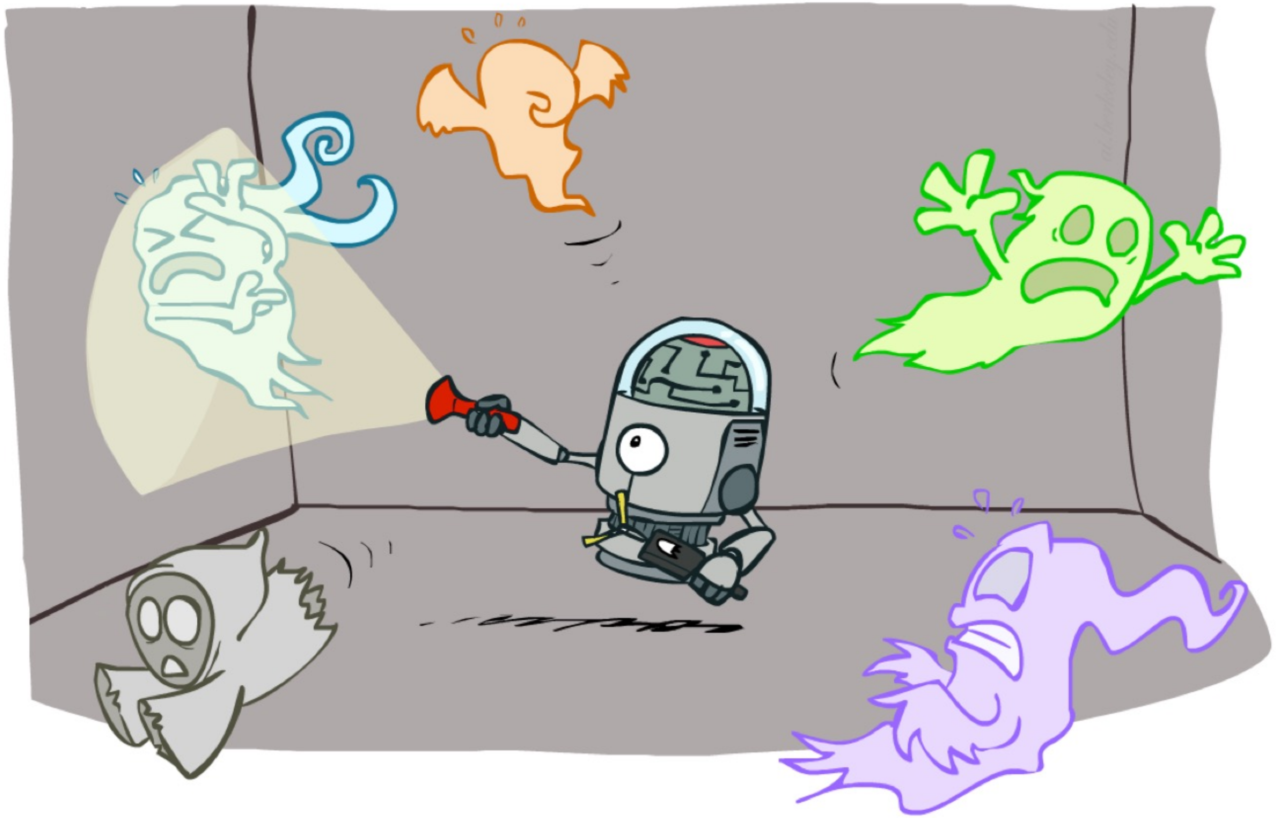
CS 188: Artificial Intelligence

Viterbi Algorithm and Particle Filters



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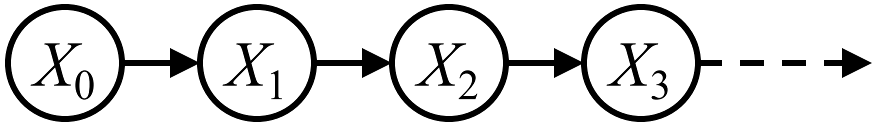
(Slides adapted from Pieter Abbeel, Dan Klein, Anca Dragan, Stuart Russell and Dawn Song)

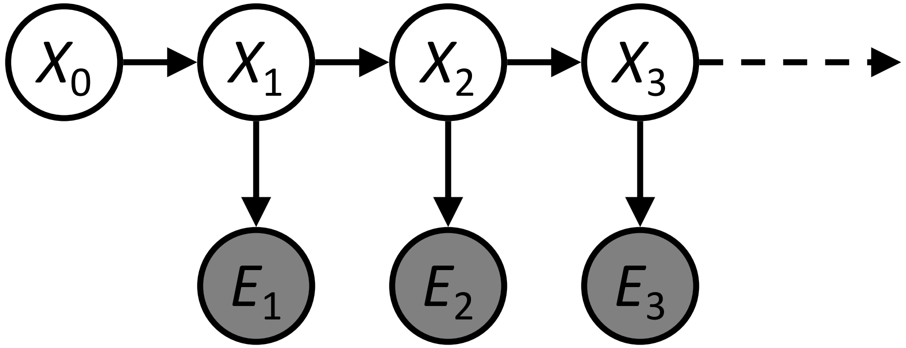
Markov Chains



$P(X_0)$ $P(X_t \mid X_{t-1})$

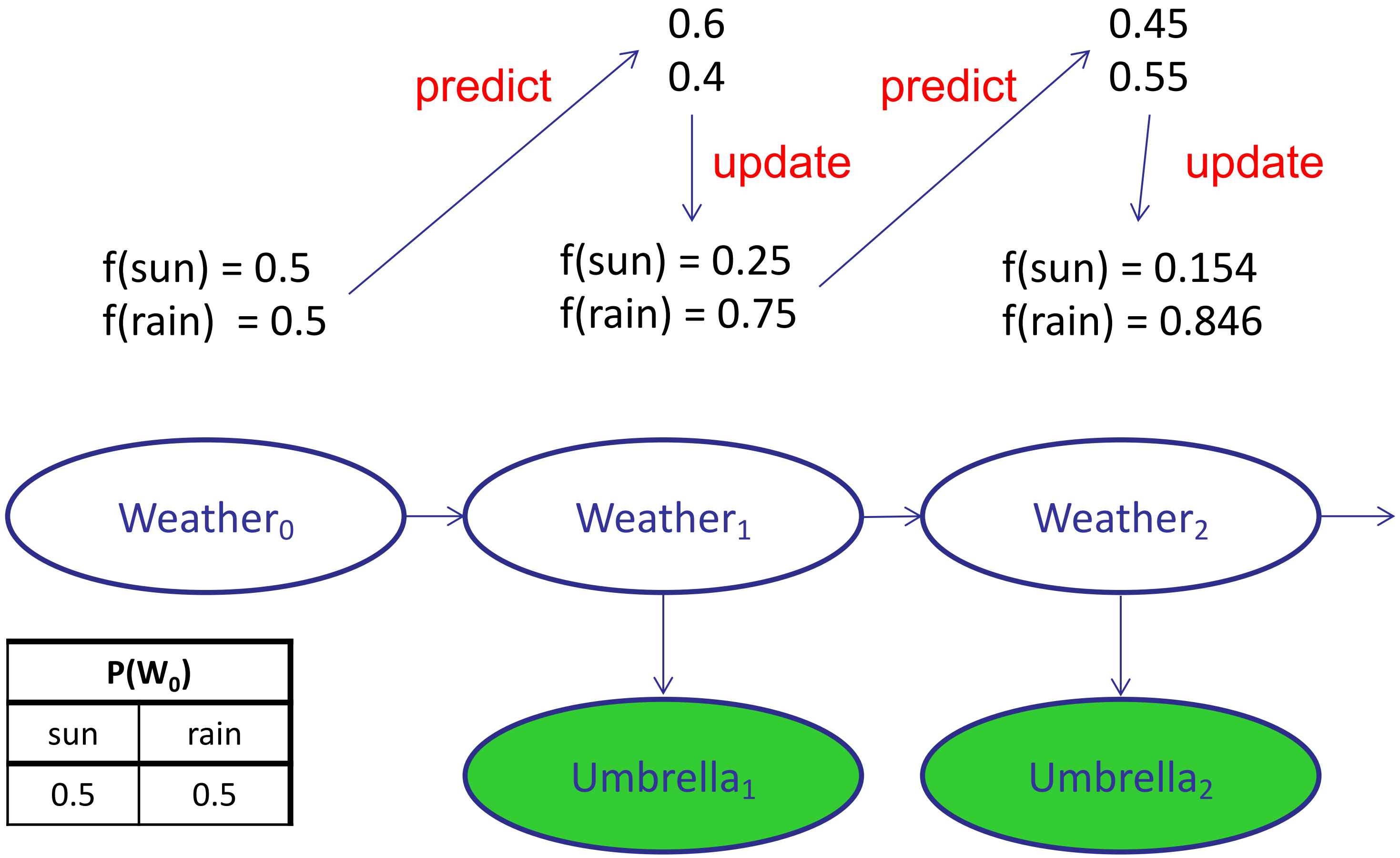
- ***Stationarity*** assumption: transition probabilities are the same at all times
- ***Markov*** assumption: "future is independent of the past given the present"
- Mini-Forward Algorithm: $P(X_t) = \sum_{x_{t-1}} P(X_{t-1}=x_{t-1})\, P(X_t \mid X_{t-1}=x_{t-1})$
- Equivalently: $P_{t+1} = T^{T} P_t$
- Stationary Distribution: $P_\infty = T^{T} P_\infty$
- Stationary distribution does not depend on the starting distribution

