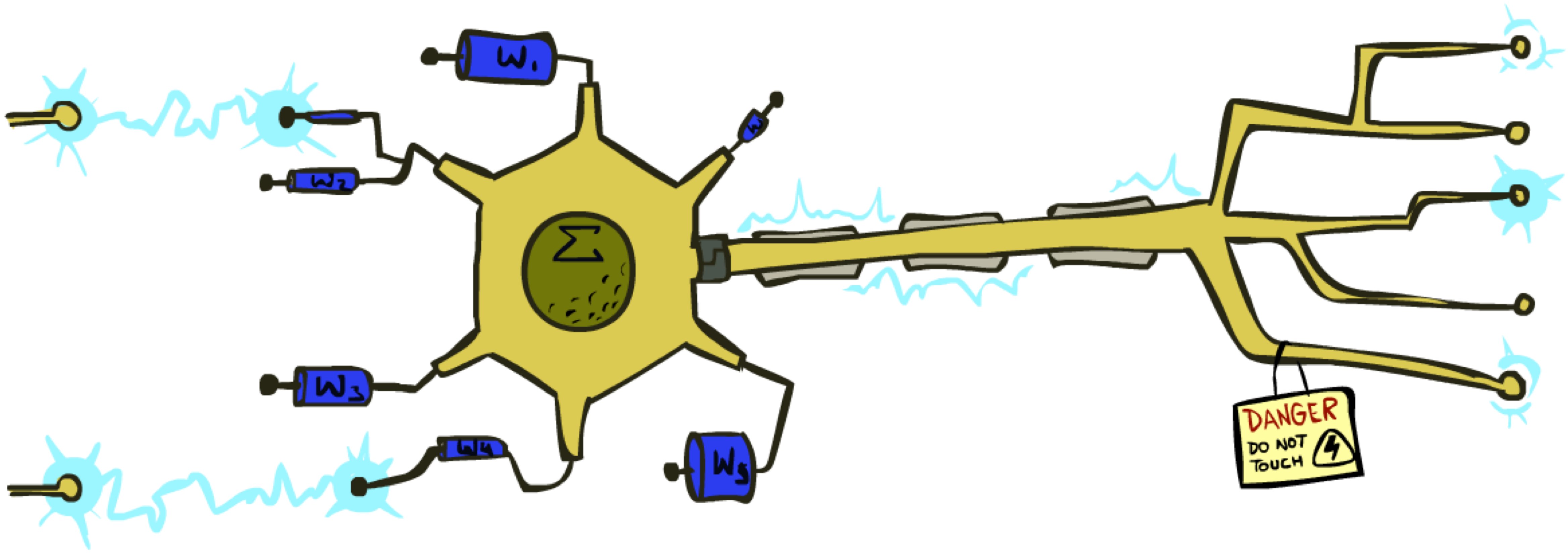
CS 188: Artificial Intelligence

Neural Networks

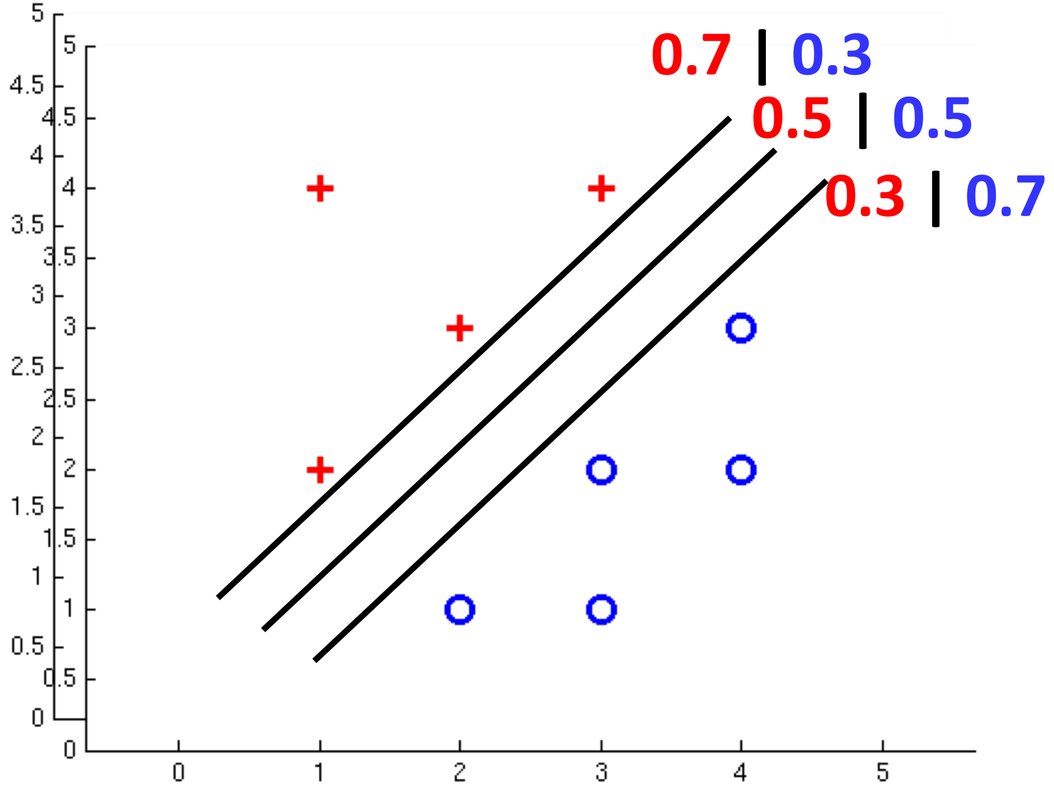


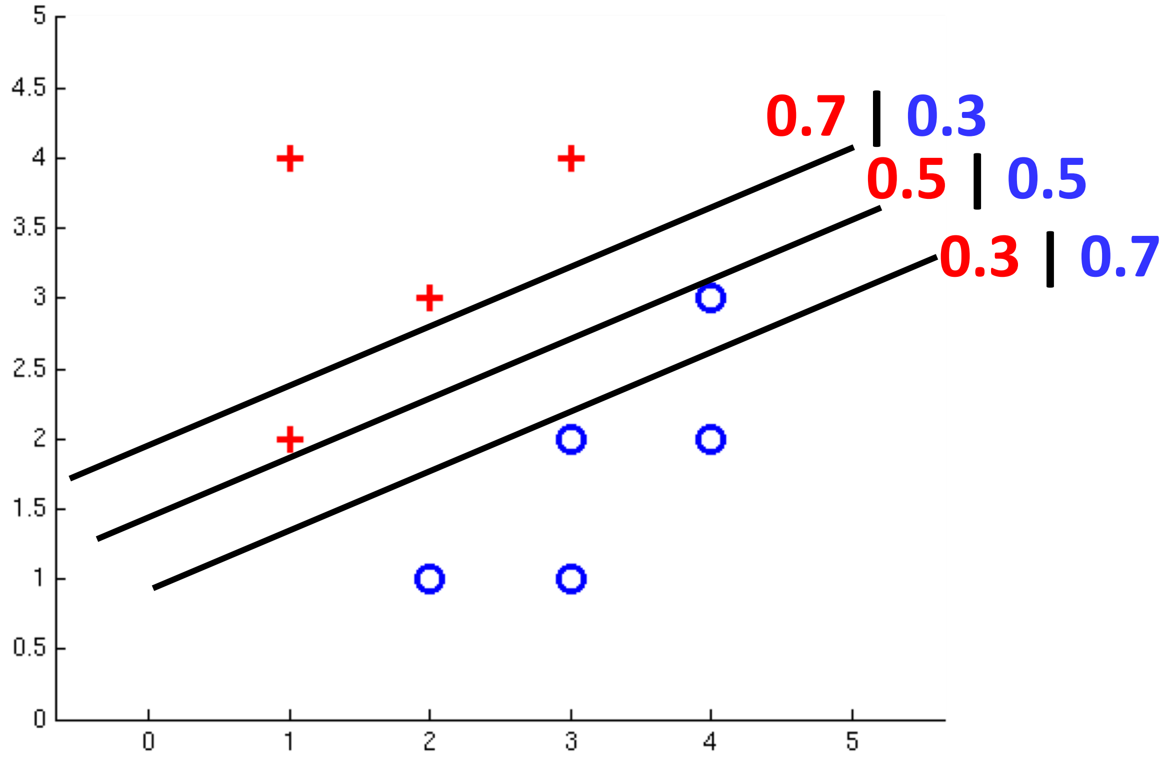
Instructors: Angela Liu and Yanlai Yang

University of California, Berkeley

(Slides adapted from Pieter Abbeel, Dan Klein, Anca Dragan, Stuart Russell and Dawn Song)

Separable Case: Probabilistic Decision – Clear Preference





Best w?

- Maximum likelihood estimation:

$$\max_w \quad ll(w) = \max_w \quad \sum_i \log P(y^{(i)}|x^{(i)}; w)$$

with:

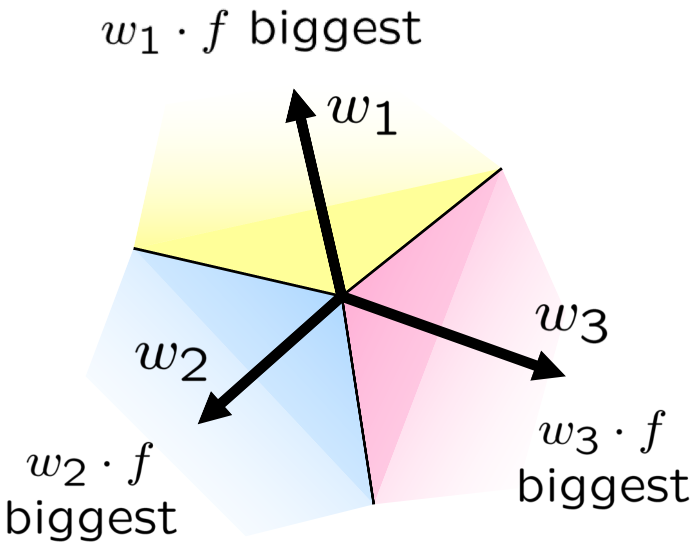
$$P(y^{(i)} = +1|x^{(i)}; w) = \frac{1}{1 + e^{-w \cdot f(x^{(i)})}}$$

$$P(y^{(i)} = -1|x^{(i)}; w) = 1 - \frac{1}{1 + e^{-w \cdot f(x^{(i)})}}$$

= Logistic Regression

Multiclass Logistic Regression

- Recall Perceptron:
  - A weight vector for each class: w_y
  - Score (activation) of a class y: $w_y \cdot f(x)$
  - Prediction highest score wins $y = \arg\max_y \; w_y \cdot f(x)$



- How to make the scores into probabilities?

$$\underbrace{z_1, z_2, z_3}_{\text{original activations}} \rightarrow \underbrace{\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}, \frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}, \frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}}_{\text{softmax activations}}$$

Best w?

- Maximum likelihood estimation:

$$\max_w \quad ll(w) = \max_w \quad \sum_i \log P(y^{(i)}|x^{(i)}; w)$$

with:
$$P(y^{(i)}|x^{(i)}; w) = \frac{e^{w_{y^{(i)}} \cdot f(x^{(i)})}}{\sum_y e^{w_y \cdot f(x^{(i)})}}$$

= Multi-Class Logistic Regression

Optimization

Optimization

i.e., how do we solve:

$$\max_w \; ll(w) = \max_w \; \sum_i \log P(y^{(i)}|x^{(i)}; w)$$

Hill Climbing

Recall from CSPs lecture: simple, general idea

Start wherever

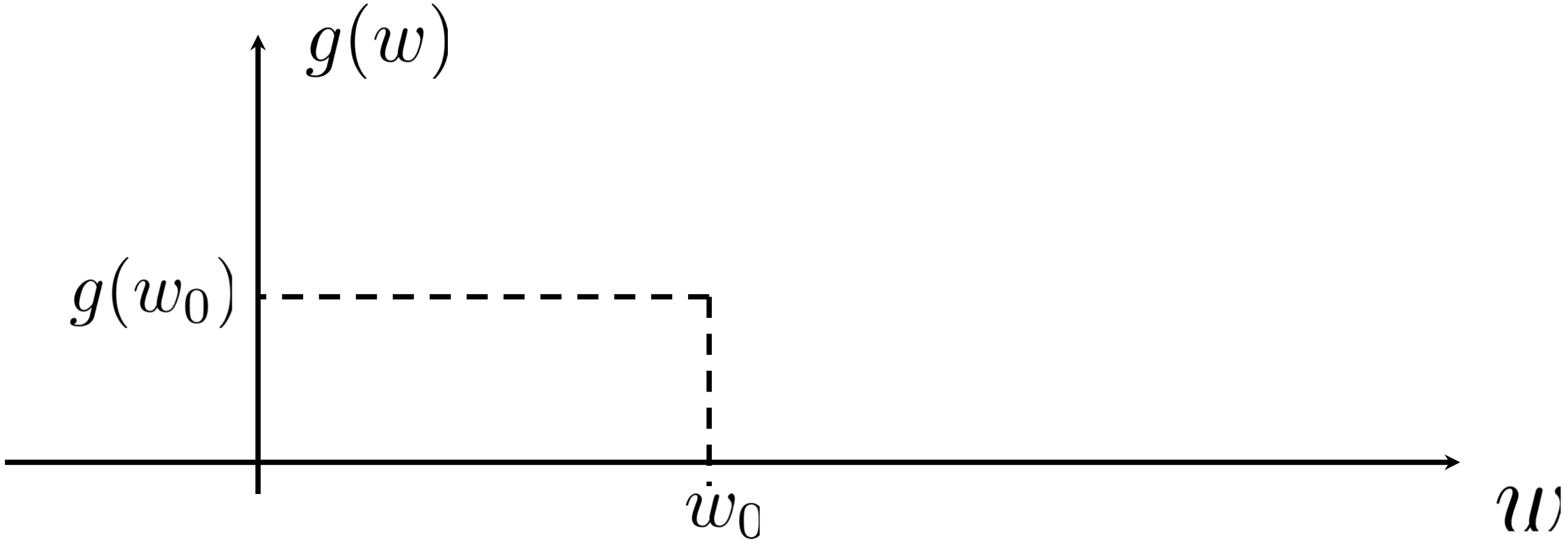
Repeat: move to the best neighboring state

If no neighbors better than current, quit

What's particularly tricky when hill-climbing for multiclass logistic regression?

