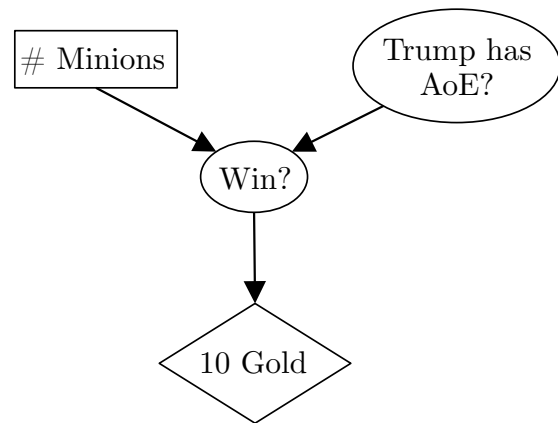


## Q1. Hearthstone Decisions

You are playing the game Hearthstone. You are up against the famous player Trump.

On your turn, you can choose between playing 0, 1, or 2 minions. You realize Trump might be holding up an Area of Effect (AoE) card, which is more devastating the more minions you play.

- If Trump has the AoE, then your chances of winning are:
  - 60% if you play 0 minions
  - 50% if you play 1 minion
  - 20% if you play 2 minions
- If Trump does NOT have the AoE, then your chances of winning are:
  - 20% if you play 0 minions
  - 60% if you play 1 minion
  - 90% if you play 2 minions



You know that there is a 50% chance that Trump has an AoE.

Winning this game is worth 10 gold and losing is worth 0.

**Solution notation:**  $A$ : Trump has AoE?,  $W$ : Win?,  $M$ : Number of minions

(a) How much gold would you expect to win choosing 0 minions?

$$\sum_w \sum_a (P(w|Minion = 0, a)P(a)R(w)) = 10 \sum_a (P(w|Minion = 0, a)P(a)) = 10(.6 \cdot .5 + .2 \cdot .5) = 4$$

(b) How much gold would you expect to win choosing 1 minion?

$$\sum_w \sum_a (P(w|Minion = 1, a)P(a)R(w)) = 10 \sum_a (P(w|Minion = 1, a)P(a)) = 10(.5 \cdot .5 + .6 \cdot .5) = 5.5$$

(c) How much gold would you expect to win choosing 2 minions?

$$\sum_w \sum_a (P(w|Minion = 2, a)P(a)R(w)) = 10 \sum_a (P(w|Minion = 2, a)P(a)) = 10(.2 \cdot .5 + .9 \cdot .5) = 5.5$$

(d) How much gold would you expect to win if you know the AoE is in Trump's hand?

$$\max_m \sum_w P(w|m, +a)R(w) = 10 \max_m P(w|m, +a) = 10 \max\{.6, .5, .2\} = 6$$

(e) How much gold would you expect to win if you know the AoE is NOT in Trump's hand?

$$\max_m \sum_w P(w|m, -a)R(w) = 10 \max_m P(w|m, -a) = 10 \max\{.2, .6, .9\} = 9$$

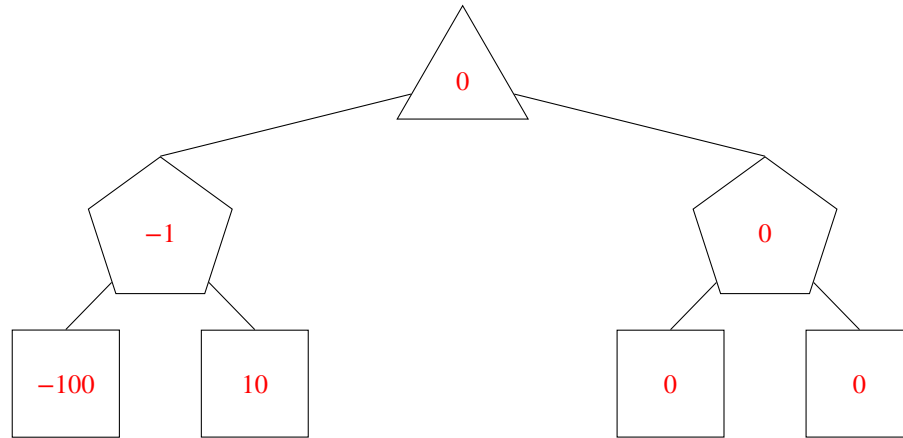
(f) How much gold would you be willing to pay for to know whether or not the AoE is in Trump's hand? (Assume your utility of gold is the same as the amount of gold.)

Two. The difference between  $MEU(\{\}) = 5.5$  and  $MEU(\{A\}) = .5 * 6 + .5 * 9 = 7.5$  is 2.

