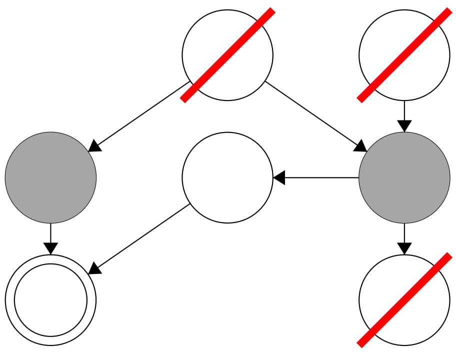
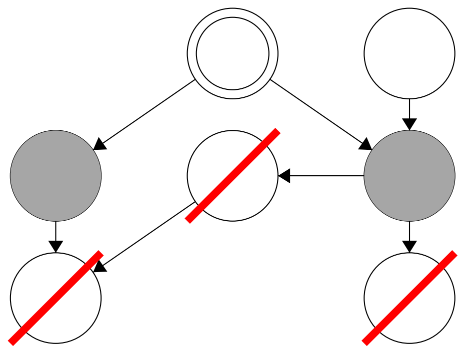
CS 188
Summer 2022

Final Review Bayes Nets Solutions

Q1. Moral Graphs

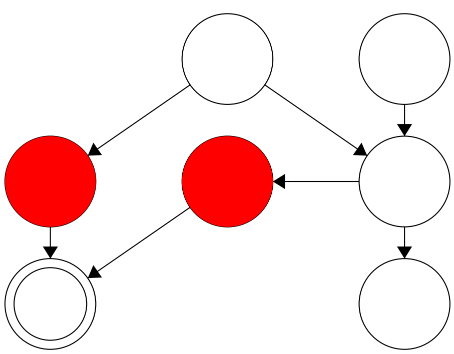
(a) For each of the following queries, we want to preprocess the Bayes net before performing variable elimination. Query variables are double-circled and evidence variables are shaded. Cross off all the variables that we can ignore in performing the query. If no variables can be ignored in one of the Bayes nets, write "None" under that Bayes net.

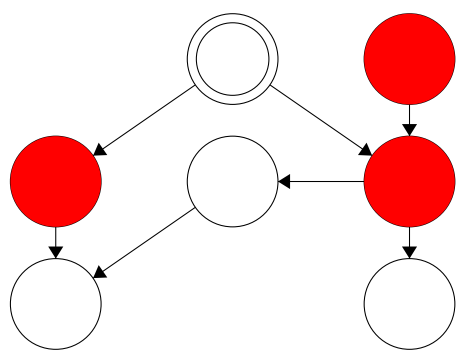




Let B be a Bayes net with a set of variables V. The **Markov blanket** of a variable $v \in V$ is the smallest set of variables $S \subset V$ such that for any variables $v' \in V$ such that $v \neq v'$ and $v' \notin S$, $v \perp\!\!\!\perp v' | S$. Less formally, v is independent from the entire Bayes net given all the variables in S.

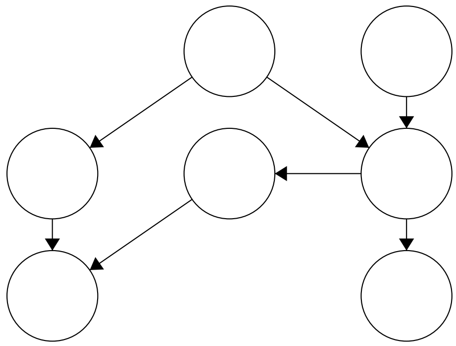
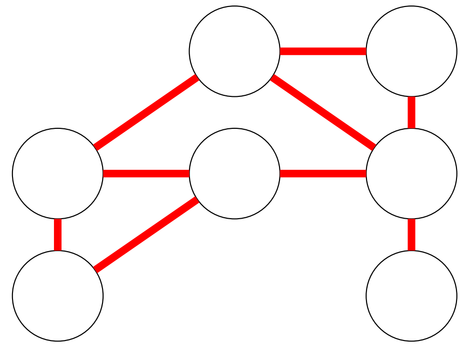
(b) In each of the following Bayes nets, shade in the Markov blanket of the double-circled variable.



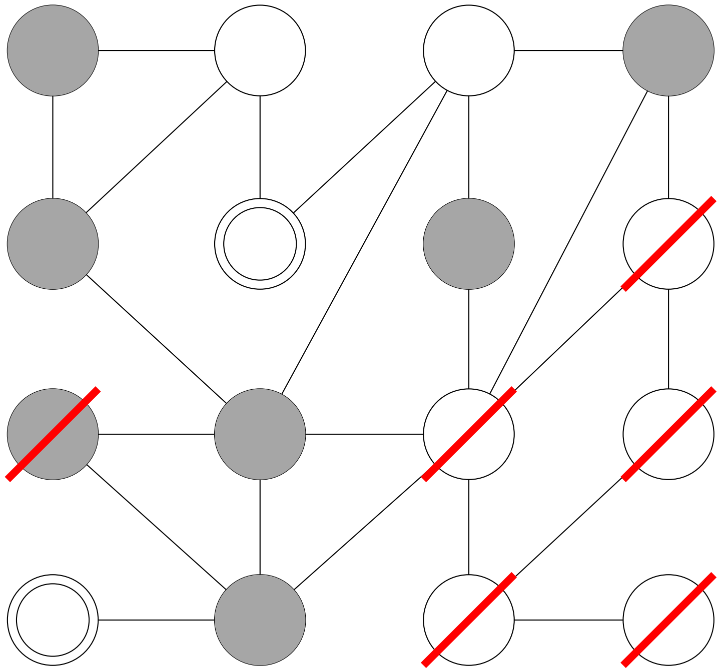


The **moral graph** of a Bayes net is an **undirected** graph with the same vertices as the Bayes net (i.e. one vertex corresponding to each variable) such that each variable has an edge connecting it to every variable in its Markov blanket.