- Optimization over a continuous space
  - Infinitely many neighbors!
  - How to do this efficiently?

1-D Optimization



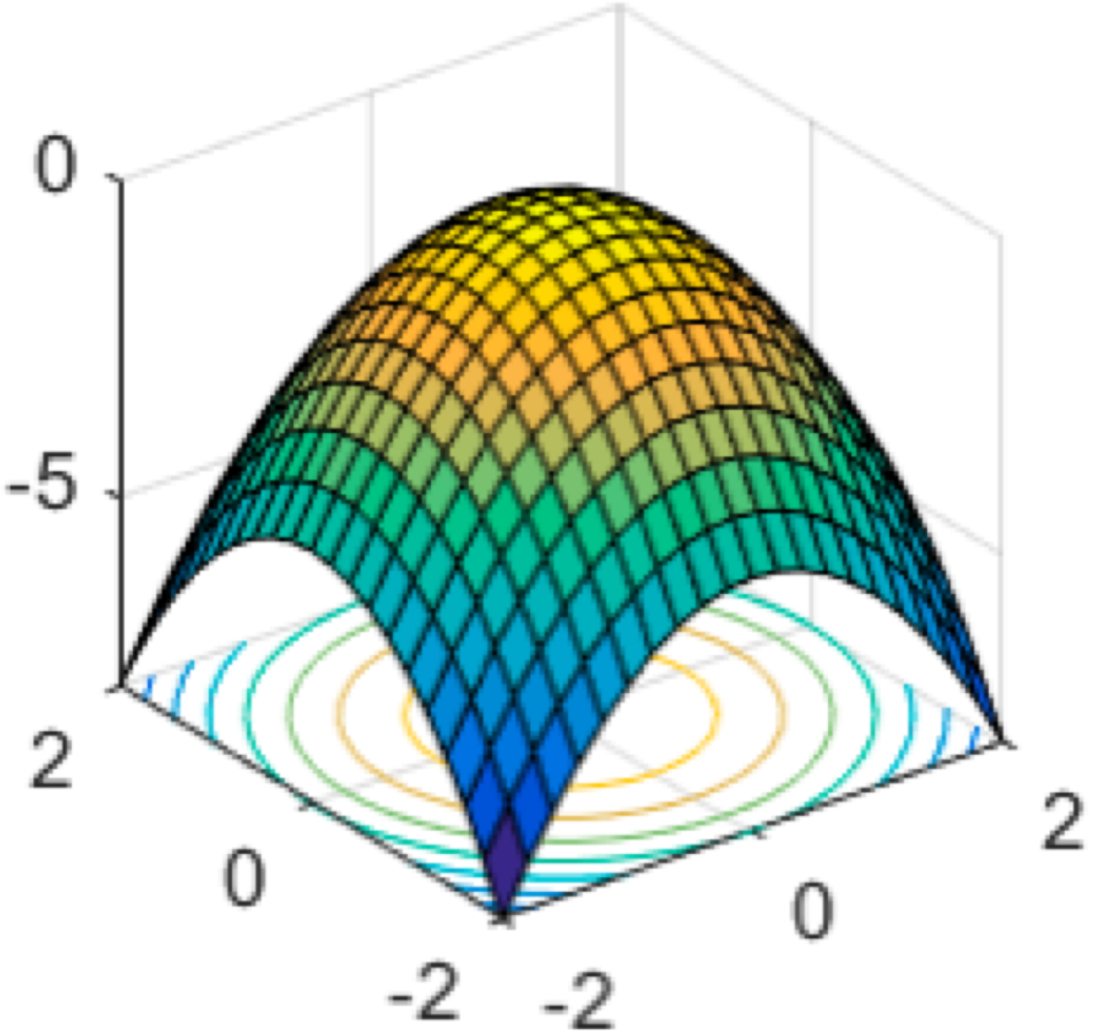
Could evaluate $g(w_0 + h)$ and $g(w_0 - h)$

Then step in best direction

Or, evaluate derivative: $\frac{\partial g(w_0)}{\partial w} = \lim_{h \to 0} \frac{g(w_0 + h) - g(w_0 - h)}{2h}$

Tells which direction to step into

2-D Optimization



Source: offconvex.org

Gradient Ascent

Perform update in uphill direction for each coordinate

The steeper the slope (i.e. the higher the derivative) the bigger the step for that coordinate

E.g., consider: $g(w_1, w_2)$

Updates:

$$w_1 \leftarrow w_1 + \alpha * \frac{\partial g}{\partial w_1}(w_1, w_2)$$

$$w_2 \leftarrow w_2 + \alpha * \frac{\partial g}{\partial w_2}(w_1, w_2)$$

- Updates in vector notation:

$$w \leftarrow w + \alpha * \nabla_w g(w)$$

with: $\nabla_w g(w) = \begin{bmatrix} \frac{\partial g}{\partial w_1}(w) \\ \frac{\partial g}{\partial w_2}(w) \end{bmatrix}$ **= gradient**

Gradient Ascent

Idea:

Start somewhere

Repeat: Take a step in the gradient direction

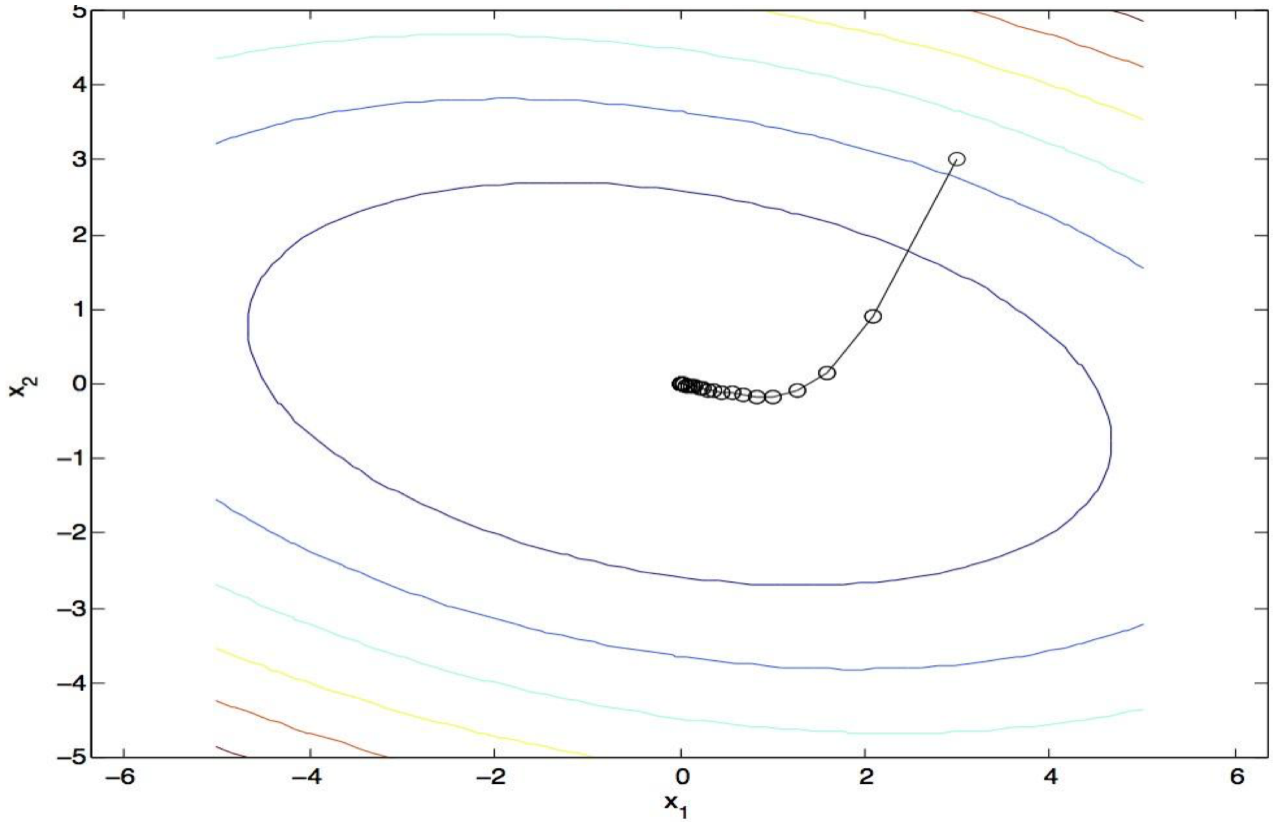


Figure source: Mathworks

Gradient in n dimensions

$$\nabla g = \begin{bmatrix} \frac{\partial g}{\partial w_1} \\ \frac{\partial g}{\partial w_2} \\ \cdots \\ \frac{\partial g}{\partial w_n} \end{bmatrix}$$

Optimization Procedure: Gradient Ascent

```
init w
for iter = 1, 2, …
```

$$w \leftarrow w + \alpha * \nabla g(w)$$

- α: learning rate --- tweaking parameter that needs to be chosen carefully

Batch Gradient Ascent on the Log Likelihood Objective

$$\max_w \quad ll(w) = \max_w \quad \underbrace{\sum_i \log P(y^{(i)}|x^{(i)}; w)}_{g(w)}$$

```
init w
for iter = 1, 2, …
```

$$w \leftarrow w + \alpha * \sum_i \nabla \log P(y^{(i)}|x^{(i)}; w)$$

Stochastic Gradient Ascent on the Log Likelihood Objective

$$\max_w \quad ll(w) = \max_w \quad \sum_i \log P(y^{(i)}|x^{(i)};w)$$

Observation: once gradient on one training example has been computed, might as well incorporate before computing next one

```
init w
for iter = 1, 2, …
  pick random j
```

$$w \leftarrow w + \alpha * \nabla \log P(y^{(j)}|x^{(j)};w)$$

Mini-Batch Gradient Ascent on the Log Likelihood Objective

$$\max_w \quad ll(w) = \max_w \quad \sum_i \log P(y^{(i)}|x^{(i)}; w)$$

Observation: gradient over small set of training examples (=mini-batch) can be computed in parallel, might as well do that instead of a single one

```
init w
for iter = 1, 2, …
   pick random subset of training examples J
```

$$w \leftarrow w + \alpha * \sum_{j \in J} \nabla \log P(y^{(j)}|x^{(j)}; w)$$

What will gradient ascent do in multi-class logistic regression?

