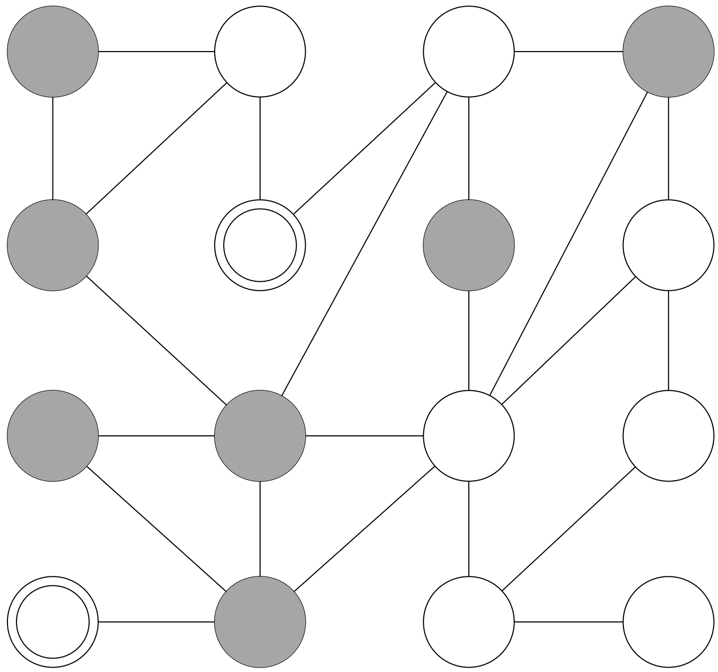


The **moral graph** of a Bayes net is an **undirected** graph with the same vertices as the Bayes net (i.e. one vertex corresponding to each variable) such that each variable has an edge connecting it to every variable in its Markov blanket.

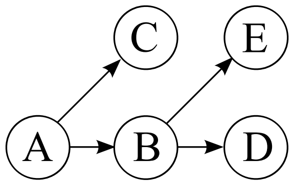
(c) Add edges to the graph on the right so that it is the moral graph of the Bayes net on the left.



(d) The following is a query in a moral graph for a larger Bayes net (the Bayes net is not shown). Cross off all the variables that we can ignore in performing the query.



Q2. Bayes' Nets



	$P(A)$
$+a$	0.25
$-a$	0.75

$P(B\|A)$	$+b$	$-b$
$+a$	0.5	0.5
$-a$	0.25	0.75

$P(C\|A)$	$+c$	$-c$
$+a$	0.2	0.8
$-a$	0.6	0.4

$P(D\|B)$	$+d$	$-d$
$+b$	0.6	0.4
$-b$	0.8	0.2

$P(E\|B)$	$+e$	$-e$
$+b$	0.25	0.75
$-b$	0.1	0.9

(a) Using the Bayes' Net and conditional probability tables above, calculate the following quantities:

(i) $P(+a, +b) =$

(ii) $P(+a | +b) =$

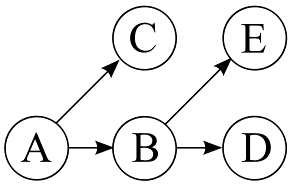
(iii) $P(+b | +a) =$

(b) Now we are going to consider variable elimination in the Bayes' Net above.

(i) Assume we have the evidence $+c$ and wish to calculate $P(E \mid +\mathrm{c})$. What factors do we have initially?

(ii) If we eliminate variable B, we create a new factor. What probability does that factor correspond to?

This is the same figure as the previous page, repeated here for your convenience:



	$P(A)$
$+a$	0.25
$-a$	0.75

$P(B\|A)$	$+b$	$-b$
$+a$	0.5	0.5
$-a$	0.25	0.75

$P(C\|A)$	$+c$	$-c$
$+a$	0.2	0.8
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$P(D\|B)$	$+d$	$-d$
$+b$	0.6	0.4
$-b$	0.8	0.2

$P(E\|B)$	$+e$	$-e$
$+b$	0.25	0.75
$-b$	0.1	0.9

(iii) What is the equation to calculate the factor we create when eliminating variable B?

(iv) After eliminating variable B, what are the new set of factors? As in (ii), write the probabilities that the factors represent. For each factor, also provide its size. Use only as many rows as you need to.

Factor	Size after elimination

(v) Now assume we have the evidence $-c$ and are trying to calculate $P(A|-c)$. What is the most efficient elimination ordering? If more than one ordering is most efficient, provide any one of them.

(vi) Once we have run variable elimination and have $f(A,-c)$ how do we calculate $P(+a \mid -c)$? (give an equation)