

Q1. CSPs

In this question, you are trying to find a four-digit number satisfying the following conditions:

1. the number is odd,
2. the number only contains the digits 1, 2, 3, 4, and 5,
3. each digit (except the leftmost) is strictly larger than the digit to its left.

(a) CSPs

We will model this as a CSP where the variables are the four digits of our number, and the domains are the five digits we can choose from. The last variable only has 1, 3, and 5 in its domain since the number must be odd. The constraints are defined to reflect the third condition above. Thus before we start executing any algorithms, the domains are

1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
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(i) Before assigning anything, enforce arc consistency. Write the values remaining in the domain of each variable after arc consistency is enforced.

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(ii) With the domains you wrote in the previous part, which variable will the MRV (Minimum Remaining Value) heuristic choose to assign a value to first? If there is a tie, choose the leftmost variable.

- The first digit (leftmost)
- The second digit
- The third digit
- The fourth digit (rightmost)

(iii) Now suppose we assign to the leftmost digit first. Assuming we will continue filtering by enforcing arc consistency, which value will LCV (Least Constraining Value) choose to assign to the leftmost digit?

Break ties from large (5) to small (1).

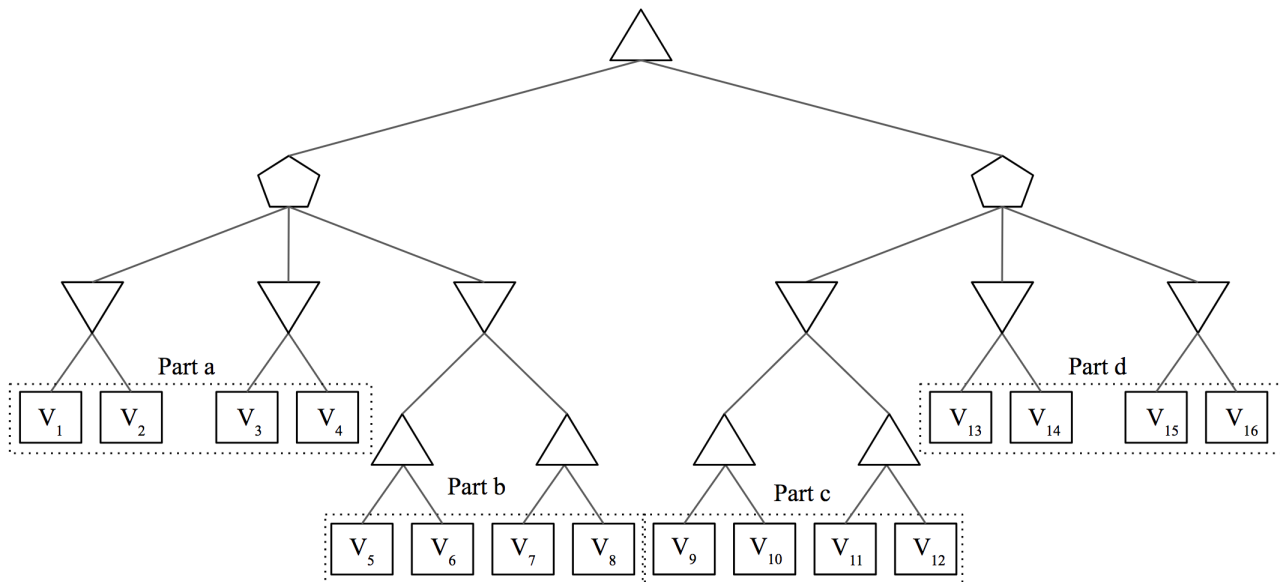
- 1
- 2
- 3
- 4
- 5

(iv) Now suppose we are running min-conflicts to try to solve this CSP. If we start with the number 1332, what will our number be after one iteration of min-conflicts? Break variable selection ties from left to right, and **break value selection ties from small (1) to large (5)**.

- (b) The following questions are completely unrelated to the above parts. Assume for these following questions, there are only binary constraints unless otherwise specified.
- (i) [*true* or *false*] When enforcing arc consistency in a CSP, the set of values which remain when the algorithm terminates does not depend on the order in which arcs are processed from the queue.
- (ii) [*true* or *false*] Once arc consistency is enforced as a pre-processing step, forward checking can be used during backtracking search to maintain arc consistency for all variables.
- (iii) In a general CSP with n variables, each taking d possible values, what is the worst case time complexity of enforcing arc consistency using the AC-3 method discussed in class?
 0 $O(1)$ $O(nd^2)$ $O(n^2d^3)$ $O(d^n)$ ∞
- (iv) In a general CSP with n variables, each taking d possible values, what is the maximum number of times a backtracking search algorithm might have to backtrack (i.e. the number of the times it generates an assignment, partial or complete, that violates the constraints) before finding a solution or concluding that none exists?
 0 $O(1)$ $O(nd^2)$ $O(n^2d^3)$ $O(d^n)$ ∞
- (v) What is the maximum number of times a backtracking search algorithm might have to backtrack in a general CSP, if it is running arc consistency and applying the MRV and LCV heuristics?
 0 $O(1)$ $O(nd^2)$ $O(n^2d^3)$ $O(d^n)$ ∞

Q2. MedianMiniMax

You're living in utopia! Despite living in utopia, you still believe that you need to maximize your utility in life, other people want to minimize your utility, and the world is a 0 sum game. But because you live in utopia, a benevolent social planner occasionally steps in and chooses an option that is a compromise. Essentially, the social planner (represented as the pentagon) is a median node that chooses the successor with median utility. Your struggle with your fellow citizens can be modelled as follows:



There are some nodes that we are sometimes able to prune. In each part, mark all of the terminal nodes such that **there exists a possible situation** for which the node **can be pruned**. In other words, you must consider **all** possible pruning situations. Assume that evaluation order is **left to right** and all V_i 's are **distinct**.

Note that as long as there exists ANY pruning situation (does not have to be the same situation for every node), you should mark the node as prunable. Also, alpha-beta pruning does not apply here, simply prune a sub-tree when you can reason that its value will not affect your final utility.

- (a) V_1
 V_2
 V_3
 V_4
 None

- (b) V_5
 V_6
 V_7
 V_8
 None

- (c) V_9
 V_{10}
 V_{11}
 V_{12}
 None

- (d) V_{13}
 V_{14}
 V_{15}
 V_{16}
 None