$$w \leftarrow w + \alpha * \sum_i \nabla \log P(y^{(i)}|x^{(i)}; w)$$

$$P(y^{(i)}|x^{(i)}; w) = \frac{e^{w_{y^{(i)}} \cdot f(x^{(i)})}}{\sum_y e^{w_y \cdot f(x^{(i)})}}$$

$$\nabla w_{y^{(i)}} f(x^{(i)}) - \nabla \log \sum_y e^{w_y f(x^{(i)})}$$

adds f to the correct class weights

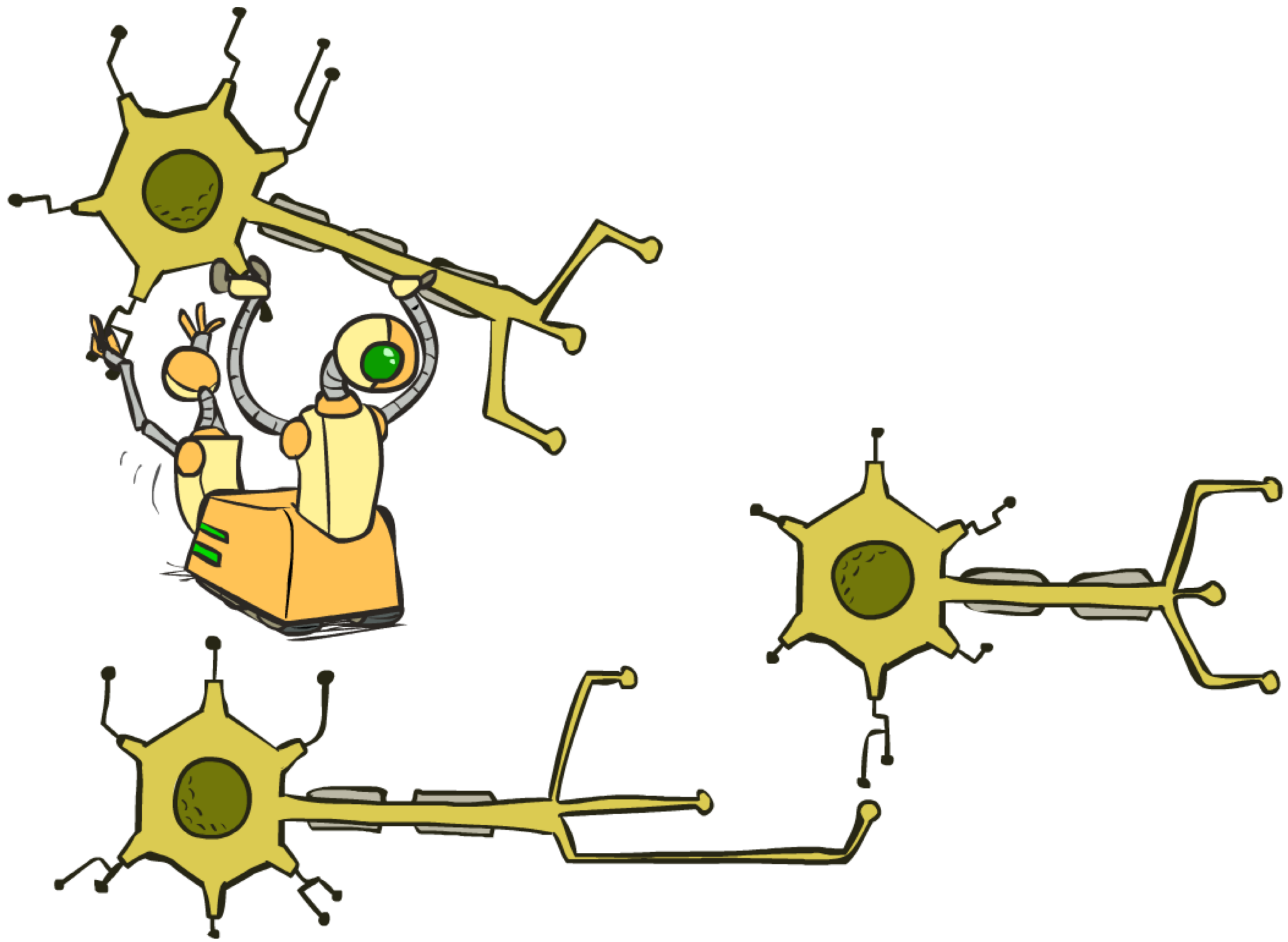
$$\frac{1}{\sum_y e^{w_y f(x^{(i)})}} \sum_y \left(e^{w_y f(x^{(i)})} [0^T f(x^{(i)})^T 0^T]^T \right)$$

for y' weights: $\frac{1}{\sum_y e^{w_y f(x^{(i)})}} e^{w_{y'} f(x^{(i)})} f(x^{(i)})$

$P(y'|x^{(i)}; w) f(x^{(i)})$

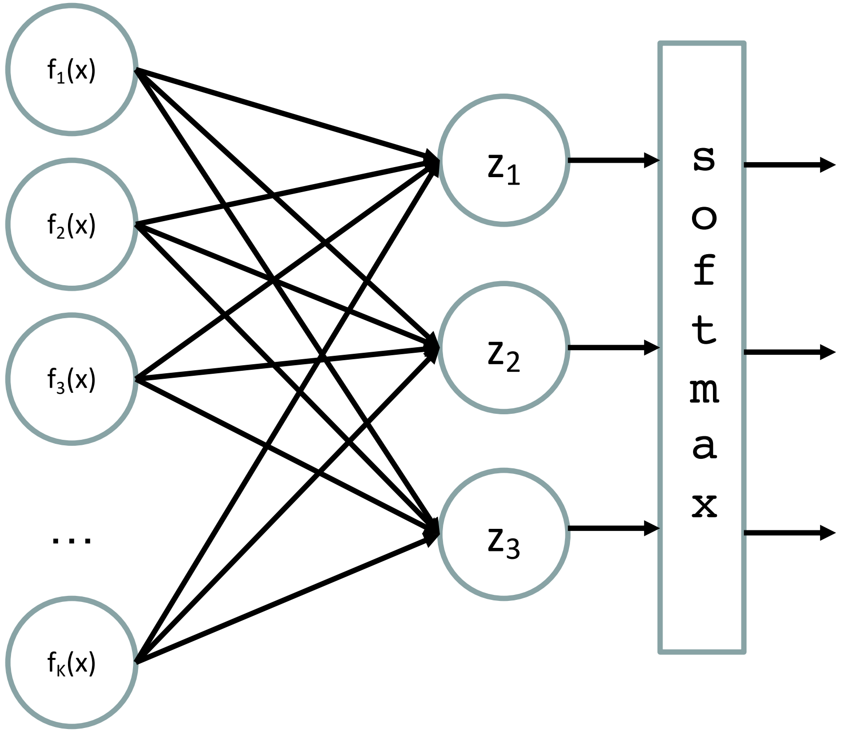
subtracts f from y' weights in proportion to the probability current weights give to y'

Neural Networks



Multi-class Logistic Regression

= special case of neural network

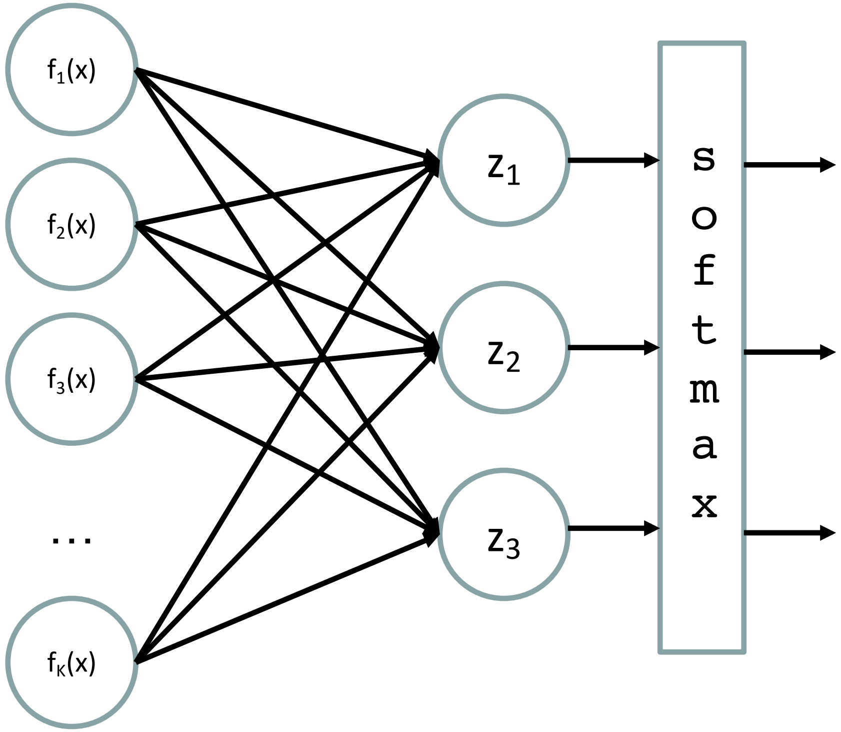


$$P(y_1|x;w) = \frac{e^{z_1}}{e^{z_1} + e^{z_2} + e^{z_3}}$$

$$P(y_2|x;w) = \frac{e^{z_2}}{e^{z_1} + e^{z_2} + e^{z_3}}$$

$$P(y_3|x;w) = \frac{e^{z_3}}{e^{z_1} + e^{z_2} + e^{z_3}}$$

Deep Neural Network = Also learn the features!



$$P(y_1|x;w) = \frac{e^{z_1}}{e^{z_1} + e^{z_2} + e^{z_3}}$$

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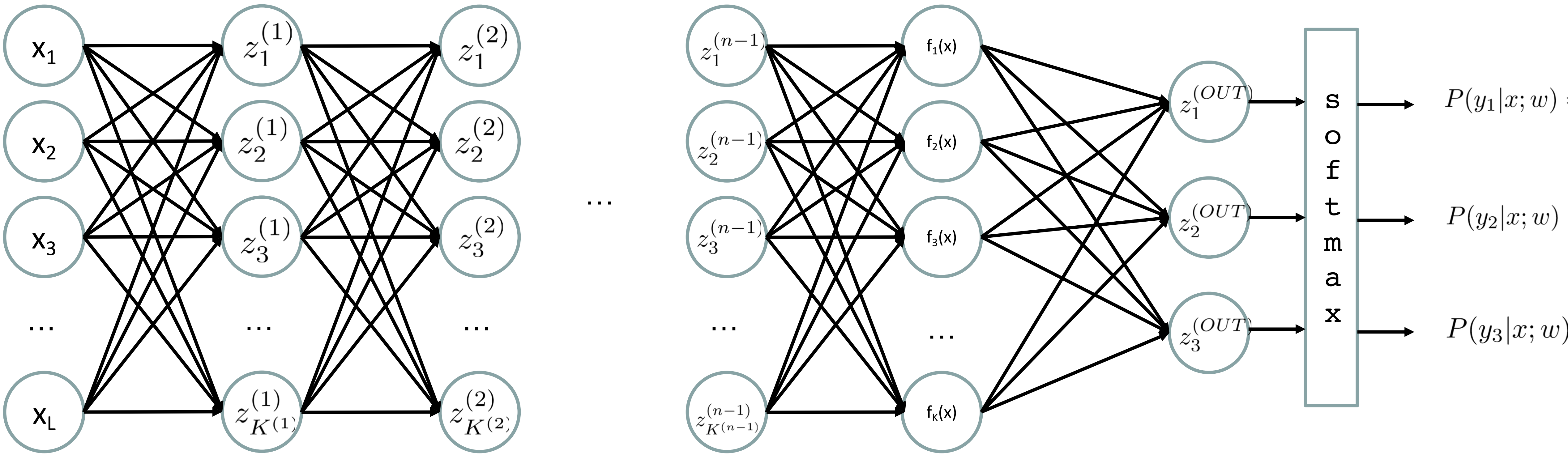
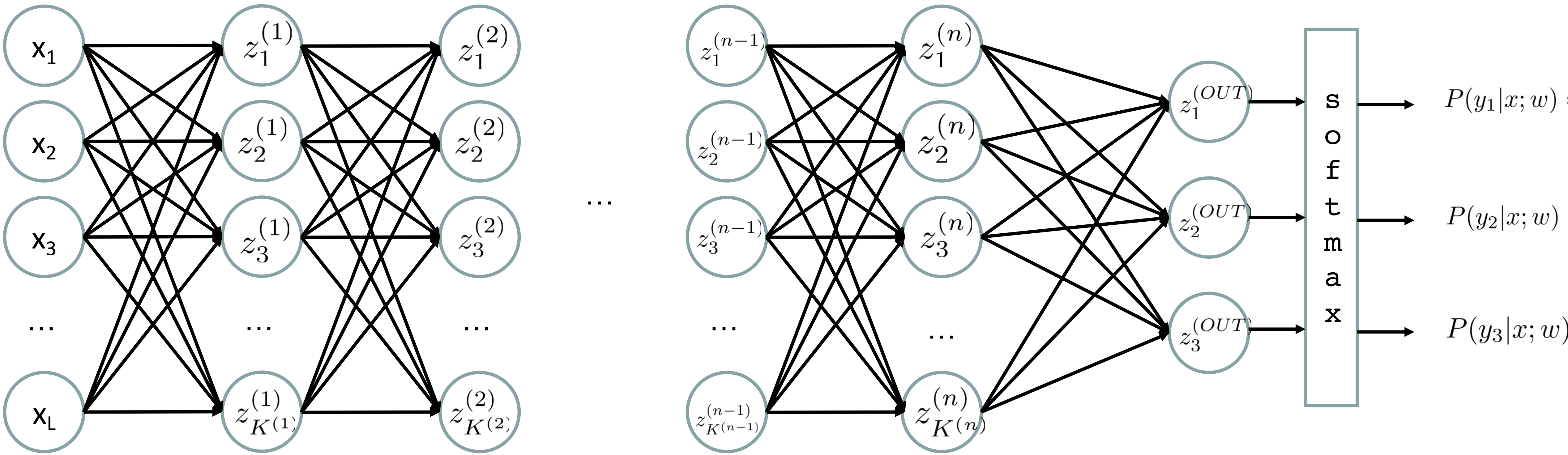
Deep Neural Network = Also learn the features!



$$z_i^{(k)} = g(\sum_j W_{i,j}^{(k-1,k)} z_j^{(k-1)})$$

g = nonlinear activation function

Deep Neural Network = Also learn the features!

