1 Independence

1. Here are two joint probability tables. In which are the variables independent?

A	B	$P(A,B)$
+a	+b	0.42
+a	–b	0.18
–a	+b	0.28
–a	–b	0.12

C	D	$P(C,D)$
+c	+d	0.4
+c	–d	0.15
–c	+d	0.4
–c	–d	0.05

2. (a) Given that $A \perp\!\!\!\perp B$, simplify $\sum_a P(a|B)P(C|a)$:

   (b) Given that $B \perp\!\!\!\perp C|A$, simplify $\frac{P(A)P(B|A)P(C|A)}{P(B|C)P(C)}$:

   (c) Given that $A \perp\!\!\!\perp B|C$, simplify $\frac{P(C,A|B)P(B)}{P(C)}$:

2 Expression Manipulation

Simplify the following expressions, as much as possible:

1. $\sum_x \sum_y f(x)h(x,y) =$

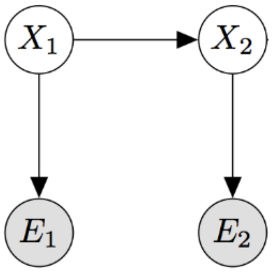
2. $\sum_a \sum_b \sum_c f(c)f(c,b)f(c,b,a) =$

3. $\sum_a \sum_b \sum_c f(b)f(c)f(a,b,c)f(a,j)f(a,m) =$

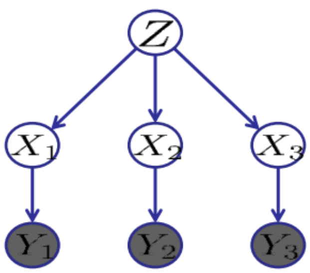
4. $\sum_d \sum_k \sum_c \sum_x f(c,k)f(x)f(c,k,x)f(k)f(c,d,k,x)f(k,x) =$

3 Bayesian Inference

For each of the following Bayes's Nets, identify the (conditional) independence assumptions it makes, and then derive expressions for the specified probabilities:



1. $P(X_1|E_1)$:
2. $P(X_2|E_1)$:
3. $P(X_2|E_1, E_2)$:



Derive $P(X_3|Y_1, Y_2, Y_3)$:

4 Expectation

Let's say we have a fair, 6-sided dice.

1. Let X_i be the result of the ith dice roll. What is $E[X_1]$?
2. What is $E[X_1^2]$?
3. What is $E[X_1 + X_2]$?
4. Let K be an arbitrary random variable, and let Y_K be the sum of the first K dice rolls. What is $E[Y|K]$?
5. We play a game where we first flip a fair coin. If the coin lands heads, then we roll the dice twice and score points equal to the sum. If the coin lands tails, then we roll three dice instead. What is our expected score?