## Q2. Decision Networks and VPI

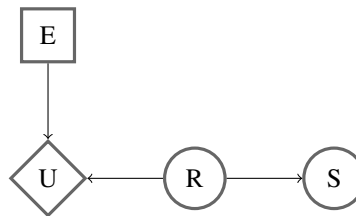
Valerie has just found a cookie on the ground. She is concerned that the cookie contains raisins, which she really dislikes but she still wants to eat the cookie. If she eats the cookie and it contains raisins she will receive a utility of  $-100$  and if the cookie doesn't contain raisins she will receive a utility of  $10$ . If she doesn't eat the cookie she will get  $0$  utility. The cookie contains raisins with probability  $0.1$ .

- (a) We want to represent this decision network as an expectimax game tree. Fill in the nodes of the tree below, with the top node representing her maximizing choice.



- (b) Should Valerie eat the cookie?  Yes  No

- (c) Valerie can now smell the cookie to judge whether it has raisins before she eats it. However, since she dislikes raisins she does not have much experience with them and cannot recognize their smell well. As a result she will incorrectly identify raisins when there are no raisins with probability  $0.2$  and will incorrectly identify no raisins when there are raisins with probability  $0.3$ . This decision network can be represented by the diagram below where E is her choice to eat, U is her utility earned, R is whether the cookie contains raisins, and S is her attempt at smelling.



Valerie has just smelled the cookie and she thinks it doesn't have raisins. Write the probability,  $X$ , that the cookie has raisins given that she smelled no raisins as a simplest form fraction or decimal.

$$X = \boxed{0.04}$$

$$P(+r|-s) = \frac{P(-s|+r)P(+r)}{P(-s)} = \frac{P(-s|+r)P(+r)}{P(-s|+r)P(+r) + P(-s|-r)P(-r)} = \frac{.3*.1}{.3*.1 + .8*.9} = \frac{.03}{.75} = .04$$

- (d) What is her maximum expected utility,  $Y$  given that she smelled no raisins? You can answer in terms of  $X$  or as a simplest form fraction or decimal.

$$Y = \boxed{-100X + 10(1 - X), 5.6}$$

$$\begin{aligned} MEU(-s) &= \max(MEU(eating|-s), MEU(not eating|-s)) = \\ &= \max(P(+r|-s) * EU(eating, +r) + P(-r|-s) * EU(eating, -r), MEU(not eating)) = \\ &= \max(X * (-100) + (1 - X) * 10, 0) = \\ &= X * 100 + (1 - X) * 10 \end{aligned}$$

- (e) What is the Value of Perfect Information (VPI) of smelling the cookie? You can answer in terms of X and Y or as a simplest form fraction or decimal.

$$VPI = \boxed{0.75 * Y, 4.2}$$

$$VPI(S) = MEU(S) - MEU(\emptyset)$$

$$MEU(S) = P(-s)MEU(-s) + P(+s)MEU(+s)$$

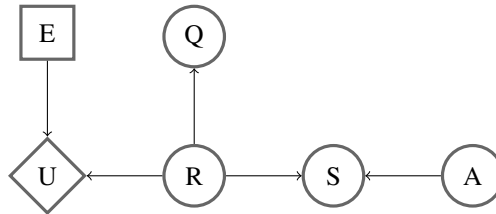
$$P(-s) = .75 \text{ from part (c), } MEU(-s) = Y$$

$MEU(+s) = 0$  because it was better for her to not eat the raisin without knowing anything, smelling raisins will only make it more likely for the cookie to have raisins and it will still be best for her to not eat and earn a utility of 0. Note this means we do not have to calculate  $P(+s)$ .

$$MEU(\emptyset) = 0$$

$$VPI(S) = .75 * Y + 0 - 0 = .75 * Y$$

- (f) Valerie is unsatisfied with the previous model and wants to incorporate more variables into her decision network. First, she realizes that the air quality (A) can affect her smelling accuracy. Second, she realizes that she can question (Q) the people around to see if they know where the cookie came from. These additions are reflected in the decision network below.



Choose one for each equation:

	Could Be True	Must Be True	Must Be False
$VPI(A, S) > VPI(A) + VPI(S)$	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
$VPI(A) = 0$	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
$VPI(Q, R) \leq VPI(Q) + VPI(R)$	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
$VPI(S, R) > VPI(R)$	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>
$VPI(Q) \geq 0$	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
$VPI(Q, A) > VPI(Q)$	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>
$VPI(S A) < VPI(S)$	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>
$VPI(A S) > VPI(A)$	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>