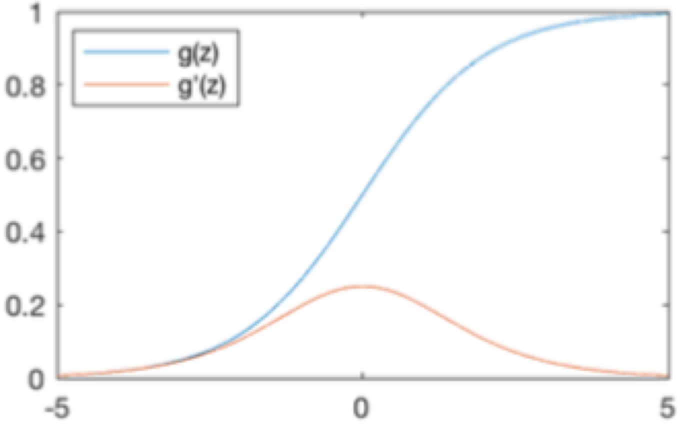


$$z_i^{(k)} = g(\sum_j W_{i,j}^{(k-1,k)} z_j^{(k-1)})$$

g = nonlinear activation function

Common Activation Functions

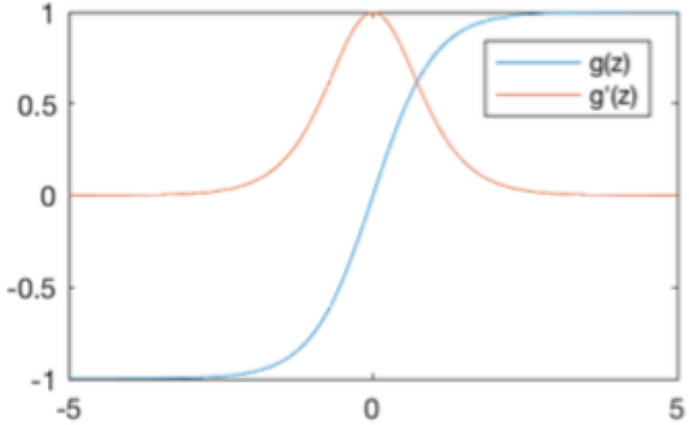
Sigmoid Function



$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = g(z)(1 - g(z))$$

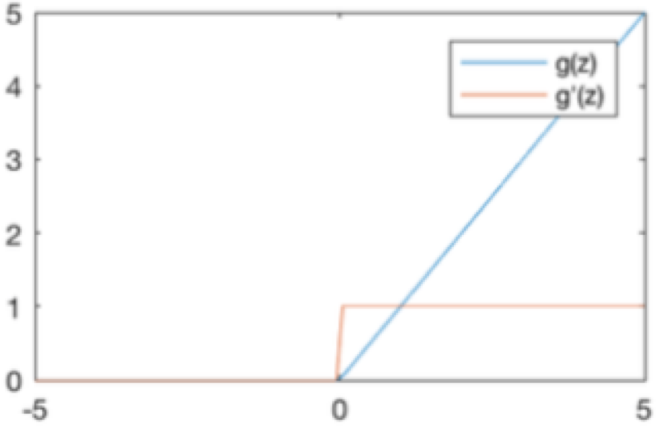
Hyperbolic Tangent



$$g(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = 1 - g(z)^2$$

Rectified Linear Unit (ReLU)



$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$

[source: MIT 6.S191 introtodeeplearning.com]

Deep Neural Network: Also Learn the Features!

Training the deep neural network is just like logistic regression:

$$\max_w \quad ll(w) = \max_w \quad \sum_i \log P(y^{(i)}|x^{(i)}; w)$$

just w tends to be a much, much larger vector

-> just run gradient ascent

+ stop when log likelihood of hold-out data starts to decrease

Neural Networks Properties

Theorem (Universal Function Approximators). A two-layer neural network with a sufficient number of neurons can approximate any continuous function to any desired accuracy.

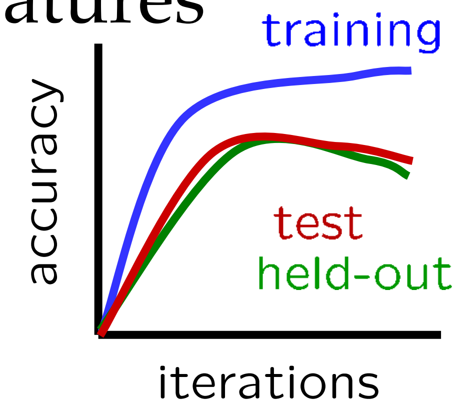
Practical considerations

Can be seen as learning the features

Large number of neurons

Danger for overfitting

(hence early stopping!)



Universal Function Approximation Theorem*

Hornik theorem 1: Whenever the activation function is *bounded and nonconstant*, then, for any finite measure μ, standard multilayer feedforward networks can approximate any function in $L^p(\mu)$ (the space of all functions on R^k such that $\int_{R^k} |f(x)|^p d\mu(x) < \infty$) arbitrarily well, provided that sufficiently many hidden units are available.

Hornik theorem 2: Whenever the activation function is *continuous, bounded and nonconstant*, then, for arbitrary compact subsets $X \subseteq R^k$, standard multilayer feedforward networks can approximate any continuous function on X arbitrarily well with respect to uniform distance, provided that sufficiently many hidden units are available.

In words: Given any continuous function f(x), if a 2-layer neural network has enough hidden units, then there is a choice of weights that allow it to closely approximate f(x).

Cybenko (1989) "Approximations by superpositions of sigmoidal functions"
Hornik (1991) "Approximation Capabilities of Multilayer Feedforward Networks"
Leshno and Schocken (1991) "Multilayer Feedforward Networks with Non-Polynomial Activation Functions Can Approximate Any Function"

Universal Function Approximation Theorem*

Math. Control Signals Systems (1989) 2: 303–314

Mathematics of Control, Signals, and Systems

Approximation by Superpositions of a Sigmoidal Function*

G. Cybenko†

Abstract. In this paper we demonstrate that finite linear combinations of compositions of a fixed, univariate function and a set of affine functionals can uniformly approximate any continuous function of n real variables with support in the unit hypercube; only mild conditions are imposed on the univariate function. Our results settle an open question about representability in the class of single hidden layer neural networks. In particular, we show that arbitrary decision regions can be arbitrarily well approximated by continuous feedforward neural networks with only a single internal, hidden layer and any continuous sigmoidal nonlinearity. The paper discusses approximation properties of other possible types of nonlinearities that might be implemented by artificial neural networks.

Key words. Neural networks, Approximation, Completeness.

1. Introduction

A number of diverse application areas are concerned with the representation of general functions of an n-dimensional real variable, $x \in \mathbb{R}^n$, by finite linear combinations of the form

$$\sum_{j=1}^{N} \alpha_j \sigma(y_j^{\mathrm{T}} x + \theta_j), \qquad (1)$$

where $y_j \in \mathbb{R}^n$ and $\alpha_j, \theta \in \mathbb{R}$ are fixed. (y^{T} is the transpose of y so that $y^{\mathrm{T}}x$ is the inner product of y and x.) Here the univariate function σ depends heavily on the context of the application. Our major concern is with so-called sigmoidal σ's:

$$\sigma(t) \to \begin{cases} 1 & \text{as } t \to +\infty, \\ 0 & \text{as } t \to -\infty. \end{cases}$$

Such functions arise naturally in neural network theory as the activation function of a neural node (or *unit* as is becoming the preferred term) [L1], [RHM]. The main result of this paper is a demonstration of the fact that sums of the form (1) are dense in the space of continuous functions on the unit cube if σ is any continuous sigmoidal

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† Center for Supercomputing Research and Development and Department of Electrical and Computer Engineering, University of Illinois, Urbana, Illinois 61801, U.S.A.

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ORIGINAL CONTRIBUTION

Approximation Capabilities of Multilayer Feedforward Networks

KURT HORNIK

Technische Universität Wien, Vienna, Austria

(Received 30 January 1990; revised and accepted 25 October 1990)

Abstract—*We show that standard multilayer feedforward networks with as few as a single hidden layer and arbitrary bounded and nonconstant activation function are universal approximators with respect to $L^p(\mu)$ performance criteria, for arbitrary finite input environment measures μ, provided only that sufficiently many hidden units are available. If the activation function is continuous, bounded and nonconstant, then continuous mappings can be learned uniformly over compact input sets. We also give very general conditions ensuring that networks with sufficiently smooth activation functions are capable of arbitrarily accurate approximation to a function and its derivatives.*

Keywords—Multilayer feedforward networks, Activation function, Universal approximation capabilities, Input environment measure, $L^p(\mu)$ approximation, Uniform approximation, Sobolev spaces, Smooth approximation.

1. INTRODUCTION

The approximation capabilities of neural network architectures have recently been investigated by many authors, including Carroll and Dickinson (1989), Cybenko (1989), Funahashi (1989), Gallant and White (1988), Hecht-Nielsen (1989), Hornik, Stinchcombe, and White (1989, 1990), Irie and Miyake (1988), Lapedes and Farber (1988), Stinchcombe and White (1989, 1990). (This list is by no means complete.)

If we think of the network architecture as a rule for computing values at l output units given values at k input units, hence implementing a class of mappings from R^k to R^l, we can ask how well arbitrary mappings from R^k to R^l can be approximated by the network, in particular, if as many hidden units as required for internal representation and computation may be employed.

How to measure the accuracy of approximation depends on how we measure closeness between functions, which in turn varies significantly with the specific problem to be dealt with. In many applications, it is necessary to have the network perform *simultaneously* well on all input samples taken from some compact input set X in R^k. In this case, closeness is measured by the uniform distance between functions on X, that is,

$$\rho_{\mu,X}(f, g) = \sup_{x \in X} |f(x) - g(x)|.$$

In other applications, we think of the inputs as random variables and are interested in the *average performance* where the average is taken with respect to the input environment measure μ, where $\mu(R^k) < \infty$. In this case, closeness is measured by the $L^p(\mu)$ distances

$$\rho_{p,\mu}(f, g) = \left[\int_{R^k} |f(x) - g(x)|^p \, d\mu(x)\right]^{1/p},$$

$1 \le p < \infty$, the most popular choice being $p = 2$, corresponding to mean square error.

Of course, there are many more ways of measuring closeness of functions. In particular, in many applications, it is also necessary that the *derivatives* of the approximating function implemented by the network closely resemble those of the function to be approximated, up to some order. This issue was first taken up in Hornik et al. (1990), who discuss the sources of need of smooth functional approximation in more detail. Typical examples arise in robotics (learning of smooth movements) and signal processing (analysis of chaotic time series); for a recent application to problems of nonparametric inference in statistics and econometrics, see Gallant and White (1989).

All papers establishing certain approximation ca-

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MULTILAYER FEEDFORWARD NETWORKS WITH NON-POLYNOMIAL ACTIVATION FUNCTIONS CAN APPROXIMATE ANY FUNCTION

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How about computing all the derivatives?

Derivatives tables:

$$\frac{d}{dx}(a) = 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(au) = a\frac{du}{dx}$$

$$\frac{d}{dx}(u + v - w) = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v}\frac{du}{dx} - \frac{u}{v^2}\frac{dv}{dx}$$

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}(\sqrt{u}) = \frac{1}{2\sqrt{u}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{u^n}\right) = -\frac{n}{u^{n+1}}\frac{du}{dx}$$

$$\frac{d}{dx}[f(u)] = \frac{d}{du}[f(u)]\frac{du}{dx}$$

$$\frac{d}{dx}[\ln u] = \frac{d}{dx}[\log_e u] = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}\left[\log_a u\right] = \log_a e \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}$$

$$\frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx}\left(u^v\right) = vu^{v-1}\frac{du}{dx} + \ln u \; u^v\frac{dv}{dx}$$

$$\frac{d}{dx}\sin u = \cos u\frac{du}{dx}$$

$$\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}$$

$$\frac{d}{dx}\tan u = \sec^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\cot u = -\csc^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\sec u = \sec u \tan u\frac{du}{dx}$$

$$\frac{d}{dx}\csc u = -\csc u \cot u\frac{du}{dx}$$

[source: http://hyperphysics.phy-astr.gsu.edu/hbase/Math/derfunc.html

How about computing all the derivatives?

