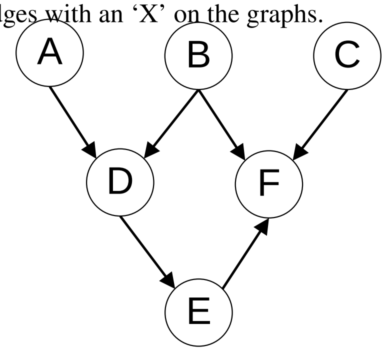
CS 188
Summer 2022

# Exam Prep 4A Solutions

## Q1. Bayes Nets

**(a)** For the following graphs, explicitly state the minimum size set of edges that must be removed such that the corresponding independence relations are guaranteed to be true.

Marked the removed edges with an 'X' on the graphs.

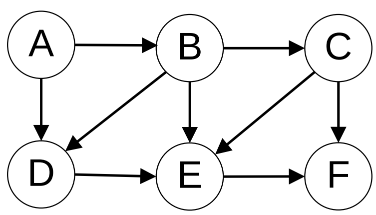



$A \perp\!\!\!\perp B | F$

$A \perp\!\!\!\perp F | D$

$B \perp\!\!\!\perp C$

**(i)** $AD$




$A \perp\!\!\!\perp D | B$

$A \perp\!\!\!\perp F | C$

$C \perp\!\!\!\perp D | B$

**(ii)** $AD$, ($EF$ OR $AB$)

**(b)** You're performing variable elimination over a Bayes Net with variables $A, B, C, D, E$. So far, you've finished joining over (but not summing out) $C$, when you realize you've lost the original Bayes Net!

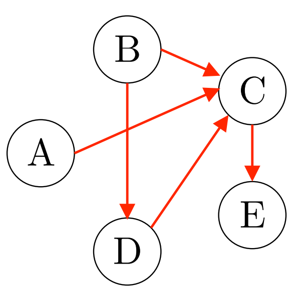
Your current factors are $f(A)$, $f(B)$, $f(B, D)$, $f(A, B, C, D, E)$. Note: these are factors, NOT joint distributions. You don't know which variables are conditioned or unconditioned.

**(i)** What's the smallest number of edges that could have been in the original Bayes Net? Draw out one such Bayes Net below.

Number of edges = 5

The original Bayes net must have had 5 factors, 1 for each node. f(A) and f(B) must have corresponded to nodes A and B, and indicate that neither A nor B have any parents. f(B, D), then, must correspond to node D, and indicates that D has only B as a parent. Since there is only one factor left, f(A, B, C, D, E), for the nodes C and E, those two nodes must have been joined while you were joining C. This implies two things: 1) E must have had C as a parent, and 2) every other node must have been a parent of either C or E.

The below figure is one possible solution that uses the fewest possible edges to satisfy the above.

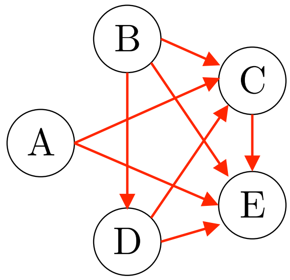


**(ii)** What's the largest number of edges that could have been in the original Bayes Net? Draw out one such Bayes Net below.

Number of edges = 8

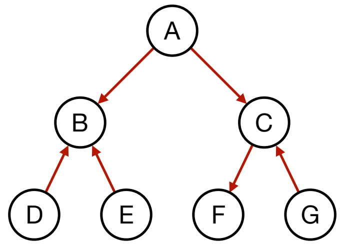
The constraints are the same as outlined in part i). To maximize the number of edges, we make each of A, B, and D a parent of both C and E, as opposed to a parent of one of them.

The below figure is the only possible solution.



# Q2. Independence

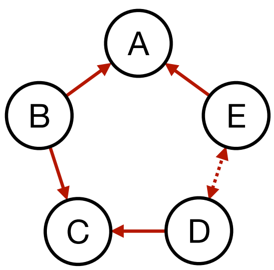
In each part of this question, you are given a Bayes' net where the edges do not have a direction. Assign a direction to every edge (by adding an arrowhead at one end of each edge) to ensure that the Bayes' Net structure implies the assumptions provided. You cannot add new edges. The Bayes' nets can imply more assumptions than listed, but they *must* imply the ones listed. There may be more than one correct solution.




**(a)**

**Assumptions:**

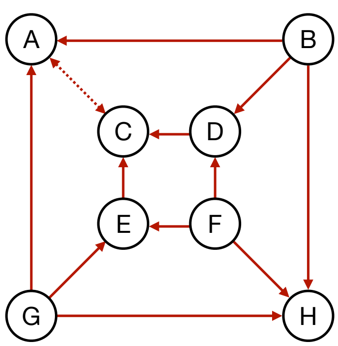
- $A \perp\!\!\!\perp G$
- $D \perp\!\!\!\perp E$
- $E \perp\!\!\!\perp F$
- $F \perp\!\!\!\perp G \mid C$




**(b)** (either direction is allowed for the edge DE)

**Assumptions:**

- $B \perp\!\!\!\perp E$
- $E \perp\!\!\!\perp C \mid D$




**(c)** (either direction is allowed for the edge AC)

**Assumptions:**

- $F \perp\!\!\!\perp G$
- $F \perp\!\!\!\perp B \mid G$
- $D \perp\!\!\!\perp E \mid F$

In order to have two nodes be independent, there must be an inactive triple along *all* paths between the two nodes.

1. $F \perp\!\!\!\perp G$, so the path $FEG$ must have $F \rightarrow E \leftarrow G$

2. $F \perp\!\!\!\perp G$, so the path $FHG$ must have $F \rightarrow H \leftarrow G$
3. $F \perp\!\!\!\perp B \mid G$, so the path $FDB$ must have $F \rightarrow D \leftarrow B$ (we must later verify that $G$ is not a descendant of $D$, but there is are no other edge directions along this path that will create an inactive triple)
4. $F \perp\!\!\!\perp B \mid G$, so the path $FHB$ must have $F \rightarrow H \leftarrow B$ (we must later verify that $G$ is not a descendant of $H$)
5. $D \perp\!\!\!\perp E \mid F$, so the path $DCE$ must have $D \rightarrow C \leftarrow E$ (we must later verify that $F$ is not a descendant of $C$)
6. $D \perp\!\!\!\perp E \mid F$, and we have already assigned some edge directions along path $DBAGE$. In particular, we have $D \leftarrow B - A - G \rightarrow E$. The only possible inactive triple we can create here is $B \rightarrow A \leftarrow G$ (we must later verify that $F$ is not a descendant of $A$).
7. The only remaining edge to assign is $A - C$. We can assign either direction to this edge, and then verify that all required assumptions hold for the completed Bayes Net.