Hidden Markov Models



- Sensor models are the same at all times
- Current evidence is independent of everything else given the current state
- Filtering: calculate the distribution $\boldsymbol{f}_{1:t} = P(X_t | e_{1:t})$.
- Forward Algorithm: Predict (Time Elapse), Update, Normalize.
- Forward Algorithm: $P(X_{t+1} | e_{1:t+1}) = \alpha\, P(e_{t+1} | X_{t+1}) \sum_{x_t} P(x_t \mid e_{1:t})\, P(X_{t+1} | x_t)$
- Equivalently: $\boldsymbol{f}_{1:t+1} = \alpha\, O_{t+1} T^{\top} \boldsymbol{f}_{1:t}$
- Most likely explanation: $\arg\max_{x_{1:t}} P(x_{1:t} \mid e_{1:t})$

Example: Weather HMM



P(W_0)	
sun	rain
0.5	0.5

W_{t-1}	P(W_t|W_{t-1})	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7

W_t	P(U_t|W_t)	
	true	false
sun	0.2	0.8
rain	0.9	0.1

Example: Passage of Time

- As time passes, uncertainty “accumulates”

(Transition model: ghosts usually go clockwise)

<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	1.00	<0.01	<0.01	<0.01
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01

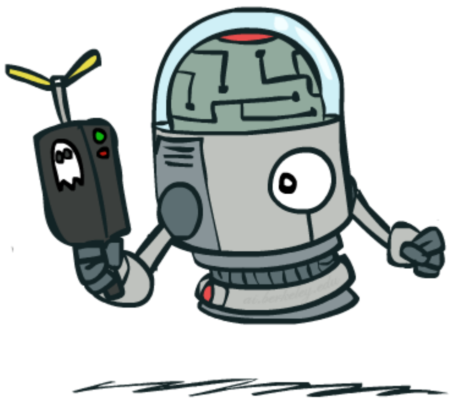
T = 1

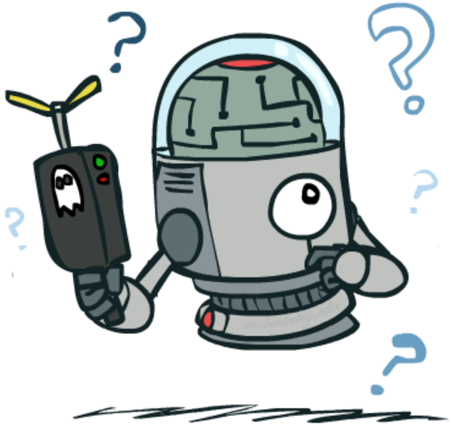
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	0.06	<0.01	<0.01	<0.01
<0.01	0.76	0.06	0.06	<0.01	<0.01
<0.01	<0.01	0.06	<0.01	<0.01	<0.01

T = 2

0.05	0.01	0.05	<0.01	<0.01	<0.01
0.02	0.14	0.11	0.35	<0.01	<0.01
0.07	0.03	0.05	<0.01	0.03	<0.01
0.03	0.03	<0.01	<0.01	<0.01	<0.01

T = 5





Example: Observation

- As we get observations, beliefs get reweighted, uncertainty "decreases"

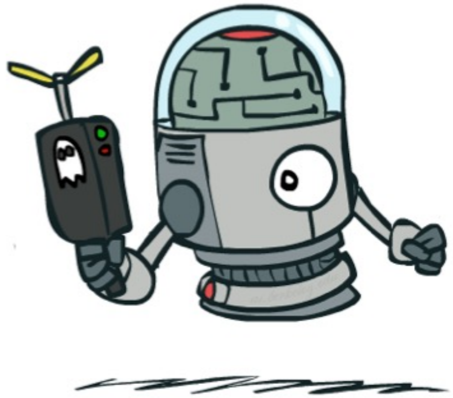
0.05	0.01	0.05	<0.01	<0.01	<0.01
0.02	0.14	0.11	0.35	<0.01	<0.01
0.07	0.03	0.05	<0.01	0.03	<0.01
0.03	0.03	<0.01	<0.01	<0.01	<0.01

Before observation

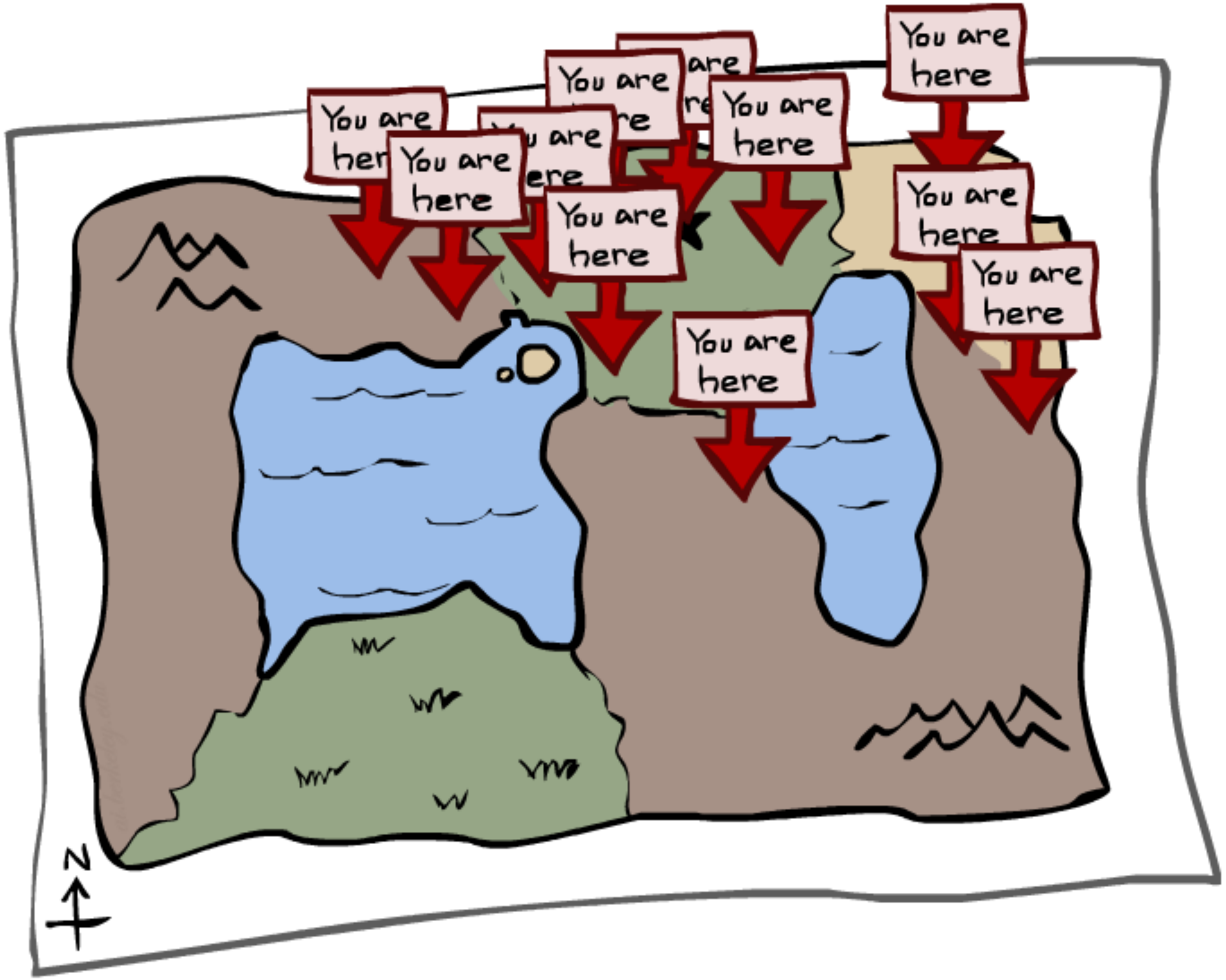
<0.01	<0.01	<0.01	<0.01	0.02	<0.01
<0.01	<0.01	<0.01	0.83	0.02	<0.01
<0.01	<0.01	0.11	<0.01	<0.01	<0.01
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01