- But neural net f is never one of those?
  - No problem: CHAIN RULE:

If $$f(x) = g(h(x))$$

Then $$f'(x) = g'(h(x))h'(x)$$

Derivatives can be computed by following well-defined procedures

Automatic Differentiation

Automatic differentiation software

e.g. TensorFlow, PyTorch, Jax

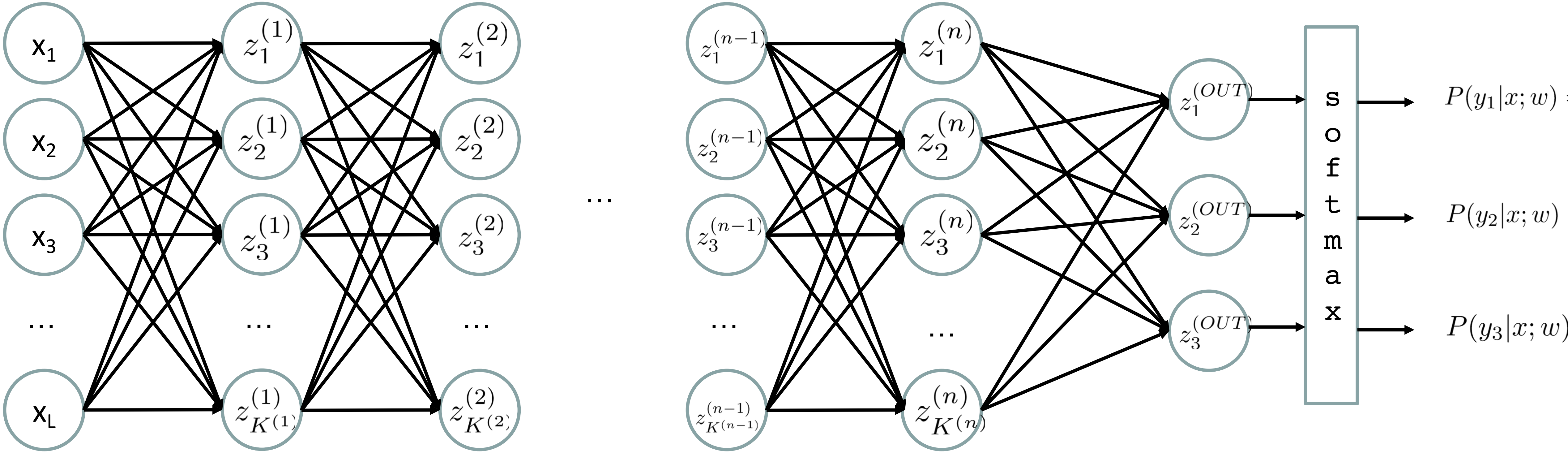
Only need to program the function g(x,y,w)

Can automatically compute all derivatives w.r.t. all entries in w

This is typically done by caching info during forward computation pass of f, and then doing a backward pass = "backpropagation"

Autodiff / Backpropagation can often be done at computational cost comparable to the forward pass

Deep Neural Network = Also learn the features!



$$z_i^{(k)} = g(\sum_j W_{i,j}^{(k-1,k)} z_j^{(k-1)})$$

g = nonlinear activation function

Training a Network (setting weights)

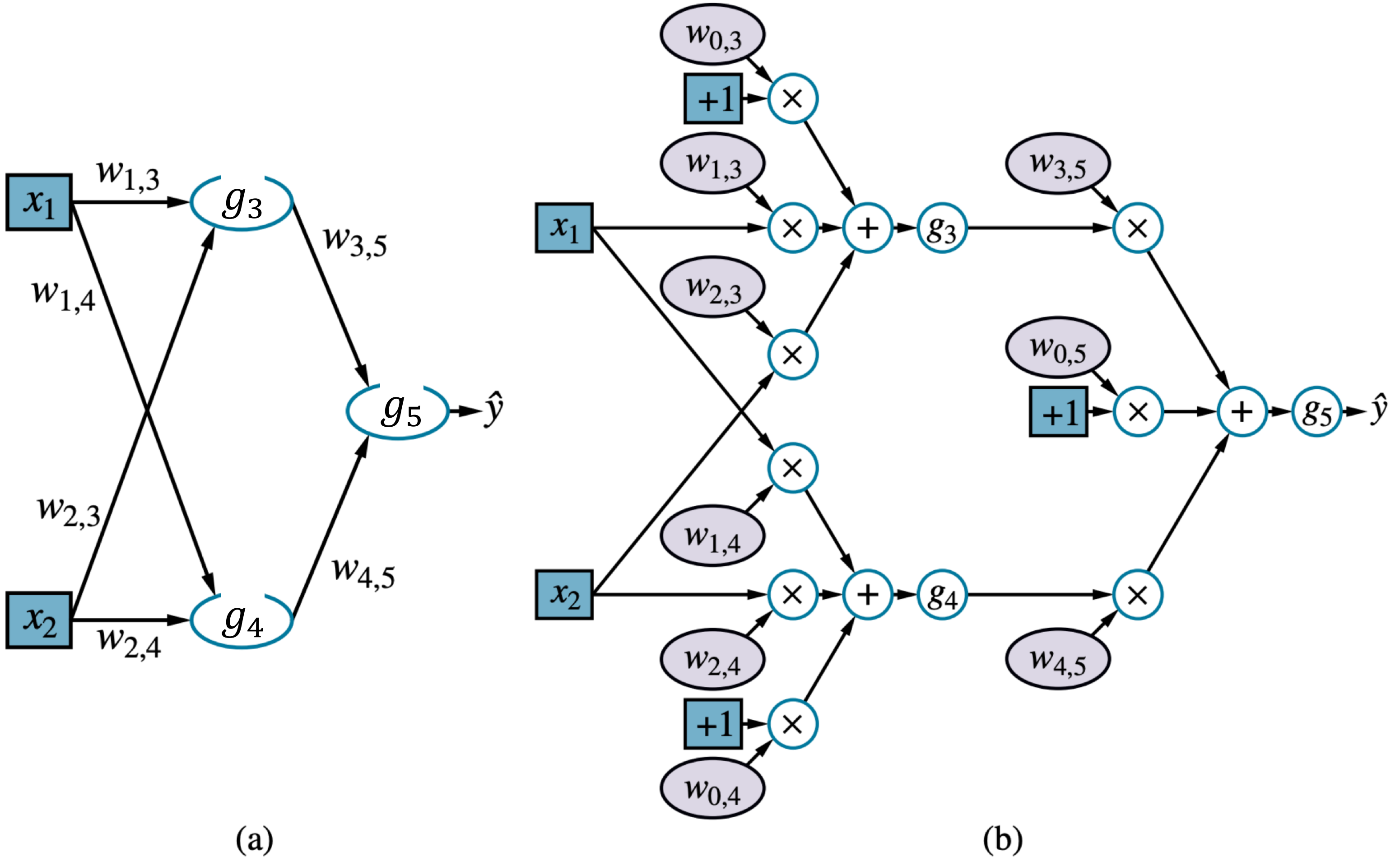
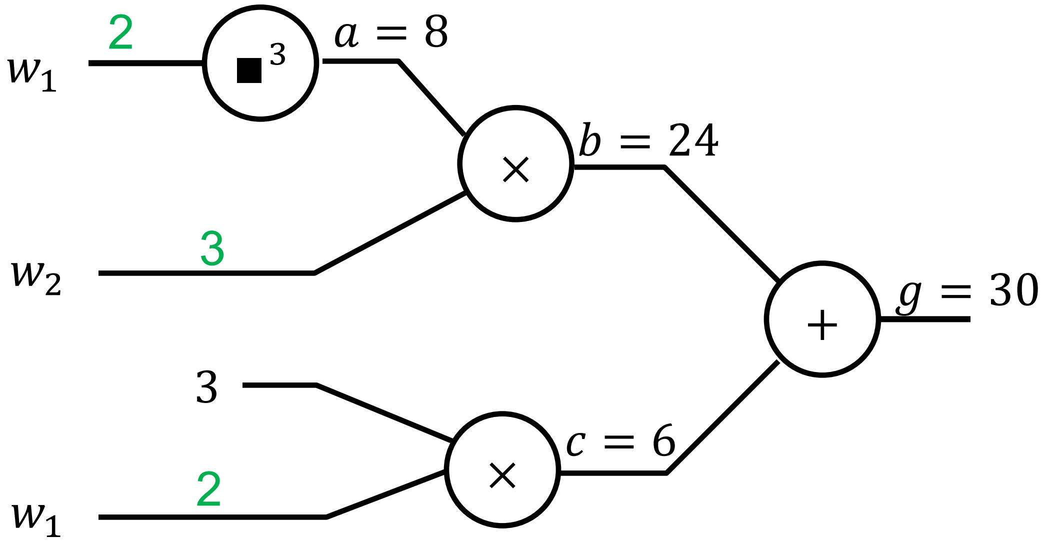


Figure 21.3 (a) A neural network with two inputs, one hidden layer of two units, and one output unit. Not shown are the dummy inputs and their associated weights. (b) The network in (a) unpacked into its full computation graph.

Back Propagation: $g(\boldsymbol{w}) = w_1^3 w_2 + 3w_1$

- Suppose we have $g(\boldsymbol{w}) = w_1^3 w_2 + 3w_1$ and want the gradient at $\boldsymbol{w} = [2, 3]$
- Think of the function as a composition of many functions.
  - Can use derivative chain rule to compute $\partial g/\partial w_1$ and $\frac{\partial g}{\partial w_2}$
  - $\frac{\partial g}{\partial w_1} =$ ______________________
  - $\frac{\partial g}{\partial w_2} =$ ______________________

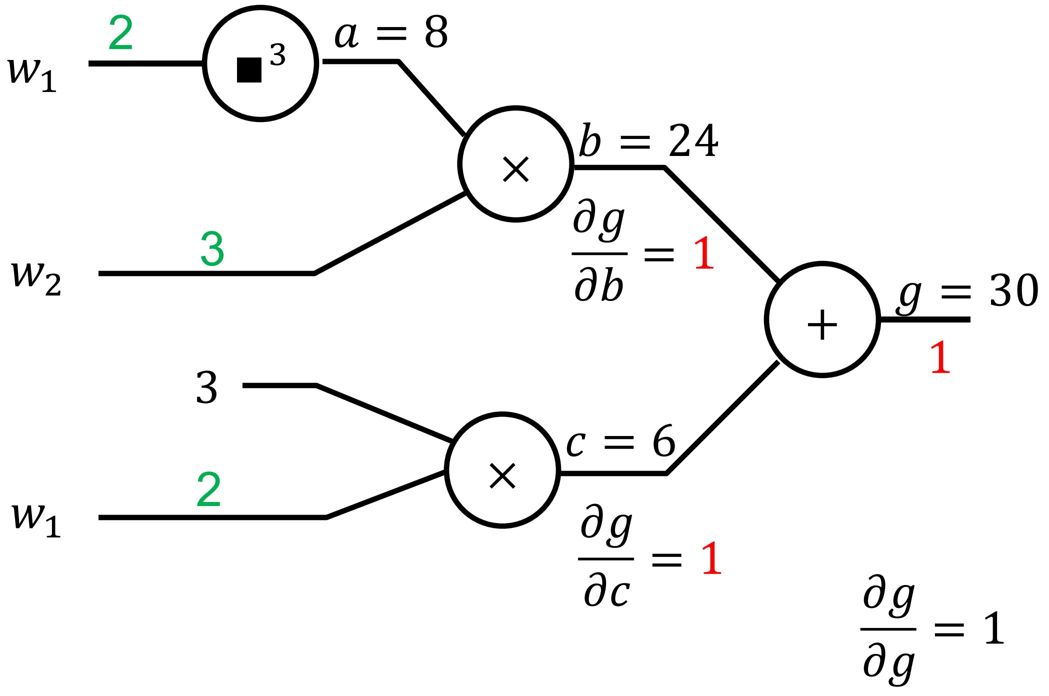


Computation Graph

- More complex to compute for more complicated functions

Back Propagation: $g(\boldsymbol{w}) = w_1^3 w_2 + 3w_1$

- Suppose we have $g(\boldsymbol{w}) = w_1^3 w_2 + 3w_1$ and want the gradient at $\boldsymbol{w} = [2, 3]$
- Think of the function as a composition of many functions.
  - Can use derivative chain rule to compute $\partial g / \partial w_1$ and $\partial g / \partial w_2$.
- $g = b + c$
  - $\frac{\partial g}{\partial b} = 1, \frac{\partial g}{\partial c} = 1$



