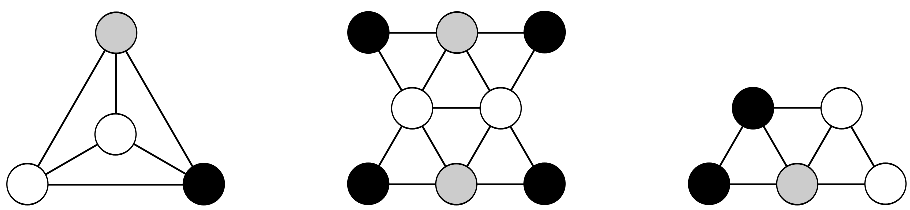
CS 188 Summer 2022 — Final Review CSPs Solutions

Q1. CSPs

In this question we are considering CSPs for map coloring. Each region on the map is a variable, and their values are chosen from {black, gray, white}. Adjacent regions cannot have the same color. The figures below show the constraint graphs for three CSPs and an assignment for each one. None of the assignments are solutions as each has a pair of adjacent variables that are white. For both parts of this question, let the score of an assignment be the number of satisfied constraints (so a higher score is better).



(1) (2) (3)

(a) Consider applying Local Search starting from each of the assignments in the figure above. For each successor function, indicate whether each configuration is a local optimum and whether it is a global optimum (note that the CSPs may not have satisfying assignments).

Successor Function	CSP	Local optimum?		Global Optimum?	
Change a single variable	(1)	**Yes**	No	**Yes**	No
	(2)	**Yes**	No	Yes	**No**
	(3)	Yes	**No**	Yes	**No**
Change a single variable, or a pair of variables	(1)	**Yes**	No	**Yes**	No
	(2)	Yes	**No**	Yes	**No**
	(3)	Yes	**No**	Yes	**No**

Q2. CSPs: Potluck Pandemonium

The potluck is coming up and the staff haven't figured out what to bring yet! They've pooled their resources and determined that they can bring some subset of the following items.

1. Pho
2. Apricots
3. Frozen Yogurt
4. Fried Rice
5. Apple Pie
6. Animal Crackers

There are five people on the course staff: Taylor, Jonathan, Faraz, Brian, and Alvin. Each of them will only bring one item to the potluck.

i. If (F)araz brings the same item as someone else, it cannot be (B)rian.

ii. (A)lvin has pho-phobia so he won't bring Pho, but he'll be okay if someone else brings it.

iii. (B)rian is no longer allowed near a stove, so he can only bring items 2, 3, or 6.

iv. (F)araz literally can't even; he won't bring items 2, 4, or 6.

v. (J)onathan was busy, so he didn't see the last third of the list. Therefore, he will only bring item 1, 2, 3, or 4.

vi. (T)aylor will only bring an item that is before an item that (J)onathan brings.

vii. (T)aylor is allergic to animal crackers, so he won't bring item 6. (If someone else brings it, he'll just stay away from that table.)

viii. (F)araz and (J)onathan will only bring items that have the same first letter (e.g. Frozen Yogurt and Fried Rice).

ix. (B)rian will only bring an item that is after an item that (A)lvin brings on the list.

x. (J)onathan and (T)aylor want to be unique; they won't bring the same item as anyone else.

(a) Which of the listed constraints are unary constraints?

i ○ ii ● iii ● iv ● v ●

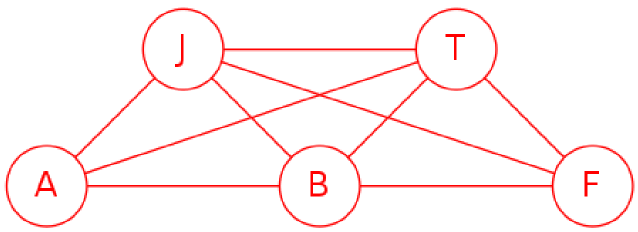
vi ○ vii ● viii ○ ix ○ x ○

(b) Rewrite implicit constraint viii. as an explicit constraint.

(F, J) ∈ { (3, 4), (4, 3),
(2, 5), (5, 2),
(2, 6), (6, 2),
(5, 6), (6, 5),
(1, 1), (2, 2), (3, 3),
(4, 4), (5, 5), (6, 6)
}

(c) How many edges are there in the constraint graph for this CSP?

There are 9 edges in this constraint graph.



(d) The table below shows the variable domains after all unary constraints have been enforced.

A		2	3	4	5	6
B		2	3			6
F	1		3		5	
J	1	2	3	4		
T	1	2	3	4	5	

Following the Minimum Remaining Values heuristic, which variable should we assign first? Break all ties alphabetically.

A ○ B ● F ○ J ○ T ○

(e) To decouple this from the previous question, assume that we choose to assign (F)araz first. In this question, we will choose which value to assign to using the Least Constraining Value method.

To determine the number of remaining values, enforce arc consistency to prune the domains. Then, count the total number of possible assignments (**not** the total number of remaining values). It may help you to enforce arc consistency twice, once before assigning values to (F)araz, and then again after assigning a value.

The domains after enforcing unary constraints are reproduced in each subquestion.

(i) Assigning F = 1 results in 0 possible assignments.

A		2	3	4	5	6
B		2	3			6
F	1		3		5	
J	1	2	3	4		
T	1	2	3	4	5	

Assigning F = 1 leaves no possible values in J's domain (due to constraint viii).

(ii) Assigning F = 3 results in 5 possible assignments.

A		2	3	4	5	6
B		2	3			6
F	1		3		5	
J	1	2	3	4		
T	1	2	3	4	5	

Assigning F = 3 leaves J's domain as $\{4\}$. Enforcing arc consistency gives A = $\{2, 3, 5\}$, B = $\{6\}$, and T = $\{1, 2\}$. Therefore, the 5 possible assignments are (A, B, F, J, T) = (2, 6, 3, 4, 1), (3, 6, 3, 4, 1), (5, 6, 3, 4, 1), (3, 6, 3, 4, 2), (5, 6, 3, 4, 2).

(iii) Assigning F = 5 results in 3 possible assignments.

A		2	3	4	5	6
B		2	3			6
F	1		3		5	
J	1	2	3	4		
T	1	2	3	4	5	

Assigning F = 5 leaves J's domain as $\{2\}$. Enforcing arc consistency gives A = $\{3, 4, 5\}$, B = $\{6\}$, and T = $\{1\}$. Therefore, the 3 possible assignments are (A, B, F, J, T) = (3, 6, 5, 2, 1), (4, 6, 5, 2, 1), (5, 6, 5, 2, 1).

(iv) Using the LCV method, which value should we assign to F? If there is a tie, choose the lower number. (e.g. If both 1 and 2 have the same value, then fill 1.)

1 ○ 2 ○ 3 ● 4 ○ 5 ○ 6 ○