After observation

$$B(X) \propto P(e|X)B'(X)$$

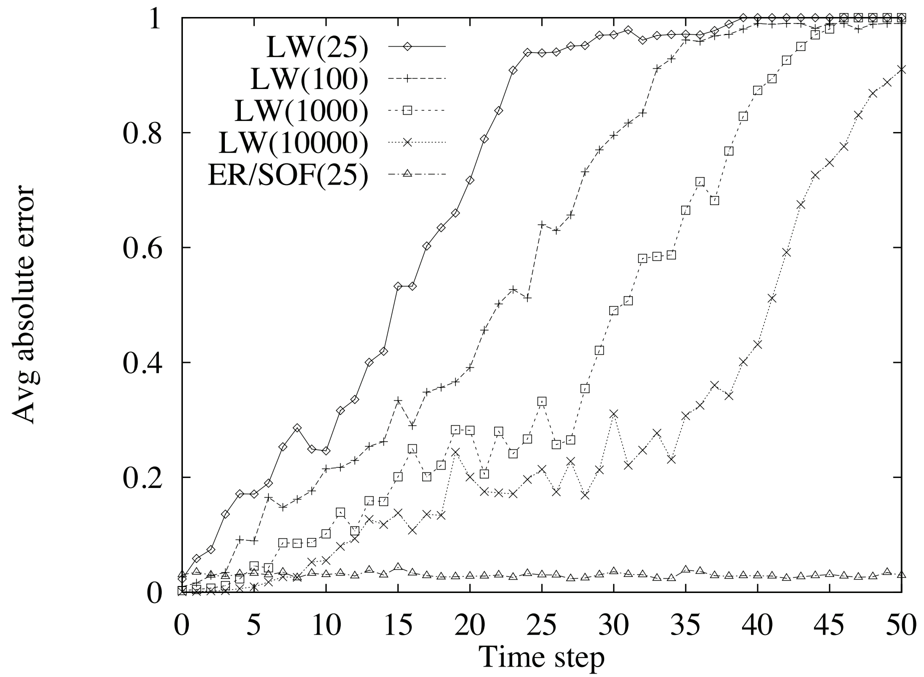


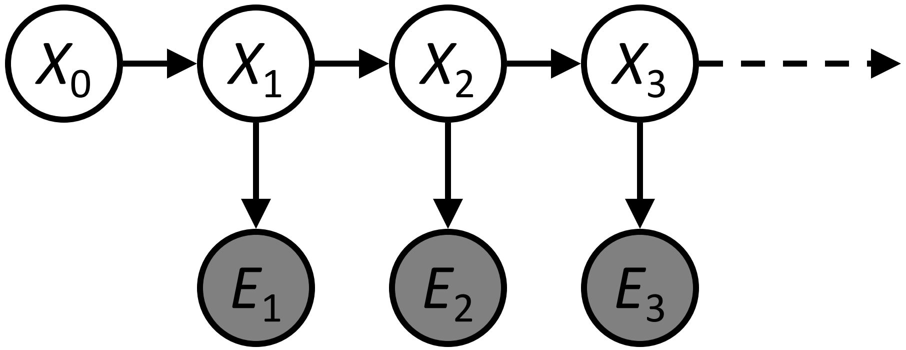
Particle Filtering



We need a new algorithm!

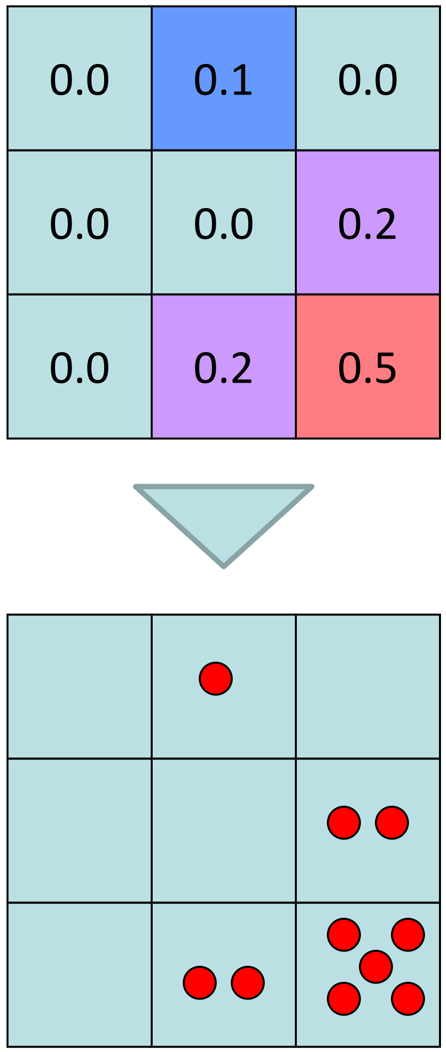
- When size of the state space is large, exact inference becomes infeasible
- Likelihood weighting also fails completely – number of samples needed grows ***exponentially*** with *T*





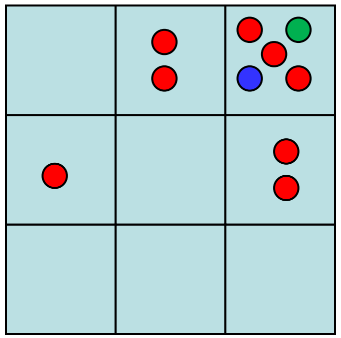
Particle Filtering

- Solution: approximate inference
  - Track samples of X, not all values
  - Samples are called particles
  - Time per step is linear in the number of samples
  - But: number needed may be large
  - In memory: list of particles, not states



Representation: Particles

- Our representation of $P(X)$ is now a list of $N << |X|$ particles
- $P(x)$ approximated by number of particles with value x
  - So, many x may have $P(x) = 0$!
  - More particles => more accuracy
- For now, all particles have a weight of 1



Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

Particle Filtering: Prediction Step

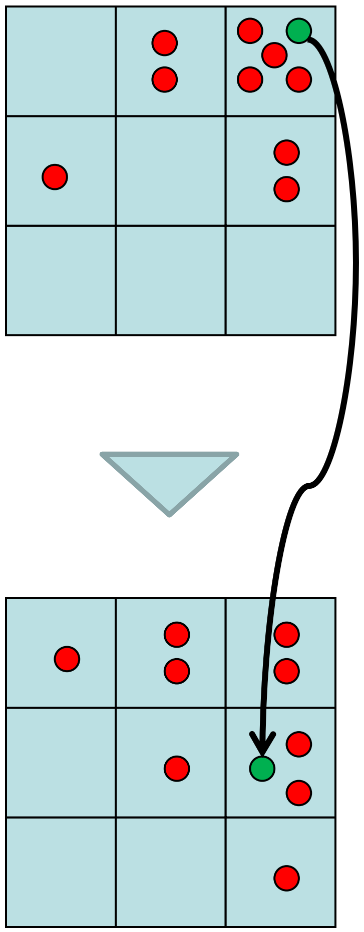
- Each particle is moved by sampling its next position from the transition model

$$x' = \text{sample}(P(X'|x))$$

  - This is like prior sampling – samples' frequencies reflect the transition probabilities
  - Here, most samples move clockwise, but some move in another direction or stay in place