Computation Graph

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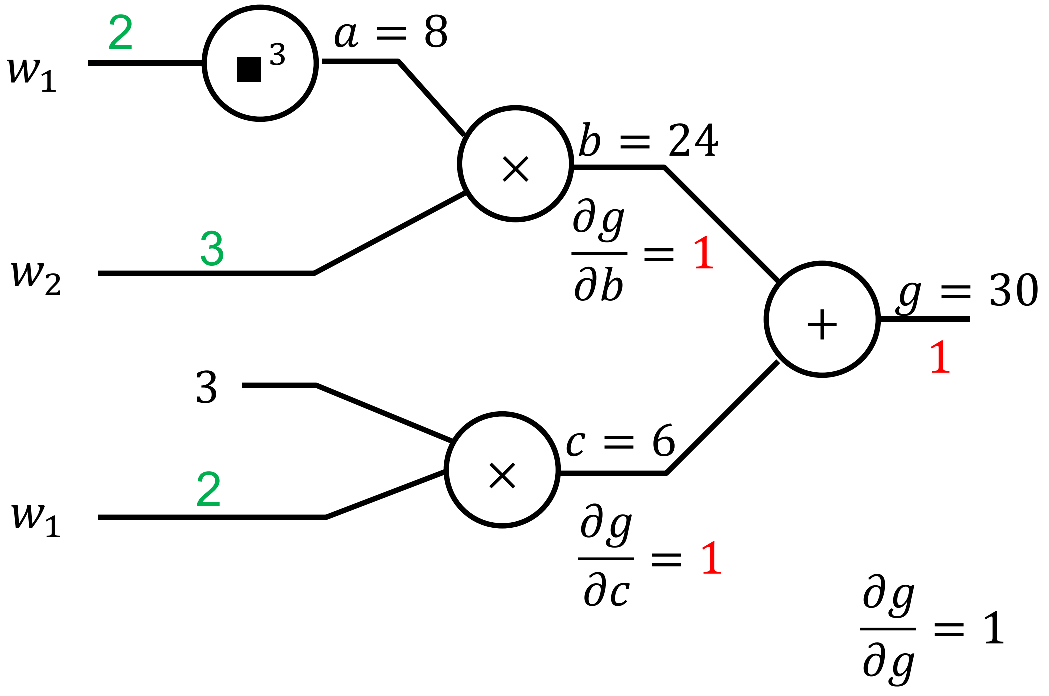

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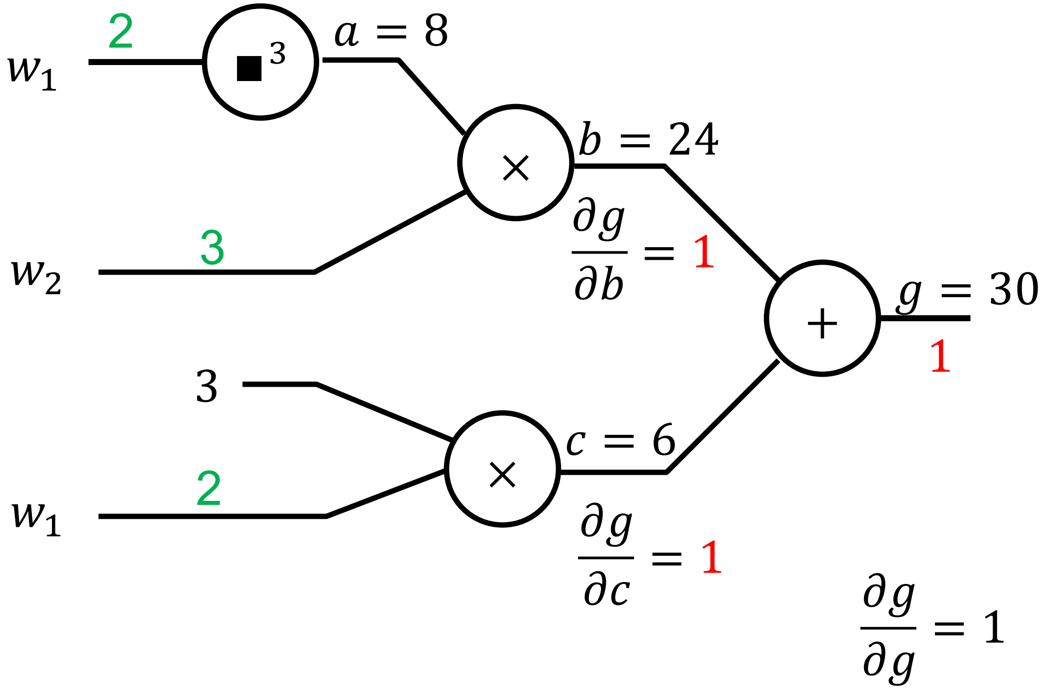

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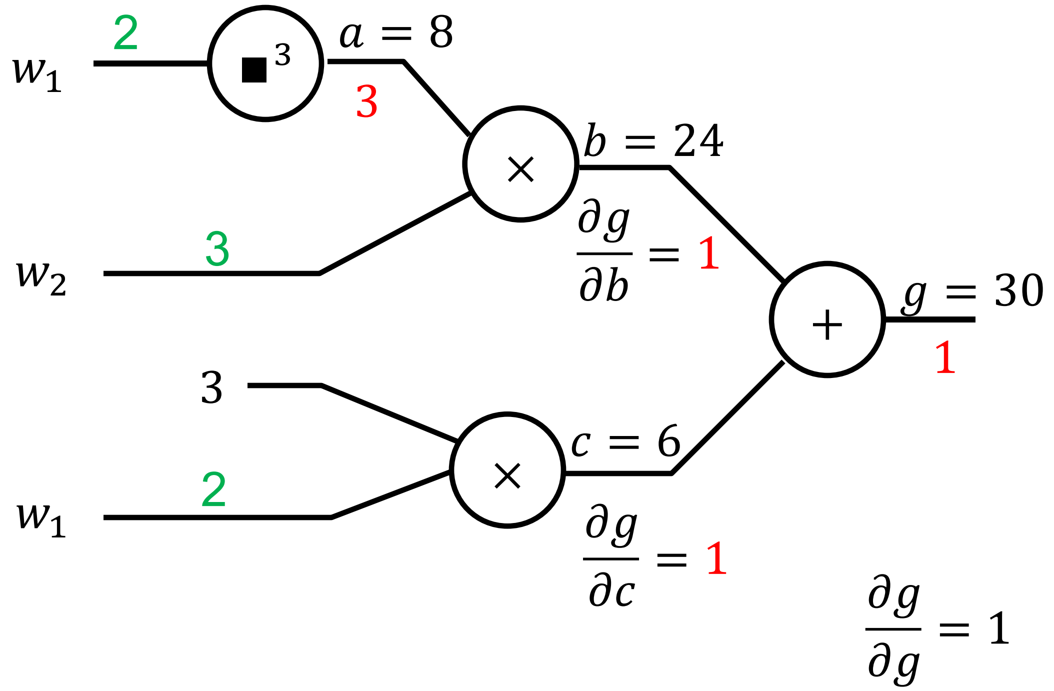

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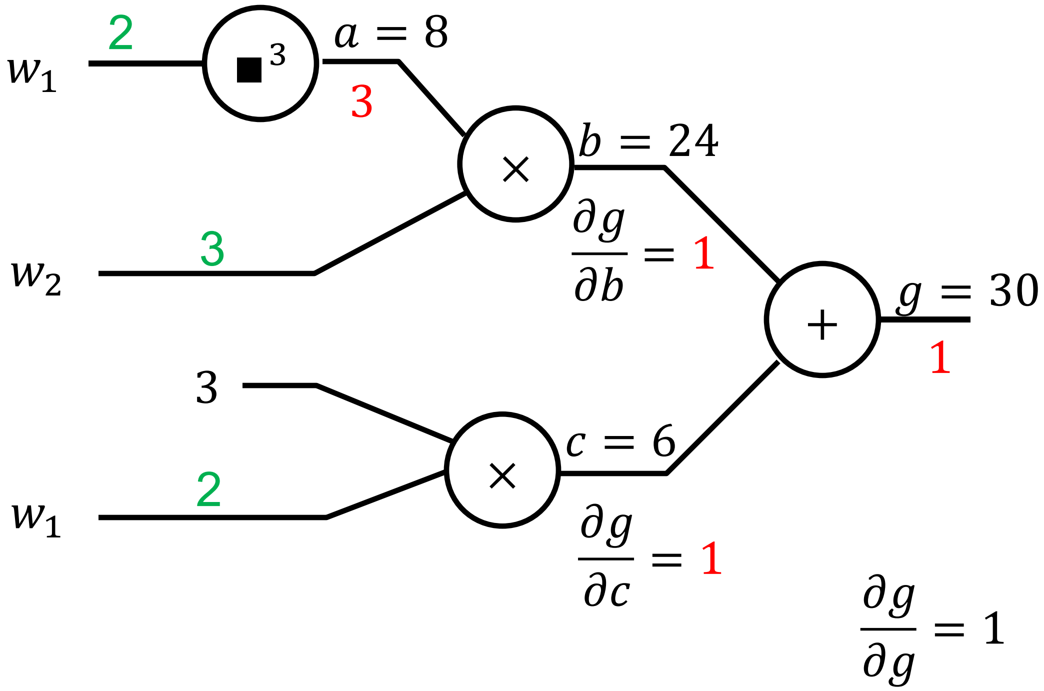

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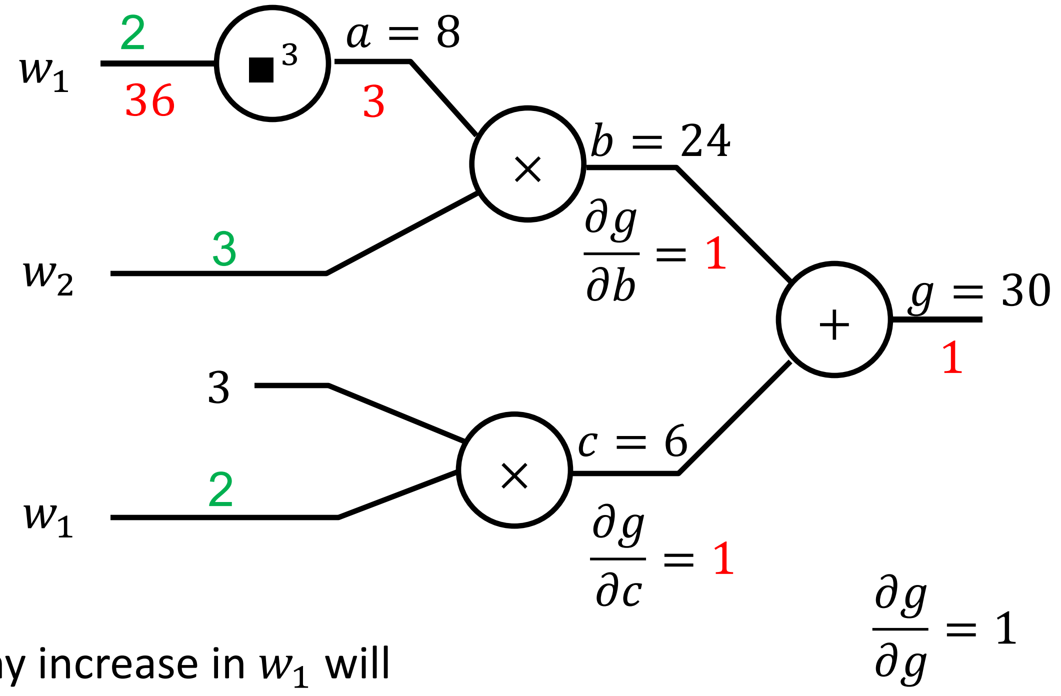
  - $\frac{\partial g}{\partial w_1} = \frac{\partial g}{\partial a}\frac{\partial a}{\partial w_1} = 3 \cdot 3w_1^2 = 36$



Interpretation: A tiny increase in w_1 will result in an approximately 36 times increase in g due to this computation path.

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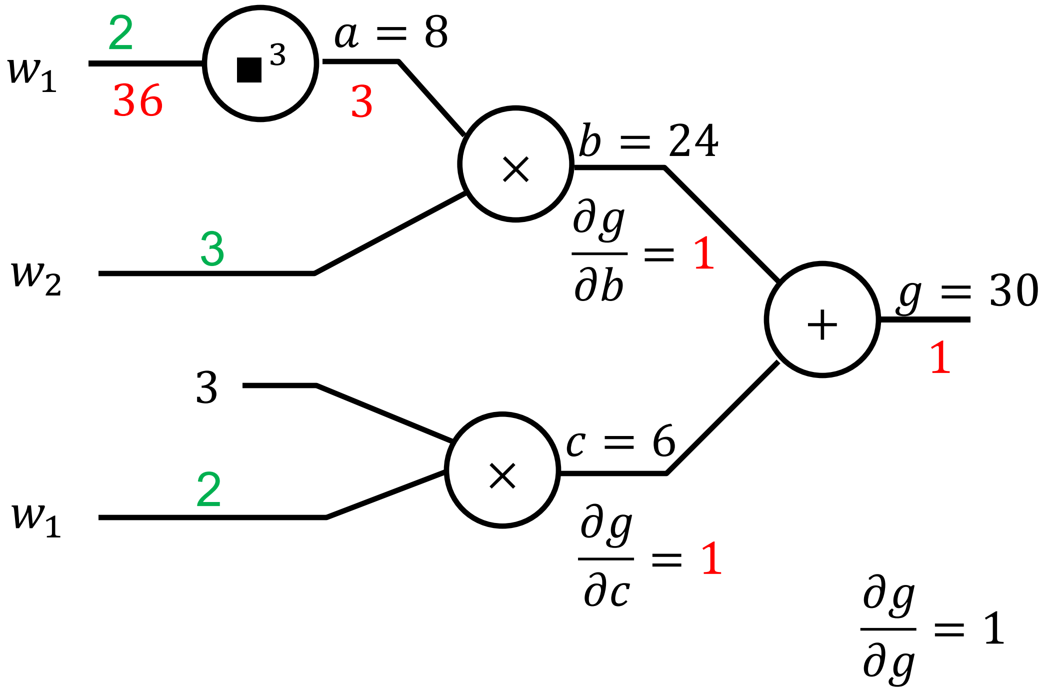
- $\frac{\partial g}{\partial w_2} =???$

Hint: $b = a \times 3$ may be useful.



