CS 188
Summer 2022

Final Review Search Solutions

Q1. Power Pellets

Consider a Pacman game where Pacman can eat 3 types of pellets:

- Normal pellets (n-pellets), which are worth one point.
- Decaying pellets (d-pellets), which are worth $max(0, 5 - t)$ points, where t is time.
- Growing pellets (g-pellets), which are worth t points, where t is time.

The location and type of each pellet is fixed. The pellet's point value stops changing once eaten. For example, if Pacman eats one g-pellet at $t = 1$ and one d-pellet at $t = 2$, Pacman will have won $1 + 3 = 4$ points.

Pacman needs to find a path to win at least 10 points but he wants to minimize distance travelled. The cost between states is equal to distance travelled.

(a) Which of the following must be including for a minimum, sufficient state space?

- [✓] Pacman's location
- ☐ Location and type of each pellet
- ☐ How far Pacman has travelled
- [✓] Current time
- ☐ How many pellets Pacman has eaten and the point value of each eaten pellet
- [✓] Total points Pacman has won
- [✓] Which pellets Pacman has eaten

A state space should include which pellets are left on the board, the current value of pellets, Pacman's location, and the total points collected so far. With this in mind:
(1) The starting location and type of each pellet are not included in the state space as this is something that does not change during the search. This is analogous to how the walls of a Pacman board are not included in the state space.
(2) How far Pacman has travelled does not need to be explicitly tracked by the state, since this will be reflected in the cost of a path.
(3) Pacman does need the current time to determine the value of pellets on the board.
(4) The number of pellets Pacman has eaten is extraneous.
(5) Pacman must track the total number of points won for the goal test.
(6) Pacman must know which pellets remain on the board, which is the complement of the pellets he has eaten.

(b) Which of the following are admissible heuristics? Let x be the number of points won so far.

- [✓] Distance to closest pellet, except if in the goal state, in which case the heuristic value is 0.
- ☐ Distance needed to win $10 - x$ points, determining the value of all pellets as if they were n-pellets.
- [✓] Distance needed to win $10 - x$ points, determining the value of all pellets as if they were g-pellets (i.e. all pellet values will be t.)
- ☐ Distance needed to win $10 - x$ points, determining the value of all pellets as if they were d-pellets (i.e. all pellet values will be $max(0, 5 - t)$.
- ☐ Distance needed to win $10 - x$ points assuming all pellets maintain current point value (g-pellets stop

increasing in value and d-pellets stop decreasing in value)

☐ None of the above

(1) Admissible; to get 10 points Pacman will always have to travel at least as far as the distance to the closest pellet, so this will always be an underestimate.
(2) Not admissible; if all the pellets are actually g-pellets, assuming they are n-pellets will lead to Pacman collecting more pellets in more locations, and thus travel further.
(3) Ambiguous; if pellets are n-pellets or d-pellets, Pacman will generally have to go further, except at the beginning of the game when d-pellets are worth more, in which case this heuristic will over-estimate the cost to the goal. However, if Pacman is allowed to stay in place with no cost, then this heuristic is admissable because the heuristic will instead calculate all pellet values as 10. This option was ignored in scoring.
(4) Not admissible; if pellets are n-pellets or g-pellets, Pacman would have an overestimate.
(5) Not admissible; if pellets are g-pellets, then using the current pellet value might lead Pacman to collect more locations, and thus travel further than necesarry.

(c) Instead of finding a path which minimizes distance, Pacman would like to find a path which minimizes the following:

$$C_{new} = a * t + b * d$$

where t is the amount of time elapsed, d is the distance travelled, and a and b are non-negative constants such that $a + b = 1$. Pacman knows an admissible heuristic when he is trying to minimize time (i.e. when $a = 1, b = 0$), h_t, and when he is trying to minimize distance, h_d (i.e. when $a = 0, b = 1$).
Which of the following heuristics is guaranteed to be admissible when minimizing C_{new}?

☐ $mean(h_t, h_d)$ [✓ $min(h_t, h_d)$] ☐ $max(h_t, h_d)$ [✓ $a * h_t + b * h_d$]

☐ None of the above

For this question, think about the inequality $C_{new} = a * t + b * d \geq a * h_t + b * h_d$. We can guarantee a heuristic h_{new} is admissible if $h_{new} \leq a * h_t + b * h_d$
(1) If $a = b$, $0.5 * h_t + 0.5 * h_d$ is not guaranteed to be less than $a * h_t + b * h_d$, so this will not be admissible.
(2) $min(h_t, h_d) = a * min(h_t, h_d) + b * min(h_t, h_d) \leq a * h_t + b * h_d$
(3) $max(h_t, h_d)$ will be greater than $a * h_t + b * h_d$ unless $h_t = h_d$, wo this will not be admissible.
(4) Admissible.

Q2. Rubik's Search

Note: You do not need to know what a Rubik's cube is in order to solve this problem.

A Rubik's cube has about 4.3×10^{19} possible configurations, but any configuration can be solved in 20 moves or less. We pose the problem of solving a Rubik's cube as a search problem, where the states are the possible configurations, and there is an edge between two states if we can get from one state to another in a single move. Thus, we have 4.3×10^{19} states. Each edge has cost 1. Note that the state space graph does contain cycles. Since we can make 27 moves from each state, the branching factor is 27. Since any configuration can be solved in 20 moves or less, we have $h^*(n) \leq 20$.

For each of the following searches, estimate the approximate number of states expanded. Mark the option that is closest to the number of states expanded by the search. Assume that the shortest solution for our start state takes exactly 20 moves. Note that 27^{20} is much larger than 4.3×10^{19}.

