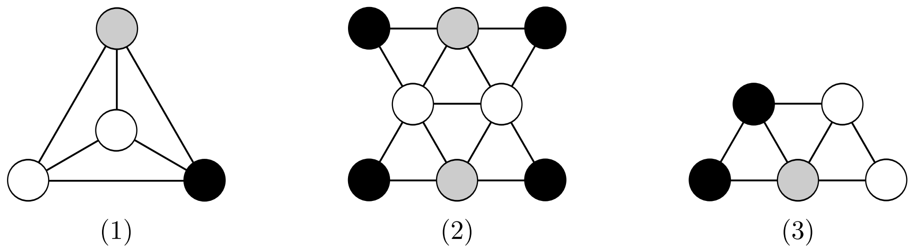
CS 188
Summer 2022

# Midterm Review CSPs

## Q1. CSPs

In this question we are considering CSPs for map coloring. Each region on the map is a variable, and their values are chosen from {black, gray, white}. Adjacent regions cannot have the same color. The figures below show the constraint graphs for three CSPs and an assignment for each one. None of the assignments are solutions as each has a pair of adjacent variables that are white. For both parts of this question, let the score of an assignment be the number of satisfied constraints (so a higher score is better).



**(a)** Consider applying Local Search starting from each of the assignments in the figure above. For each successor function, indicate whether each configuration is a local optimum and whether it is a global optimum (note that the CSPs may not have satisfying assignments).

| Successor Function | CSP | Local optimum? | Global Optimum? |
|---|---|---|---|
| Change a single variable | (1) | Yes No | Yes No |
| | (2) | Yes No | Yes No |
| | (3) | Yes No | Yes No |
| Change a single variable, or a pair of variables | (1) | Yes No | Yes No |
| | (2) | Yes No | Yes No |
| | (3) | Yes No | Yes No |

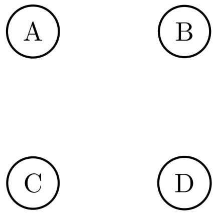
# Q2. CSPs

Four people, A, B, C, and D, are all looking to rent space in an apartment building. There are three floors in the building, 1, 2, and 3 (where 1 is the lowest floor and 3 is the highest). Each person must be assigned to some floor, but it's ok if more than one person is living on a floor. We have the following constraints on assignments:

- $A$ and $B$ must not live together on the same floor.
- If $A$ and $C$ live on *the same* floor, they must both be living on floor 2.
- If $A$ and $C$ live on *different* floors, one of them must be living on floor 3.
- $D$ must not live on the same floor as anyone else.
- $D$ must live on a higher floor than $C$.

We will formulate this as a CSP, where each person has a variable and the variable values are floors.

**(a)** Draw the edges for the constraint graph representing this problem. Use binary constraints only. You do not need to label the edges.



**(b)** Suppose we have assigned C = 2. Apply forward checking to the CSP, filling in the boxes next to the values for each variable that are eliminated:

| | | | |
|---|---|---|---|
| A | ☐ 1 | ☐ 2 | ☐ 3 |
| B | ☐ 1 | ☐ 2 | ☐ 3 |
| C | | ☐ 2 | |
| D | ☐ 1 | ☐ 2 | ☐ 3 |

**(c)** Starting from the original CSP with full domains (i.e. without assigning any variables or doing the forward checking in the previous part), enforce arc consistency for the entire CSP graph, filling in the boxes next to the values that are eliminated for each variable:

| | | | |
|---|---|---|---|
| A | ☐ 1 | ☐ 2 | ☐ 3 |
| B | ☐ 1 | ☐ 2 | ☐ 3 |
| C | ☐ 1 | ☐ 2 | ☐ 3 |
| D | ☐ 1 | ☐ 2 | ☐ 3 |

**(d)** Suppose that we were running local search with the min-conflicts algorithm for this CSP, and currently have the following variable assignments.

| | |
|---|---|
| A | 3 |
| B | 1 |
| C | 2 |
| D | 3 |

Which variable would be reassigned, and which value would it be reassigned to? Assume that any ties are broken alphabetically for variables and in numerical order for values.

|  |  |  |  |
|---|---|---|---|
| The variable | ◯ A | will be assigned the new value | ◯ 1 |
|  | ◯ B |  | ◯ 2 |
|  | ◯ C |  | ◯ 3 |
|  | ◯ D |  |  |

## Q3. Satisfying Search

Consider a search problem $(S, A, Succ, s_0, G)$, where all actions have cost 1. $S$ is the set of states, $A(s)$ is the set of legal actions from a state $s$, $Succ(s, a)$ is the state reached after taking action $a$ in state $s$, $s_0$ is the start state, and $G(s)$ is true if and only if $s$ is a goal state.

Suppose we have a search problem where we know that the solution cost is exactly $k$, but we do not know the actual solution. The search problems has $|S|$ states and a branching factor of $b$.

**(a)** **(i)** Since the costs are all 1, we decide to run breadth-first tree search. Give the tightest bound on the worst-case running time of breadth-first tree search in terms of $|S|$, $b$, and $k$.