- This captures the passage of time
  - If enough samples, close to exact values before and after (consistent)

Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

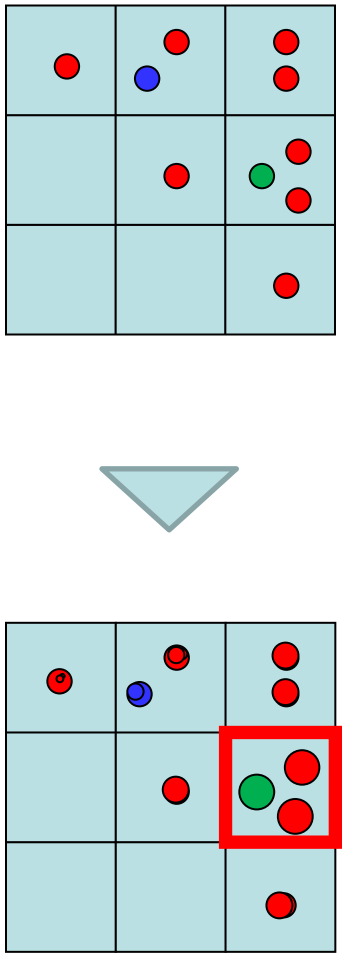


Particles:
(3,2)
(2,3)
(3,2)
(3,1)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(2,2)

Particle Filtering: Update step

- After observing e_{t+1} :
  - As in likelihood weighting, weight each sample based on the evidence
    - $w^{(j)} = P(e_{t+1} | x_{t+1}^{(j)})$
  - Normalize the weights: particles that fit the data better get higher weights, others get lower weights

Particles:
(3,2)
(2,3)
(3,2)
(3,1)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(2,2)



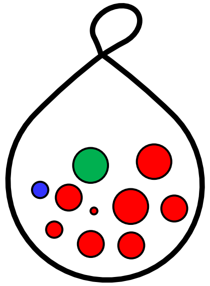
Particles:
(3,2) w=.9
(2,3) w=.2
(3,2) w=.9
(3,1) w=.4
(3,3) w=.4
(3,2) w=.9
(1,3) w=.1
(2,3) w=.2
(3,2) w=.9
(2,2) w=.4

Particle Filtering: Resample

- Rather than tracking weighted samples, we ***resample***

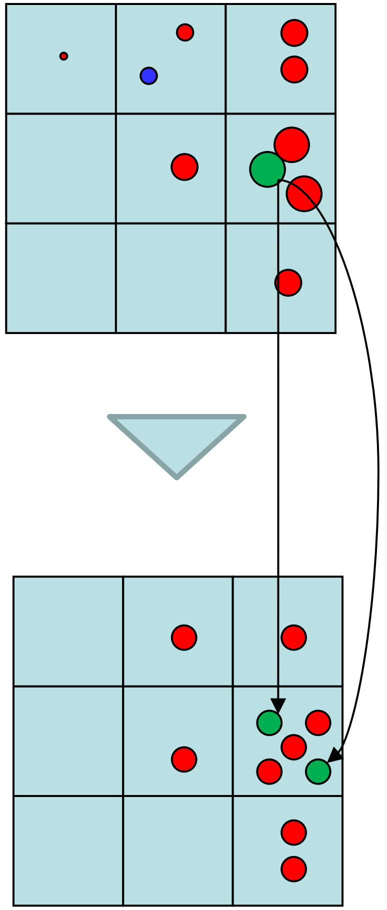
- *N* times, we choose from our weighted sample distribution (i.e., draw with replacement)

- Now the update is complete for this time step, continue with the next one (with weights reset to 1)

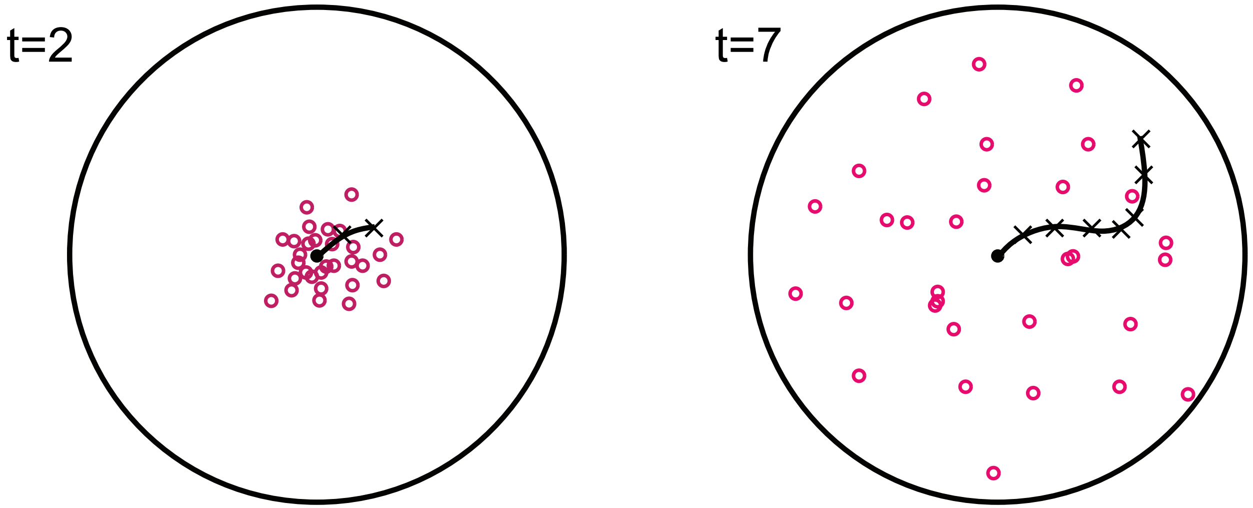


Particles:
(3,2) w=.9
(2,3) w=.2
(3,2) w=.9
(3,1) w=.4
(3,3) w=.4
(3,2) w=.9
(1,3) w=.1
(2,3) w=.2
(3,2) w=.9
(2,2) w=.4

(New) Particles:
(3,2)
(2,2)
(3,2)
(2,3)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(3,2)



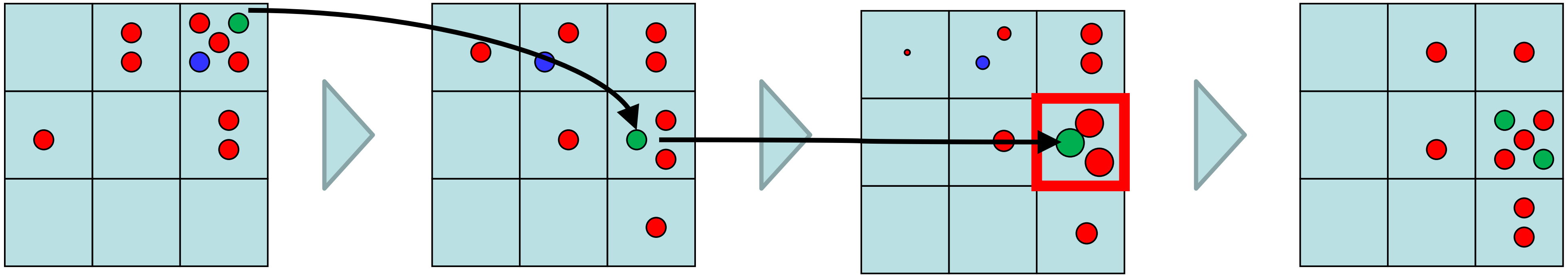
Particle Filtering: Resample



- The problem of likelihood weighting: sample state trajectories go off into low-probability regions; too few “reasonable” samples
- Solution: kill the bad ones, make more of the good ones
- This way the population of samples stays in the high-probability region

