1 Independent Probability Tables

Suppose you have two random variables, C and N. C is the result of flipping a biased coin that lands on heads with probability 0.8. N is the number of heads that result from flipping a fair coin twice. Fill in the probability tables for $P(C)$, $P(N)$, and $P(C,N)$.

C	$P(C)$
h	0.8
t	0.2

N	$P(N)$
0	0.25
1	0.5
2	0.25

C	N	$P(C,N)$
h	0	0.2
h	1	0.4
h	2	0.2
t	0	0.05
t	1	0.1
t	2	0.05

Note that in general, tables cannot be combined in this way. This works because of our simplifying assumption that the coin flips are independent.

2 Joining Probability Tables and Bayes's Rule

Consider two binary random variables, L and T. L takes on values $+l$ and l, corresponding to whether or not youre late for work. T takes on values $+t$ and t and corresponds to whether or not theres a traffic jam. For example, $+l,+t$ means youre late for work and theres a traffic jam. We are given the following probability tables:

T	$P(T)$
+t	0.4
–t	0.6

L	T	$P(L\|T)$
+l	+t	0.8
+l	–t	0.25
–l	+t	0.2
–l	–t	0.75

Use the above tables to fill in the following tables:

L	T	$P(L,T)$
+l	+t	0.32
+l	–t	0.15
–l	+t	0.08
–l	–t	0.45

L	T	$P(T\|L)$
+l	+t	0.68
+l	–t	0.32
–l	+t	0.15
–l	–t	0.85

3 Querying a Joint Distribution

A	B	C	$P(A,B,C)$
+a	+b	+c	1/16
+a	+b	–c	1/8
+a	–b	+c	1/16
+a	–b	–c	1/4
–a	+b	+c	1/4
–a	+b	–c	1/16
–a	–b	+c	1/16
–a	–b	–c	1/8

Compute the following values:

(a) $P(+c) = 7/16$

(b) $P(-b,+a) = 5/16$

(c) $P(-b|+a) = 5/8$

(d) $P(+c|-a,+b) = 4/5$

(e) $P(-a,+c|-b) = 1/8$

Compute the following tables:

(a) $P(B)$

B	$P(B)$
+b	0.5
–b	0.5

(b) $P(+b|+a,C)$

C	$P(+b|+a,C)$
+c	1/2
–c	1/3

(c) $P(A,C|+b)$

A	C	$P(A,C|+b)$
+a	+c	1/8
+a	–c	1/4
–a	+c	1/2
–a	–c	1/8