(c) Add edges to the graph on the right so that it is the moral graph of the Bayes net on the left.

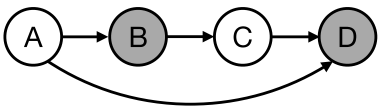
 

(d) The following is a query in a moral graph for a larger Bayes net (the Bayes net is not shown). Cross off all the variables that we can ignore in performing the query.



Q2. Bayes Nets: Sampling

Consider the following Bayes Net, where we have observed that $B = +b$ and $D = +d$.



$P(A)$	
$+a$	0.5
$-a$	0.5

$P(B\|A)$		
$+a$	$+b$	0.8
$+a$	$-b$	0.2
$-a$	$+b$	0.4
$-a$	$-b$	0.6

$P(C\|B)$		
$+b$	$+c$	0.1
$+b$	$-c$	0.9
$-b$	$+c$	0.7
$-b$	$-c$	0.3

$P(D\|A,C)$			
$+a$	$+c$	$+d$	0.6
$+a$	$+c$	$-d$	0.4
$+a$	$-c$	$+d$	0.1
$+a$	$-c$	$-d$	0.9
$-a$	$+c$	$+d$	0.2
$-a$	$+c$	$-d$	0.8
$-a$	$-c$	$+d$	0.5
$-a$	$-c$	$-d$	0.5

(a) Consider doing Gibbs sampling for this example. Assume that we have initialized all variables to the values $+a, +b, +c, +d$. We then unassign the variable C, such that we have $A = +a$, $B = +b$, $C = ?$, $D = +d$. Calculate the probabilities for new values of C at this stage of the Gibbs sampling procedure.

$P(C = +c$ at the next step of Gibbs sampling$) =$ $\frac{0.1\cdot 0.6}{0.1\cdot 0.6+0.9\cdot 0.1} = \boxed{\frac{2}{5}}$

$P(C = -c$ at the next step of Gibbs sampling$) =$ $\frac{0.9\cdot 0.1}{0.1\cdot 0.6+0.9\cdot 0.1} = \boxed{\frac{3}{5}}$

(b) Consider a sampling scheme that is a hybrid of rejection sampling and likelihood-weighted sampling. Under this scheme, we first perform rejection sampling for the variables A and B. We then take the sampled values for A and B and extend the sample to include values for variables C and D, using likelihood-weighted sampling.

(i) Below is a list of candidate samples. Mark the samples that would be rejected by the rejection sampling portion of the hybrid scheme.

■ $-a$ $-b$
☐ $+a$ $+b$
■ $+a$ $-b$
☐ $-a$ $+b$

(ii) To decouple from part (i), you now receive a *new* set of samples shown below. Fill in the weights for these samples under our hybrid scheme.

				Weight
$-a$	$+b$	$-c$	$+d$	$\boxed{0.5}$
$+a$	$+b$	$-c$	$+d$	$\boxed{0.1}$
$+a$	$+b$	$-c$	$+d$	$\boxed{0.1}$
$-a$	$+b$	$+c$	$+d$	$\boxed{0.2}$
$+a$	$+b$	$+c$	$+d$	$\boxed{0.6}$

(iii) Use the weighted samples from part (ii) to calculate an estimate for $P(+a| +b, +d)$.

The estimate of $P(+a| +b, +d)$ is $\frac{0.1+0.1+0.6}{0.5+0.1+0.1+0.2+0.6} = \boxed{\frac{8}{15}}$

(c) We now attempt to design an alternative hybrid sampling scheme that combines elements of likelihood-weighted and rejection sampling. For each proposed scheme, indicate whether it is valid, i.e. whether the weighted samples it produces correctly approximate the distribution $P(A, C| + b, +d)$.

(i) *First collect a likelihood-weighted sample for the variables A and B. Then switch to rejection sampling for the variables C and D. In case of rejection, the values of A and B and the sample weight are **thrown away**. Sampling then restarts from node **A**.*

● Valid ○ Invalid

(ii) *First collect a likelihood-weighted sample for the variables A and B. Then switch to rejection sampling for the variables C and D. In case of rejection, the values of A and B and the sample weight are **retained**. Sampling then restarts from node **C**.*

○ Valid ● Invalid

The sampling procedure in part (i) is the correct way of combining likelihood-weighted and rejection sampling: any time a node gets rejected, the sample must be thrown out in its entirety. In part (ii), however, the evidence that $D = +d$ has no effect on which values of A are sampled or on the sample weights. This means that values for A would be sampled according to $P(A| + b)$, not $P(A| + b, +d)$.

As an extreme case, suppose node D had a different probability table where $P(+d| + a) = 0$. Following the procedure from part (ii), we might start by sampling $(+a, +b)$ and assigning a weight according to $P(+b| + a)$. However, when we move on to rejection sampling we will be forced to continuously reject all possible values because our evidence $+d$ is inconsistent with our the assignment of $A = +a$. This means that the procedure from part (ii) is flawed to the extent that it might fail to generate a sample altogether!