Summary: Particle Filtering

- Particles: track samples of states rather than an explicit distribution



Particles:	Particles:	Particles:	(New) Particles:
(3,3)	(3,2)	(3,2) w=.9	(3,2)
(2,3)	(2,3)	(2,3) w=.2	(2,2)
(3,3)	(3,2)	(3,2) w=.9	(3,2)
(3,2)	(3,1)	(3,1) w=.4	(2,3)
(3,3)	(3,3)	(3,3) w=.4	(3,3)
(3,2)	(3,2)	(3,2) w=.9	(3,2)
(1,2)	(1,3)	(1,3) w=.1	(1,3)
(3,3)	(2,3)	(2,3) w=.2	(2,3)
(3,3)	(3,2)	(3,2) w=.9	(3,2)
(2,3)	(2,2)	(2,2) w=.4	(3,2)

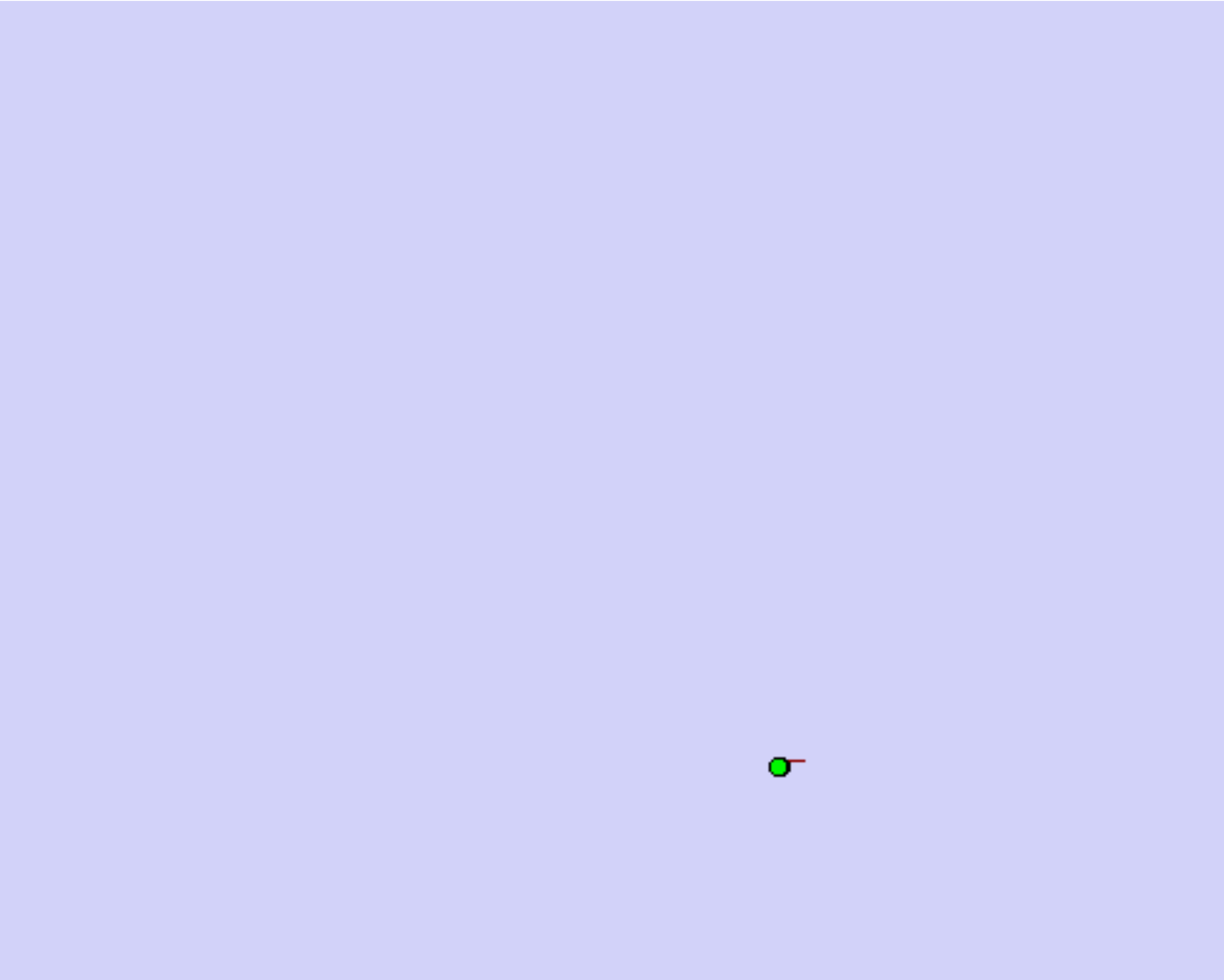
Consistency: see proof in AIMA Ch. 14

[Demos: ghostbusters particle filtering (L15D3,4,5)]

Particle Filter Localization (Sonar)

[Video: global-sonar-uw-annotated.avi]

Particle Filter SLAM



[Demo: PARTICLES-SLAM-fastslam.avi]

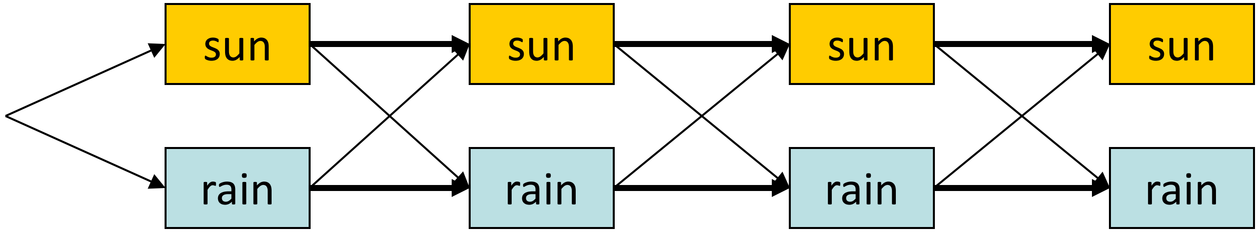
Most Likely Explanation

Most Likely Explanation

- ***Most likely explanation***: $\arg\max_{x_{1:t}} P(x_{1:t} \mid e_{1:t})$

Most likely explanation = most probable path

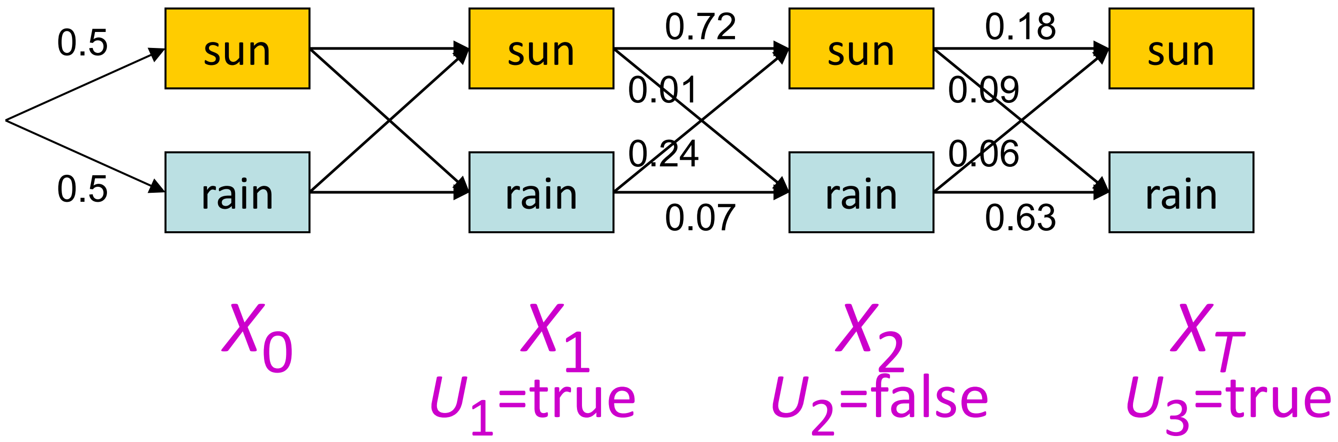
- ***State trellis***: graph of states and transitions over time