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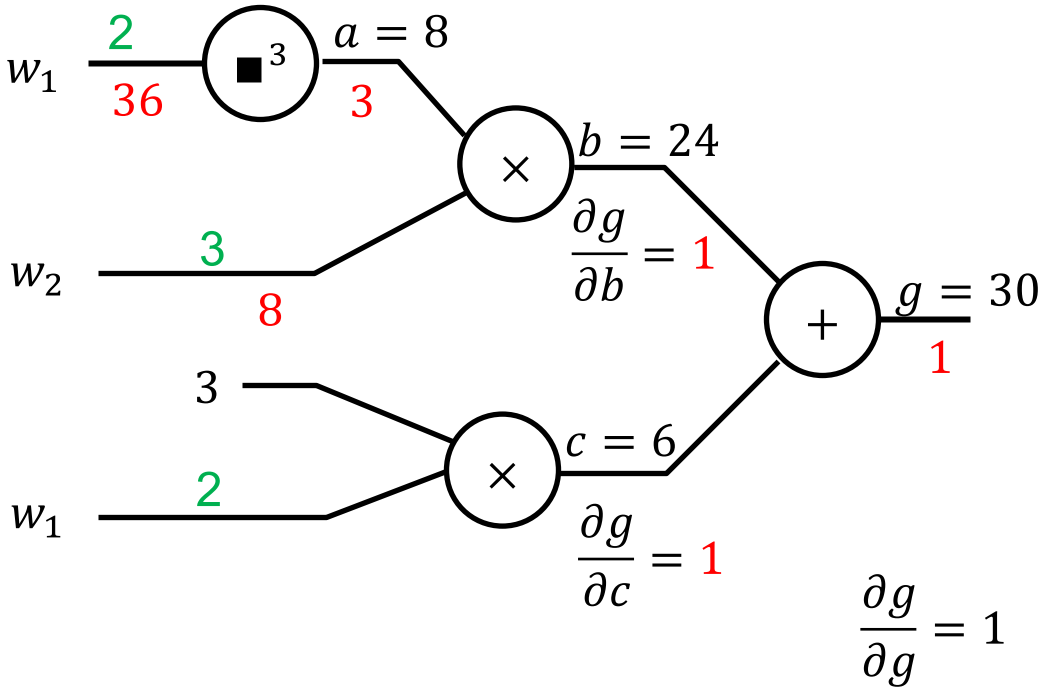

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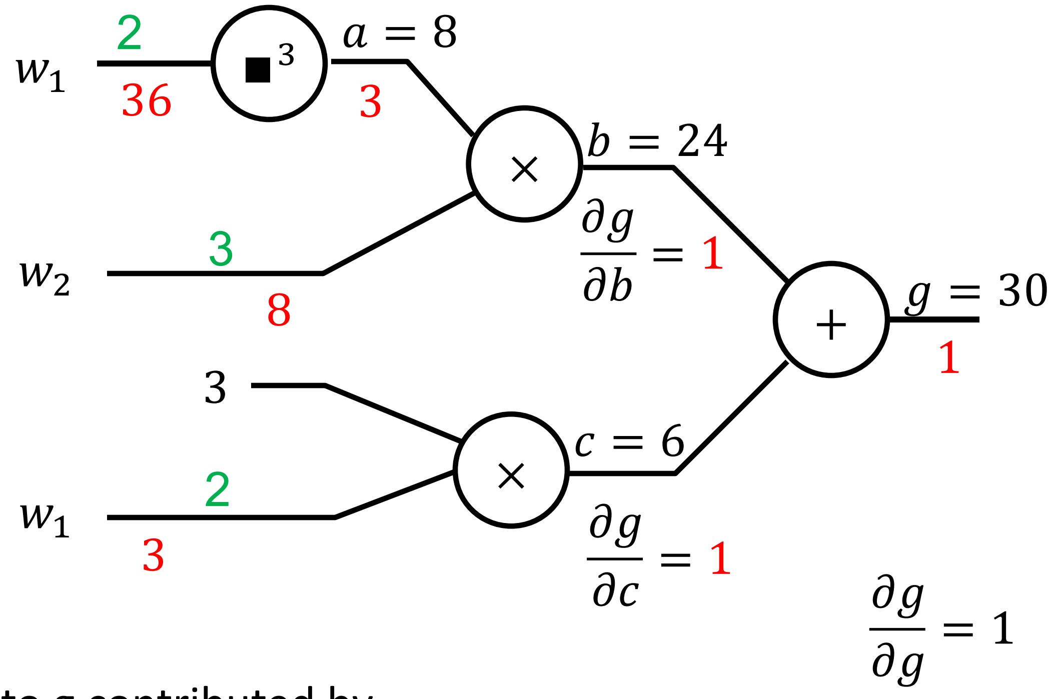
Adding the changes to g contributed by change in w_1 together



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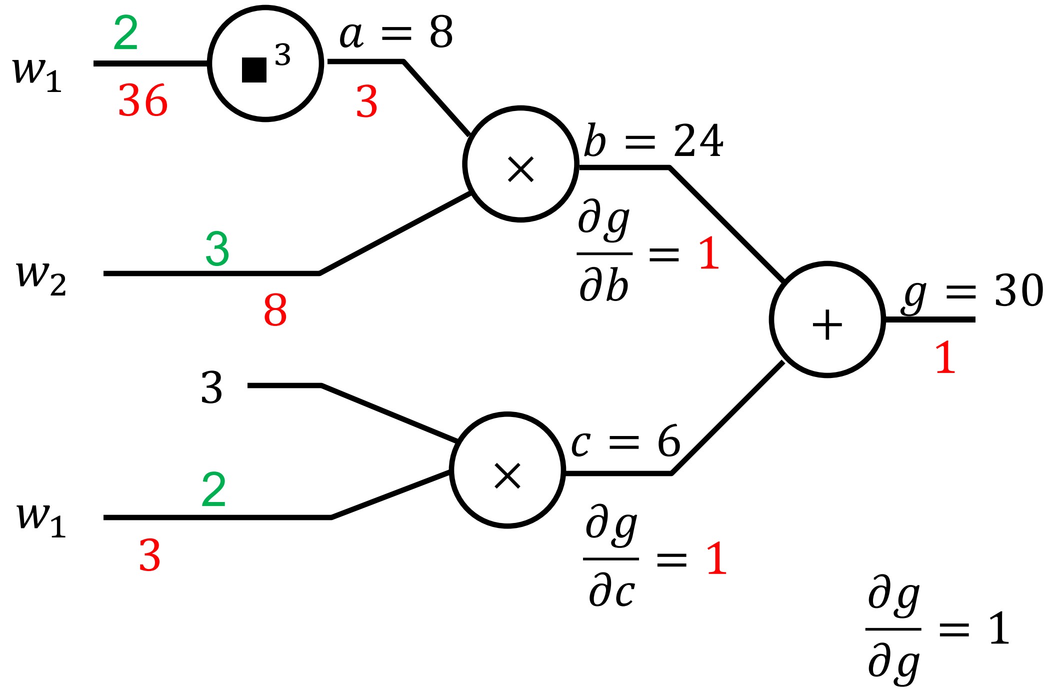
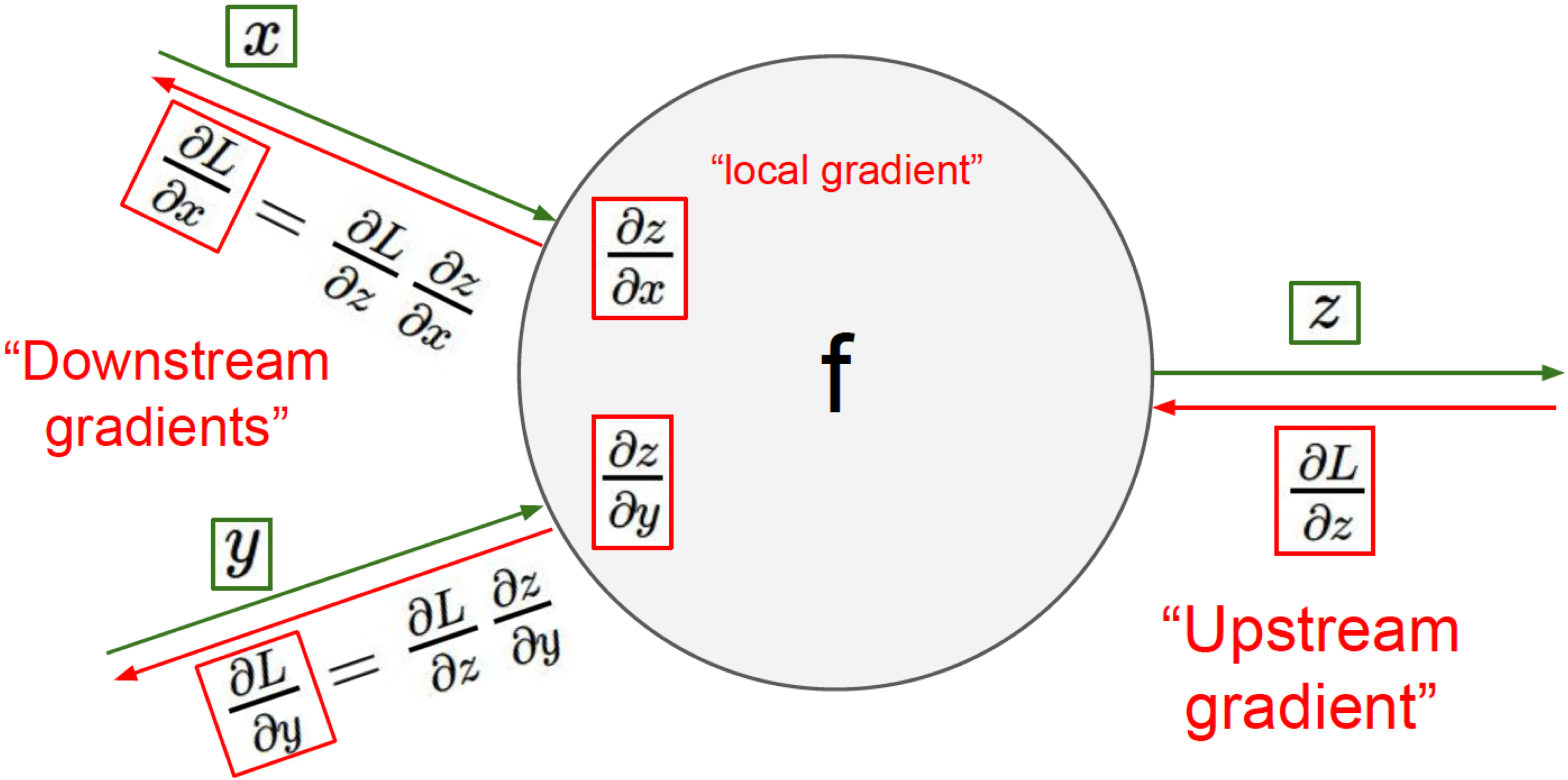


$\nabla g = \left[\frac{\partial g}{\partial w_1}, \frac{\partial g}{\partial w_2}\right] = [39, 8]$



x
$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
"local gradient"
$\frac{\partial z}{\partial x}$
f
z
"Downstream gradients"
$\frac{\partial z}{\partial y}$
$\frac{\partial L}{\partial z}$
y
$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$
"Upstream gradient"