The running time is $O($ ______________________________ $)$

**(ii)** Unfortunately, we get an out of memory error when we try to use breadth first search. Which of the following algorithms is the best one to use instead?

- ○ Depth First Search
- ○ Depth First Search limited to depth $k$
- ○ Iterative Deepening
- ○ Uniform Cost Search

Instead of running a search algorithm to find the solution, we can phrase this as a CSP:

Variables: $X_0, X_1, X_2, \cdots X_k$

Domain of each variable: $S$, the set of all possible states

Constraints:

1. $X_0$ is the start state, that is, $X_0 = s_0$.
2. $X_k$ must be a goal state, that is, $G(X_k)$ has to be true.
3. For every $0 \leq i < k$, $(X_i, X_{i+1})$ is an edge in the search graph, that is, there exists an action $a \in A(X_i)$ such that $X_{i+1} = Succ(X_i, a)$.

With these constraints, when we get a solution $(X_0 = s_0, X_1 = s_1, \cdots X_k = s_k)$, the solution to our original search problem is the path $s_0 \rightarrow s_1 \rightarrow \cdots \rightarrow s_k$.

**(b)** This is a tree-structured CSP. Illustrate this by drawing the constraint graph for $k = 3$ and providing a linearization order. (For $k = 3$, the states should be named $X_0$, $X_1$, $X_2$, and $X_3$.)

Constraint Graph:

Linearization Order:______________________________________________

**(c)** We can solve this CSP using the tree-structured CSP algorithm. You can make the following assumptions:

1. For any state $s$, computing $G(s)$ takes $O(1)$ time.
2. Checking consistency of *a single arc* $F \to G$ takes $O(fg)$ time, where $f$ is the number of remaining values that $F$ can take on and $g$ is the number of remaining values that $G$ can take on.

Remember that the search problem has a solution cost of exactly $k$, $|S|$ states, and a branching factor of $b$.

**(i)** Give the tightest bound on the time taken to enforce unary constraints, in terms of $|S|$, $b$, and $k$.

The running time to enforce unary constraints is $O($ ________________________________ $)$

**(ii)** Give the tightest bound on the time taken to run the backward pass, in terms of $|S|$, $b$, and $k$.

The running time for the backward pass is $O($ ________________________________ $)$

**(iii)** Give the tightest bound on the time taken to run the forward pass, in terms of $|S|$, $b$, and $k$.

The running time for the forward pass is $O($ ________________________________ $)$

**(d)** Suppose $s_0 \to s_1 \to \cdots \to s_k$ is a solution to the search problem. Mark all of the following options that are *guaranteed* to be true after enforcing unary constraints and running arc consistency.

☐ The remaining values of $X_i$ will be $s_i$ and possibly other values.
☐ The remaining values of $X_i$ will be $s_i$ and nothing else.
☐ A solution can be found by setting each $X_i$ to any of the remaining states in its domain.
☐ A solution can be found by executing the forward pass of the tree-structured CSP algorithm.
☐ None of the above

**(e)** Suppose you have a heuristic $h(s)$. You decide to add more constraints to your CSP (with the hope that it speeds up the solver by eliminating many states quickly). Mark all of the following options that are valid constraints that can be added to the CSP, under the assumption that $h(s)$ is (a) any function (b) admissible and (c) consistent. *Recall that the cost of every action is* 1.

| | Any $h(s)$ | $h(s)$ is admissible | $h(s)$ is consistent |
|---|---|---|---|
| For every $0 \le i \le k$, $h(X_i) \le i$ | ☐ | ☐ | ☐ |
| For every $0 \le i \le k$, $h(X_i) \le k - i$ | ☐ | ☐ | ☐ |
| For every $0 \le i < k$, $h(X_{i+1}) \le h(X_i) - 1$ | ☐ | ☐ | ☐ |
| For every $0 \le i < k$, $h(X_{i+1}) \ge h(X_i) - 1$ | ☐ | ☐ | ☐ |

☐ None of the above

**(f)** Now suppose we only know that the solution will have $\le k$ moves. We do not need to find the optimal solution - we only need to find some solution of cost $\le k$. Mark all of the following options such that if you make single change described in that line it will correctly modify the CSP to find some solution of cost $\le k$. *Remember, the CSP can only have unary and binary constraints.*

☐ Remove the constraints "$(X_i, X_{i+1})$ is an edge in the search graph". Instead, add the constraints "$(X_i, X_{i+1})$ is an edge in the search graph, OR $X_i = X_{i+1}$".
☐ Remove the constraints "$(X_i, X_{i+1})$ is an edge in the search graph". Instead, add the constraints "$(X_i, X_{i+1})$ is an edge in the search graph, AND $X_i = X_{i+1}$".
☐ Remove the constraint "$X_k$ is a goal state." Instead, add the constraint "For every $0 \le i \le k$, $X_i$ is a goal state".
Remove the constraint "$X_k$ is a goal state." Instead, add the constraint "There is some $i$, $0 \le i \le k$, such

that $X_i$ is a goal state".

☐ None of the above