X_0 X_1 … X_T

- $\arg\max_{x_{1:t}} P(x_{1:t} \mid e_{1:t}) = \arg\max_{x_{1:t}} P(x_0) \prod_t P(x_t \mid x_{t-1}) P(e_t \mid x_t)$
- Each arc represents some transition $x_{t-1} \rightarrow x_t$
- Each arc has weight $P(x_t \mid x_{t-1}) P(e_t \mid x_t)$ (arcs to initial states have weight $P(x_0)$)
- The ***product*** of weights on a path is proportional to that state sequence's probability
- Forward algorithm computes sums of paths, ***Viterbi algorithm*** computes best paths

Viterbi algorithm

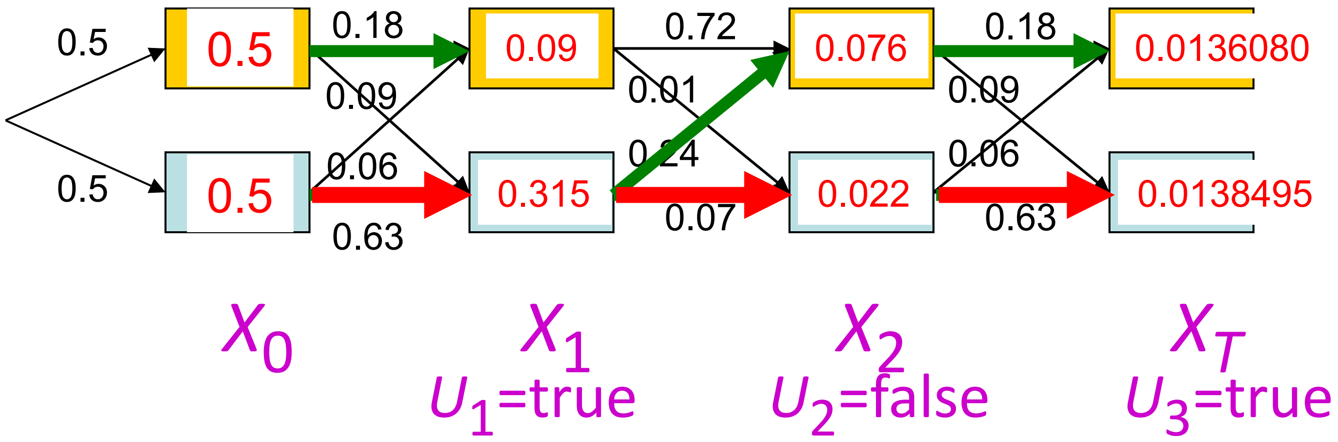


W_{t-1}	$P(W_t \mid W_{t-1})$	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7

W_t	$P(U_t \mid W_t)$	
	true	false
sun	0.2	0.8
rain	0.9	0.1

- Each arc has weight $P(x_t \mid x_{t-1})\, P(e_t \mid x_t)$ (arcs to initial states have weight $P(x_0)$)
- The ***product*** of weights on a path is proportional to that state sequence's probability
- The best way to go to a state S in timestep t+1 is first going to some state S' in timestep t with the best way, and then go from S' to S at timestep t+1.

Viterbi algorithm contd.

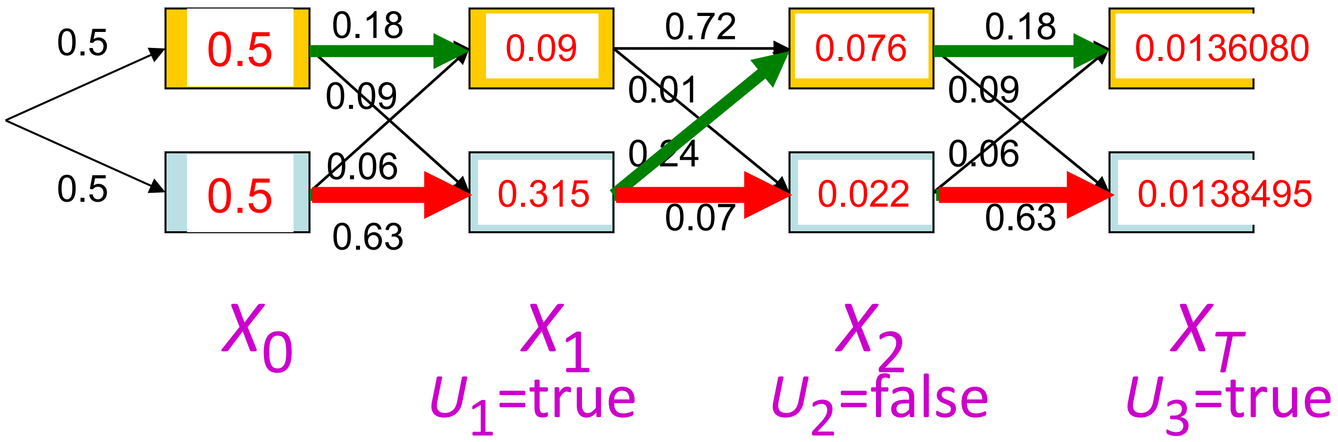


W_{t-1}	$P(W_t \mid W_{t-1})$	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7

W_t	$P(U_t \mid W_t)$	
	true	false
sun	0.2	0.8
rain	0.9	0.1

- Each arc has weight $P(x_t \mid x_{t-1})\, P(e_t \mid x_t)$ (arcs to initial states have weight $P(x_0)$)
- The ***product*** of weights on a path is proportional to that state sequence's probability
- The best way to go to a state S in timestep t+1 is first going to some state S' in timestep t with the best way, and then go from S' to S at timestep t+1.

Viterbi algorithm contd.



W_{t-1}	$P(W_t \mid W_{t-1})$	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7

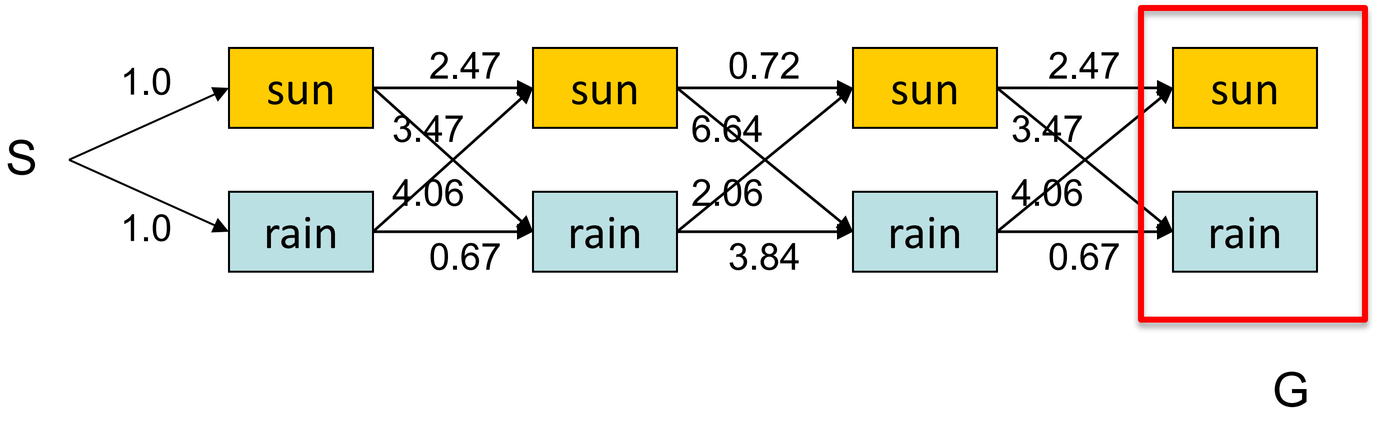
W_t	$P(U_t \mid W_t)$	
	true	false
sun	0.2	0.8
rain	0.9	0.1