(a) DFS Tree Search

(i) Best Case: [✓ 20] ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

(ii) Worst Case: ○ 20 ○ 4.3×10^{19} ○ 27^{20} [✓ ∞ (never finishes)]

(b) DFS graph search

(i) Best Case: [✓ 20] ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

(ii) Worst Case: ○ 20 [✓ 4.3×10^{19}] ○ 27^{20} ○ ∞ (never finishes)

(c) BFS tree search

(i) Best Case: ○ 20 ○ 4.3×10^{19} [✓ 27^{20}] ○ ∞ (never finishes)

(ii) Worst Case: ○ 20 ○ 4.3×10^{19} [✓ 27^{20}] ○ ∞ (never finishes)

(d) BFS graph search

(i) Best Case: ○ 20 [✓ 4.3×10^{19}] ○ 27^{20} ○ ∞ (never finishes)

(ii) Worst Case: ○ 20 [✓ 4.3×10^{19}] ○ 27^{20} ○ ∞ (never finishes)

(e) A* tree search with a perfect heuristic, h*(n), Best Case

[✓ 20] ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

(f) A* tree search with a bad heuristic, h(n) = 20 - h*(n), Worst Case

○ 20 ○ 4.3×10^{19} [✓ 27^{20}] ○ ∞ (never finishes)

(g) A* graph search with a perfect heuristic, h*(n), Best Case

[✓ 20] ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

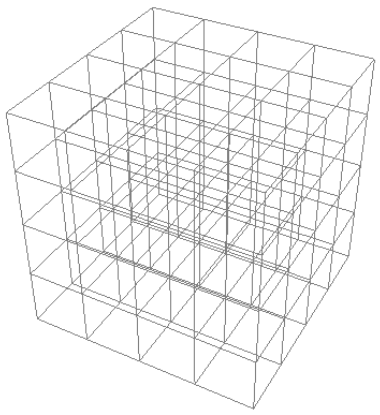
(h) A* graph search with a bad heuristic, h(n) = 20 - h*(n), Worst Case

○ 20 [✓ 4.3×10^{19}] ○ 27^{20} ○ ∞ (never finishes)

Q3. **[Optional]** Search in 3D Maze

Imagine you are the Spider man. Your friend Superman was captured by the evil Spider man somewhere in this mystical 3D maze (Fig 1), (x_t, y_t, z_t). This maze is a *infinite* 3D grid world. You are located at $(0, 0, 0)$ right now and want to come up with a plan to rescue the Superman. Even though you are the Spider man, you can only travel along the wires, not through the space.

3D Maze



(a) What is the branch factor b in this space?

6

(b) How many distinct states can you reach at depth k?

$4k + 4k(k-1) + 2 = 4k^2 + 2$

We're essentially asked the number of solutions to the equation: $|x| + |y| + |z| = k$. To do this, we can split this into three different cases and use a balls and bins argument:

1. Case 1: Any two of x, y, z are 0. In this case, there are $3 * 2 = 6$ solutions, where 3 is because we can have $|x| = k, |y| = k, |z| = k$ and 2 is because if $|x| = k$, then x can be $+k$ or $-k$.
2. Case 2: One of x, y, z is 0. From this there are $3 * \binom{k-1}{1} * 4 = 12k - 12$ solutions.
3. Case 3: None of x, y, z are 0. Then there are $\binom{k-1}{2} * 8 = 4(k-1)(k-2) = 4k^2 - 12k + 8$ solutions

Summing these results, we get $6 + 12k - 12 + 4k^2 - 12k + 8 = 4k^2 + 2$

(c) If you run BFS-tree search, how many nodes would you have expanded up to the goal state? What about BFS-graph search?

Without checking repeated states, the BFS will expand exponentially many nodes on the order of $6^{x_t+y_t+z_t}$. On the other hand, BFS-graph search would expand on the order of $(x_t + y_t + z_t)^3$ many states.

(d) Assume each edge has a cost of 1. Let the state be (u, v, w). Which of the following heuristics are admissible? Select all that apply.

- 🟥 $h1(u, v, w) = \sqrt{(x_t - u)^2 + (y_t - v)^2 + (z_t - w)^2}$
- 🟥 $h2(u, v, w) = |x_t - u| + |y_t - v| + |z_t - w|$
- 🟥 $h3(u, v, w) = \sqrt{|x_t - u|} + \sqrt{|y_t - v|} + \sqrt{|z_t - w|}$
- ☐ $h1(u, v, w) = (x_t - u)^2 + (y_t - v)^2 + (z_t - w)^2$

(e) Approximately how many nodes would you expand if you use heuristic $h1$?

○ $|x_t y_t z_t|$ ○ $(x_t)^2 + (y_t)^2 + (z_t)^2$ ○ $\sqrt{(x_t - u)^2 + (y_t - v)^2 + (z_t - w)^2}$ [✓ $|x_t| + |y_t| + |z_t|$]

What about $h2$?

[✓ $|x_t y_t z_t|$] ○ $(x_t)^2 + (y_t)^2 + (z_t)^2$ ○ $\sqrt{(x_t - u)^2 + (y_t - v)^2 + (z_t - w)^2}$ ○ $|x_t| + |y_t| + |z_t|$

(f) If the evil Spider man destroys half of the links in this grid, would the heuristics h_1 and h_2 be admissible?

Yes, they are, because removing links only cause more detours.

(g) In expectation (assume random tie-breaking), how many nodes would you expand before hitting the goal if you use UCS tree search, DFS graph search, or greedy search instead of A* search? Assume each path has cost 1 and heuristic $h1$.

UCS tree search would explore $6^{x_t+y_t+z_t}$ many nodes. DFS graph search could go to infinity because this is an infinite grid world. Greedy search would have the same complexity as A* in this case, which is $|x_t| + |y_t| + |z_t|$