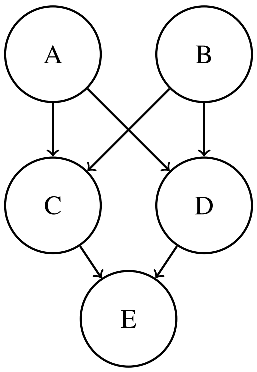
CS 188
Summer 2022

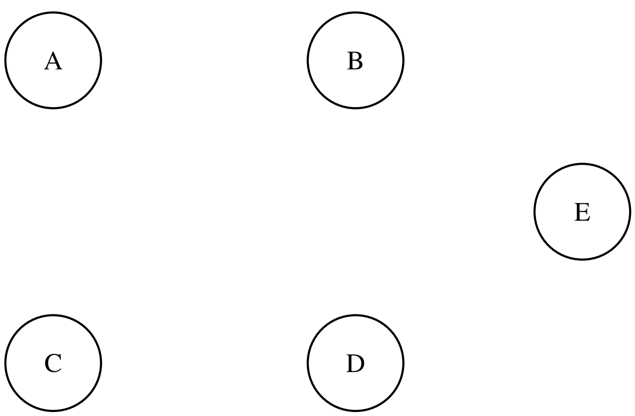
Exam Prep 3B

Q1. Bayes Nets and Joint Distributions

(a) Write down the joint probability distribution associated with the following Bayes Net. Express the answer as a product of terms representing individual conditional probabilities tables associated with this Bayes Net:



(b) Draw the Bayes net associated with the following joint distribution:
$P(A) \cdot P(B) \cdot P(C|A,B) \cdot P(D|C) \cdot P(E|B,C)$



(c) Do the following products of factors correspond to a valid joint distribution over the variables A, B, C, D? (Circle FALSE or TRUE.)

(i) FALSE TRUE $P(A) \cdot P(B) \cdot P(C|A) \cdot P(C|B) \cdot P(D|C)$

(ii) FALSE TRUE $P(A) \cdot P(B|A) \cdot P(C) \cdot P(D|B,C)$

(iii) FALSE TRUE $P(A) \cdot P(B|A) \cdot P(C) \cdot P(C|A) \cdot P(D)$

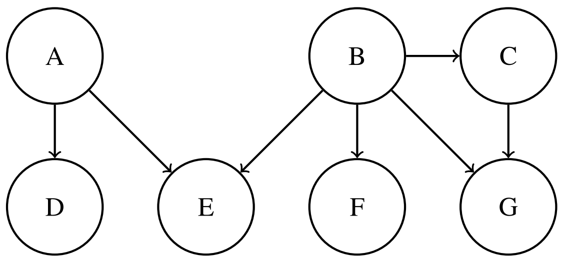
(iv) FALSE TRUE $P(A|B) \cdot P(B|C) \cdot P(C|D) \cdot P(D|A)$

(d) What factor can be multiplied with the following factors to form a valid joint distribution? (Write "none" if the given set of factors can't be turned into a joint by the inclusion of exactly one more factor.)

(i) $P(A) \cdot P(B|A) \cdot P(C|A) \cdot P(E|B, C, D)$

(ii) $P(D) \cdot P(B) \cdot P(C|D, B) \cdot P(E|C, D, A)$

(e) **[Optional - After Learning Variable Elimination]** Answer the next questions based off of the Bayes Net below: All variables have domains of {-1, 0, 1}



(i) Before eliminating any variables or including any evidence, how many entries does the factor at G have?

(ii) Now we observe $e = 1$ and want to query $P(D|e = 1)$, and you get to pick the first variable to be eliminated.

- Which choice would create the **largest** factor f_1?

- Which choice would create the **smallest** factor f_1?

Q2. Probability and Bayes Nets

(a) A, B, and C are random variables with binary domains. How many entries are in the following probability tables and what is the sum of the values in each table? Write a “?" in the box if there is not enough information given.

Table	Size	Sum
$P(A, B \mid C)$		
$P(A \mid +b, +c)$		
$P(+a \mid B)$		

(b) Circle true if the following probability equalities are valid and circle false if they are invalid (leave it blank if you don’t wish to risk a guess). Each True/False question is worth 1 points. Leaving a question blank is worth 0 points. **Answering incorrectly is worth −1 points.**

No independence assumptions are made.

(i) [*true* or *false*] $P(A, B) = P(A \mid B)P(A)$

(ii) [*true* or *false*] $P(A \mid B)P(C \mid B) = P(A, C \mid B)$

(iii) [*true* or *false*] $P(B, C) = \sum_{a \in A} P(B, C \mid A)$

(iv) [*true* or *false*] $P(A, B, C, D) = P(C)P(D \mid C)P(A \mid C, D)P(B \mid A, C, D)$

(c) Space Complexity of Bayes Nets

Consider a joint distribution over N variables. Let k be the domain size for all of these variables, and let d be the maximum indegree of any node in a Bayes net that encodes this distribution.

(i) What is the space complexity of storing the entire joint distribution? Give an answer of the form $O(\cdot)$.

(ii) Draw an example of a Bayes net over four binary variables such that it takes less space to store the Bayes net than to store the joint distribution.

(iii) Draw an example of a Bayes net over four binary variables such that it takes more space to store the Bayes net than to store the joint distribution.