Time complexity?
$O(|X|^2 T)$

Space complexity?
$O(|X| T)$

Number of paths?
$O(|X|^T)$

Viterbi in negative log space



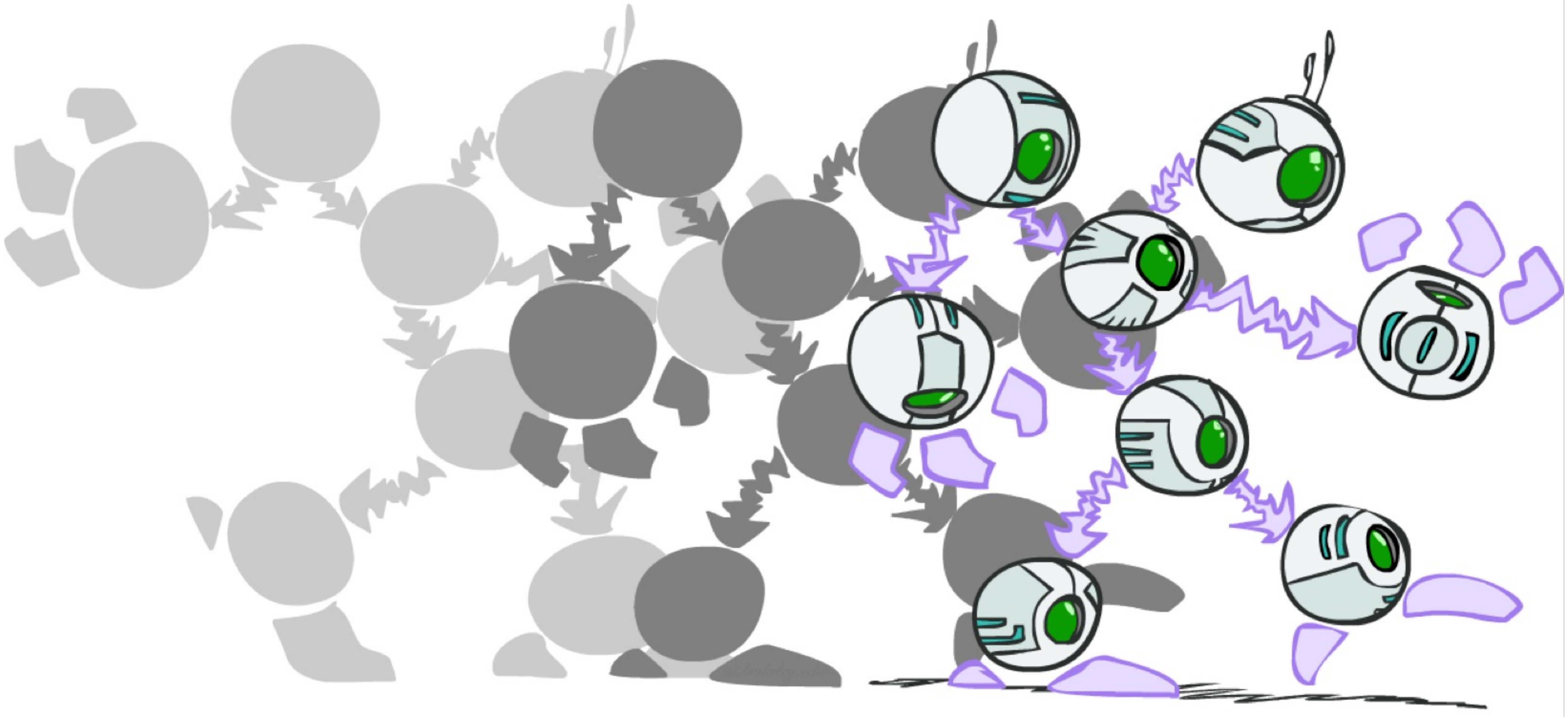
W_{t-1}	$P(W_t \mid W_{t-1})$	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7

W_t	$P(U_t \mid W_t)$	
	true	false
sun	0.2	0.8
rain	0.9	0.1

argmax of product of probabilities

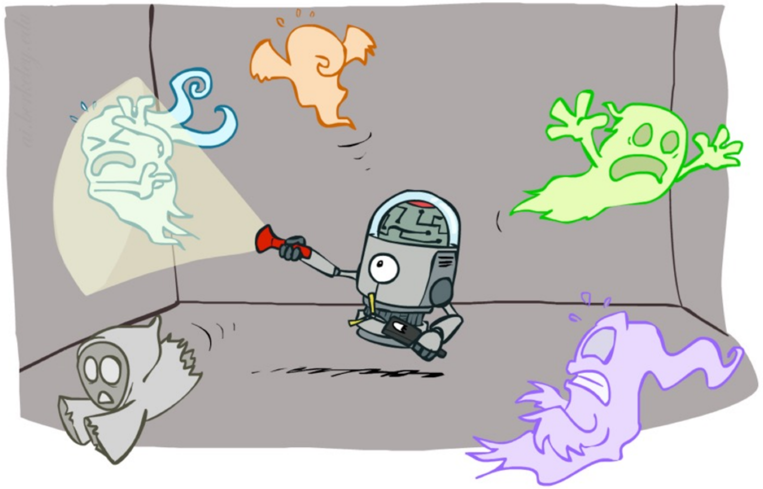
= argmin of sum of negative log probabilities

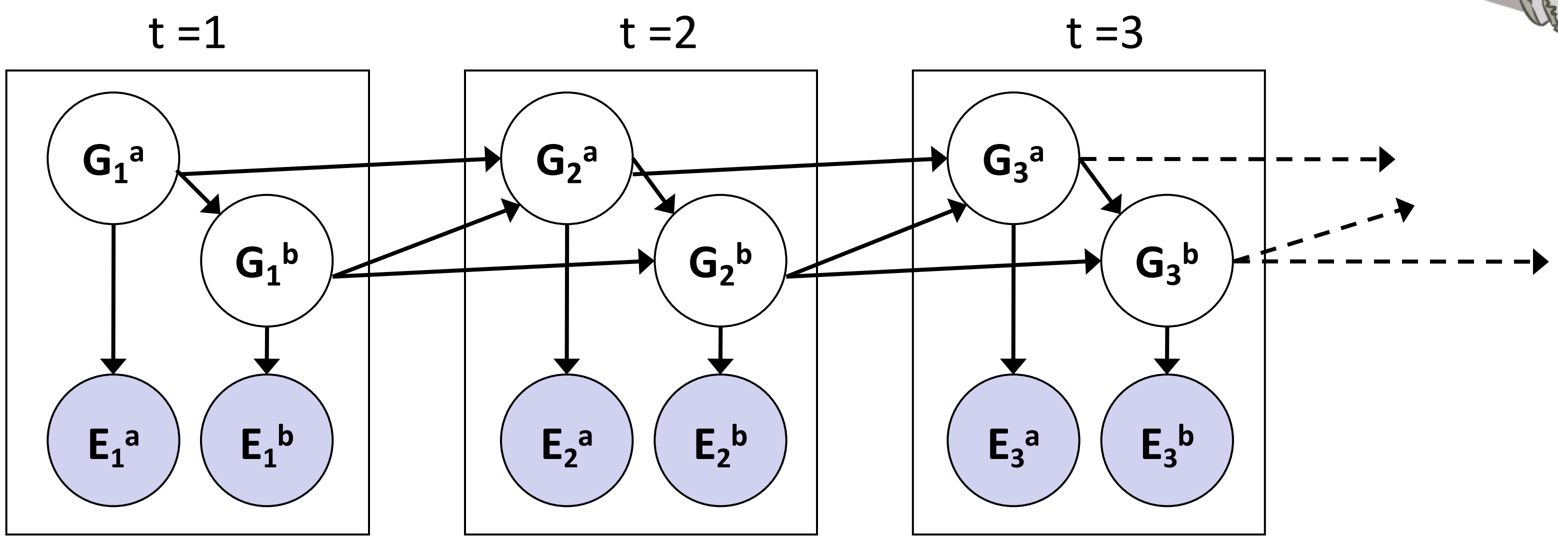
Dynamic Bayes Nets

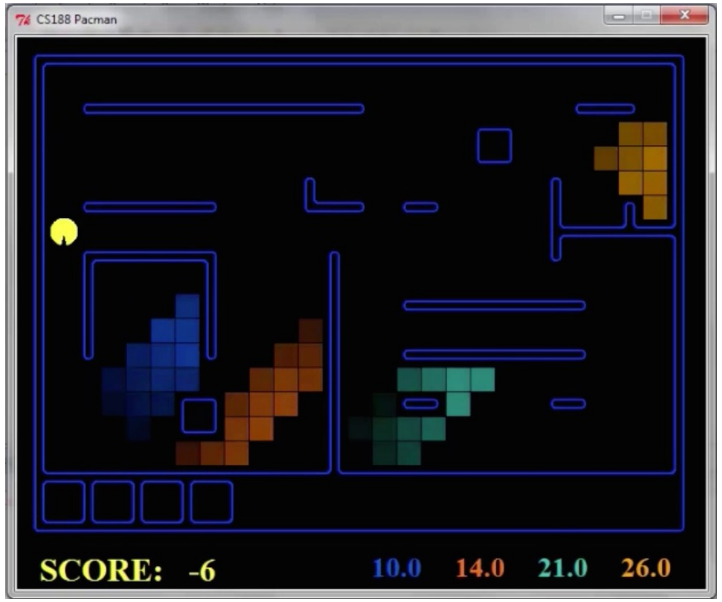


Dynamic Bayes Nets (DBNs)

- We want to track multiple variables over time, using multiple sources of evidence
- Idea: Repeat a fixed Bayes net structure at each time
- Variables at time *t* can have parents at time *t-1*

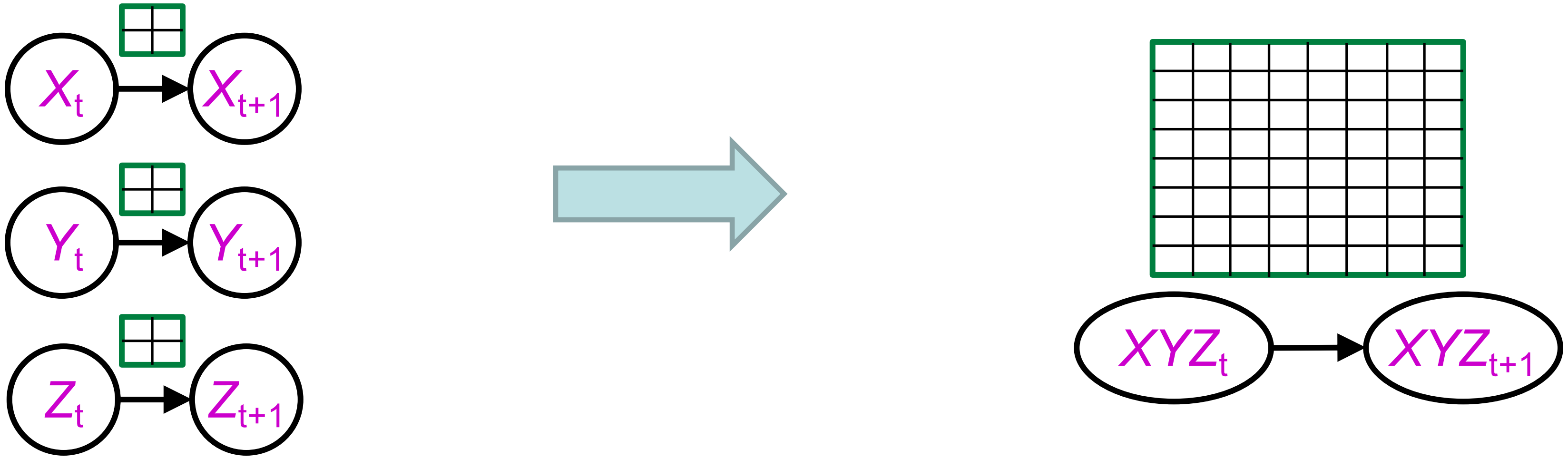






DBNs and HMMs

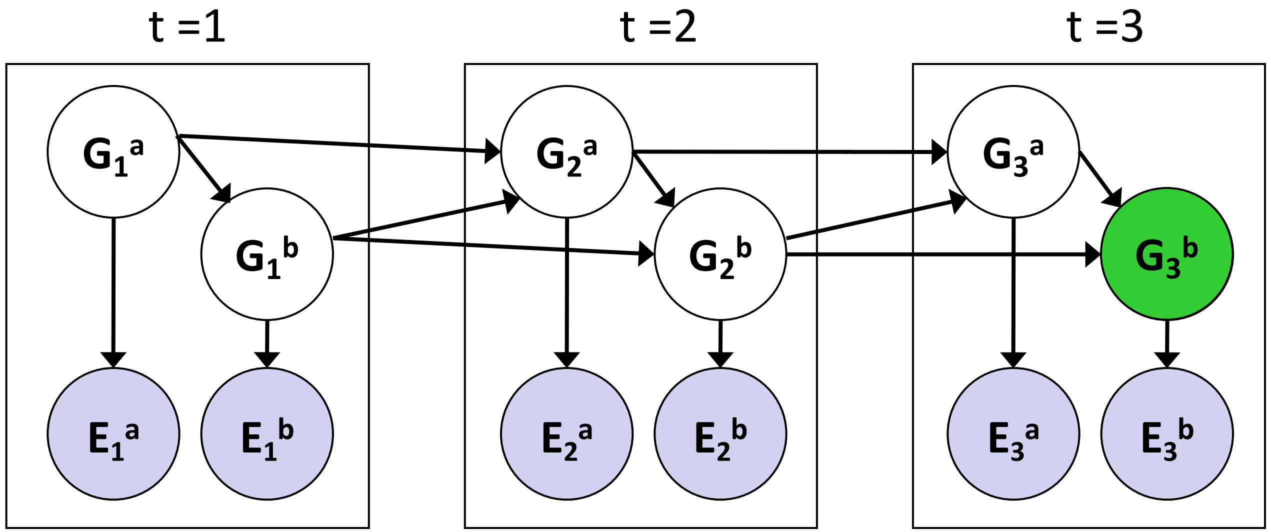
- Every HMM is a single-variable DBN
- Every discrete DBN is an HMM
  - HMM state is Cartesian product of DBN state variables



- Sparse dependencies => exponentially fewer parameters in DBN
  - E.g., 20 state variables, 3 parents each;
    DBN has $20 \times 2^3 = 160$ parameters, HMM has $2^{20} \times 2^{20} =\sim 10^{12}$ parameters

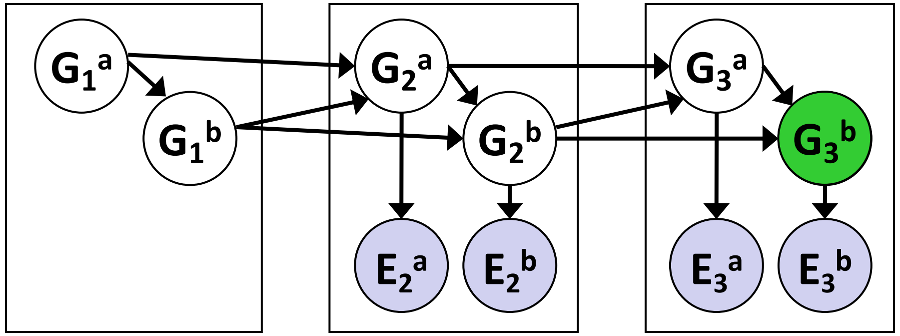
Exact Inference in DBNs

- Variable elimination applies to dynamic Bayes nets
- Offline: "unroll" the network for T time steps, then eliminate variables to find $P(X_T|e_{1:T})$



- Online: eliminate all variables from the previous time step; store factors for current time only
- Problem: largest factor contains all variables for current time (plus a few more)

DBN Particle Filters



- A particle is a complete sample for a time step
- **Initialize**: Generate prior samples for the t=1 Bayes net
  - Example particle: $\mathbf{G_1^a}$ = (3,3) $\mathbf{G_1^b}$ = (5,3)
- **Elapse time**: Sample a successor for each particle
  - Example successor: $\mathbf{G_2^a}$ = (2,3) $\mathbf{G_2^b}$ = (6,3)
- **Observe**: Weight each *entire* sample by the likelihood of the evidence conditioned on the sample
  - Likelihood: $P(\mathbf{E_2^a} \mid \mathbf{G_2^a}) * P(\mathbf{E_2^b} \mid \mathbf{G_2^b})$
- **Resample:** Select prior samples (tuples of values) in proportion to their likelihood