Summary of Key Ideas

Optimize probability of label given input

$$\max_w \quad ll(w) = \max_w \quad \sum_i \log P(y^{(i)}|x^{(i)}; w)$$

Continuous optimization

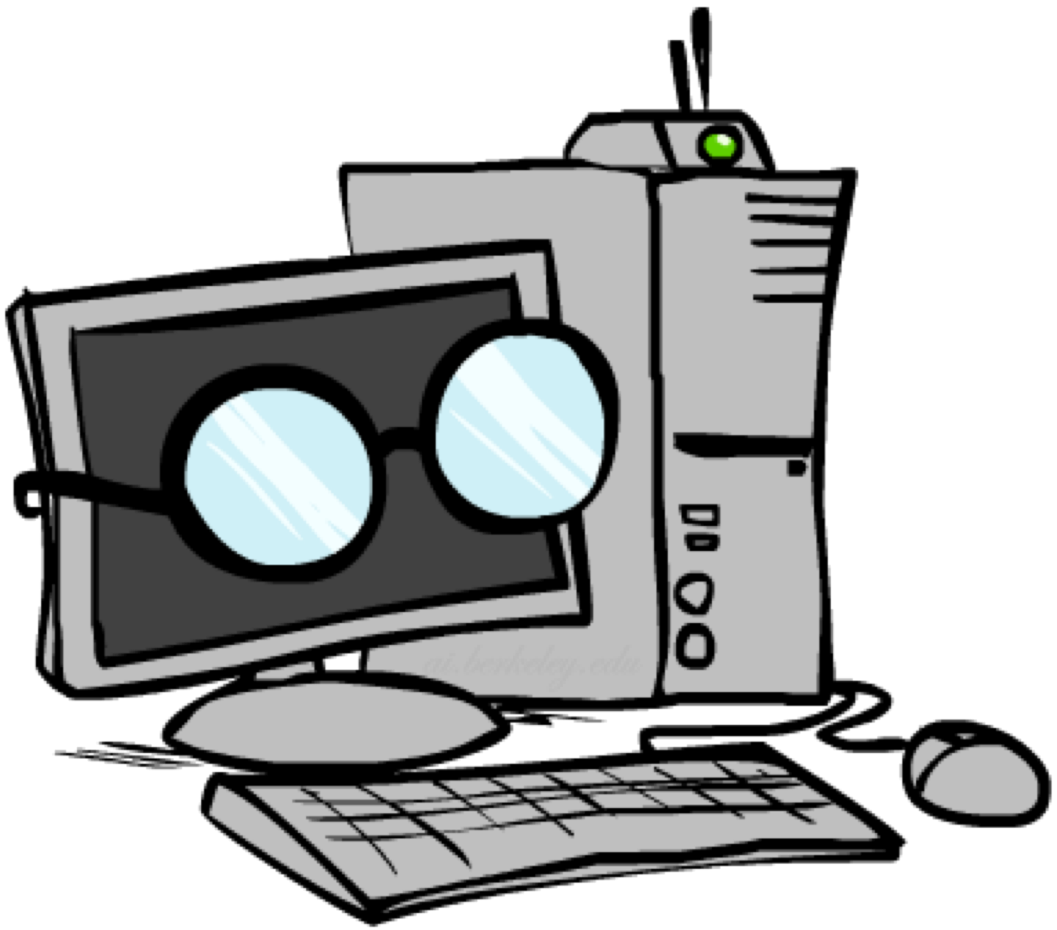
- Gradient ascent:
  - Compute steepest uphill direction = gradient (= just vector of partial derivatives)
  - Take step in the gradient direction
  - Repeat (until held-out data accuracy starts to drop = "early stopping")

Deep neural nets

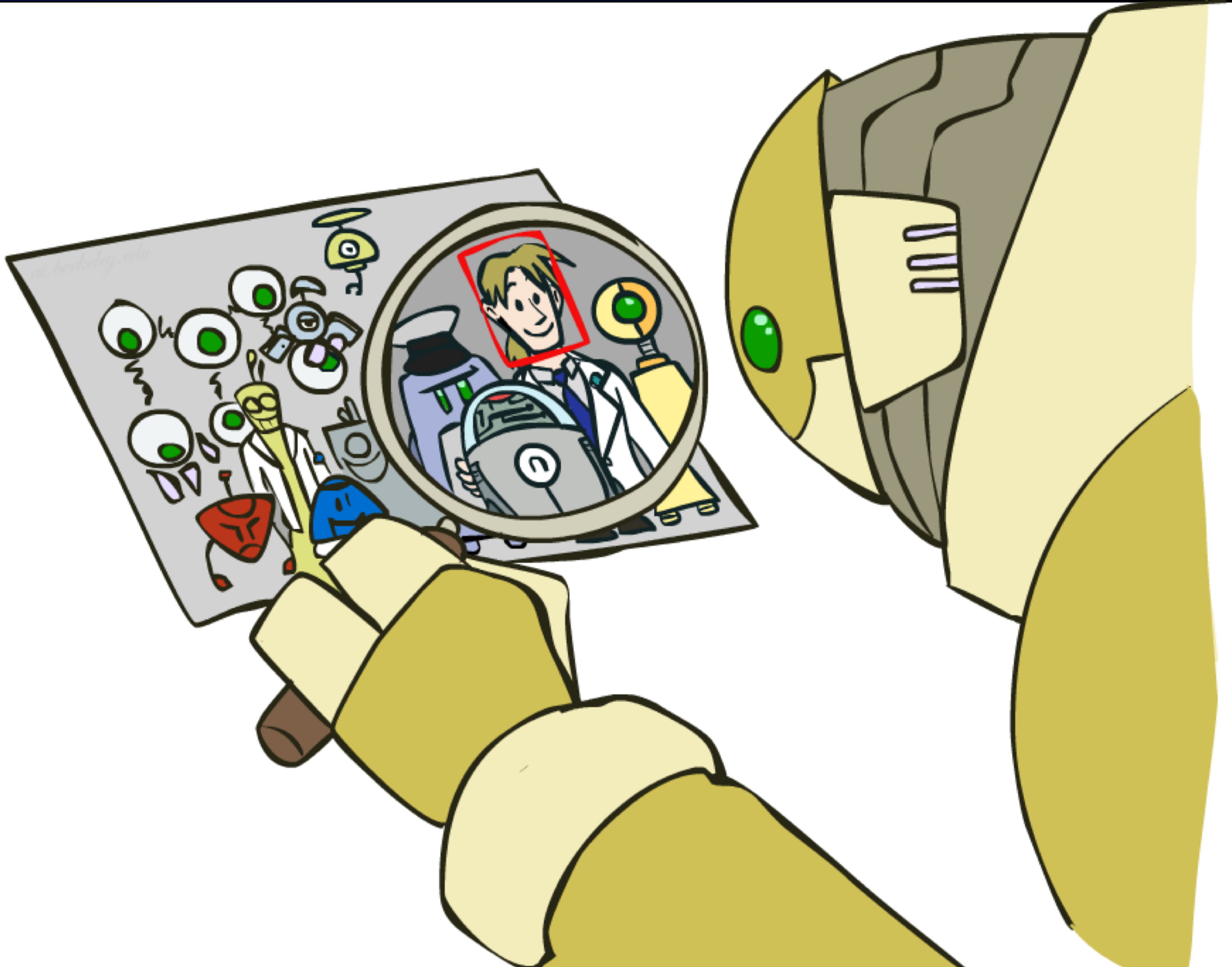
- Last layer = still logistic regression
- Now also many more layers before this last layer
  - = computing the features
  - the features are learned rather than hand-designed
- Universal function approximation theorem
  - `If` neural net is large enough
  - `Then` neural net can represent any continuous mapping from input to output with arbitrary accuracy
  - But remember: need to avoid overfitting / memorizing the training data → early stopping!
- Automatic differentiation gives the derivatives efficiently

How well does it work?

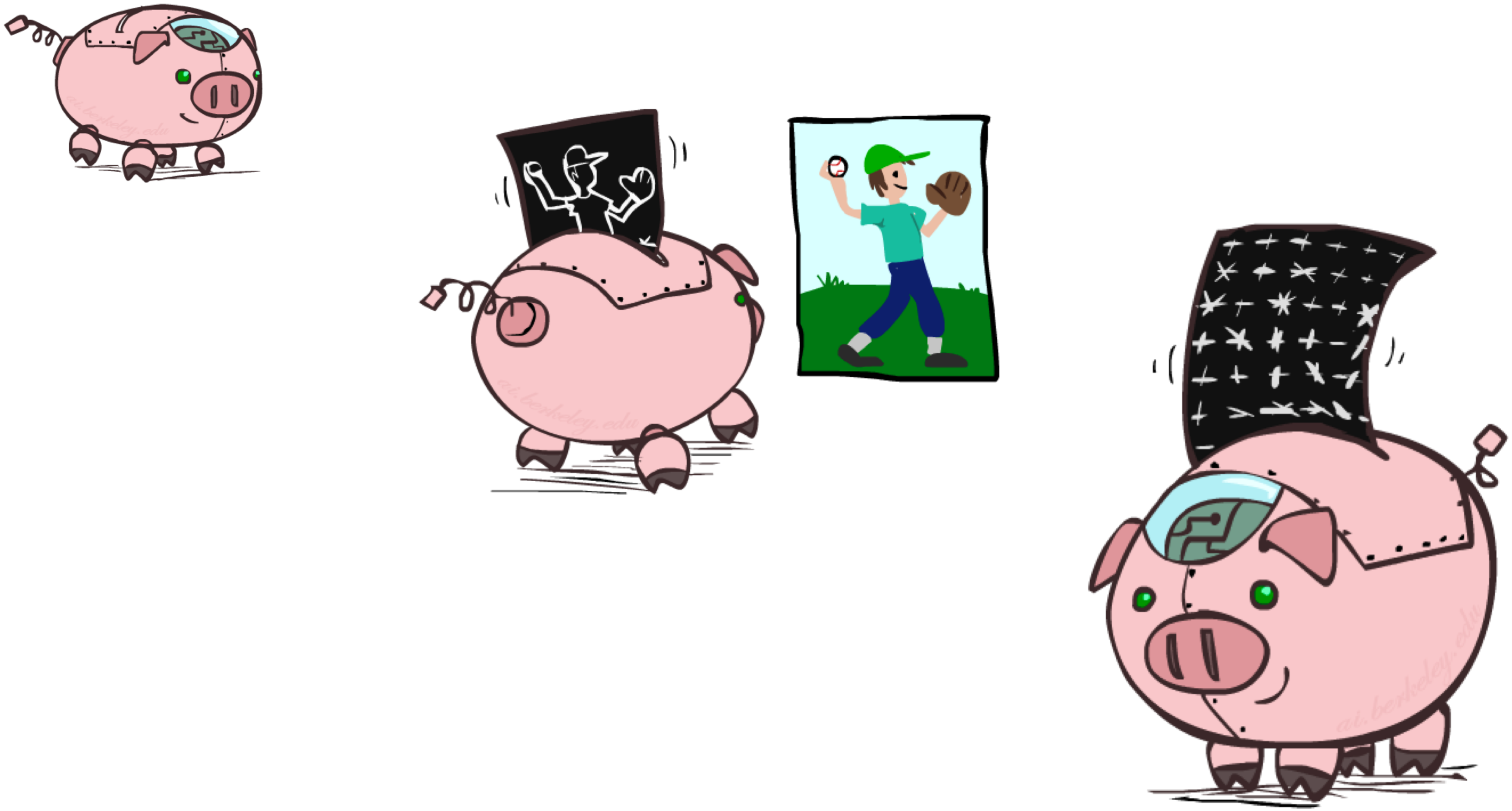
Computer Vision



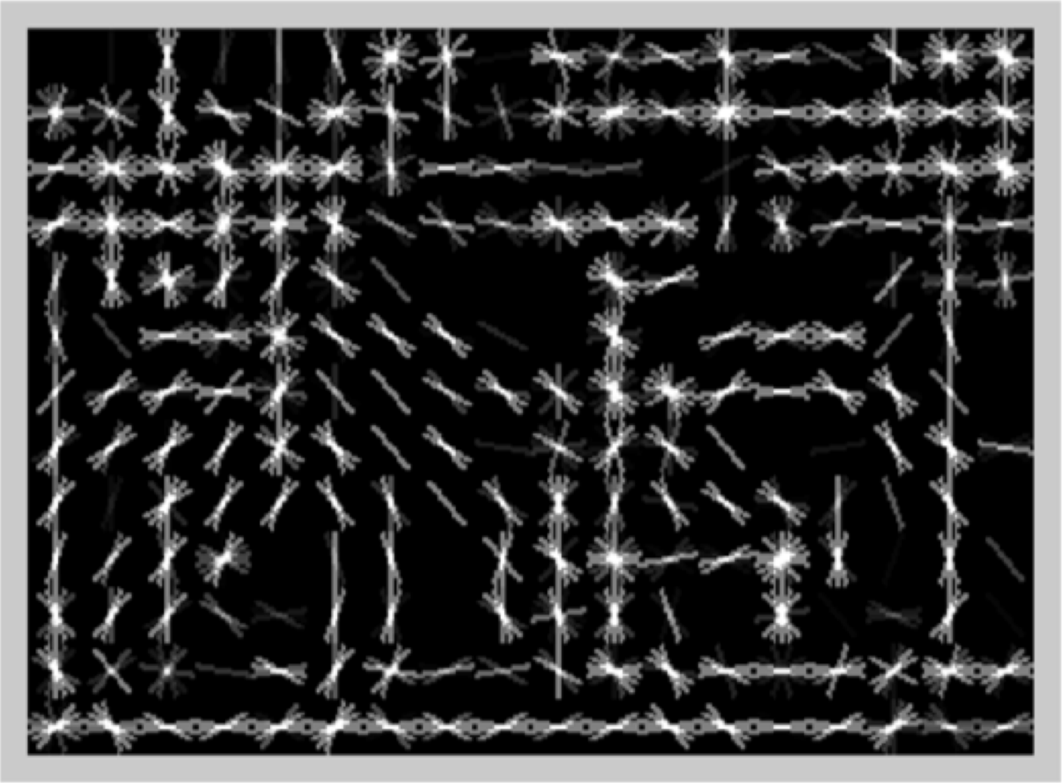
Object Detection



Manual Feature Design



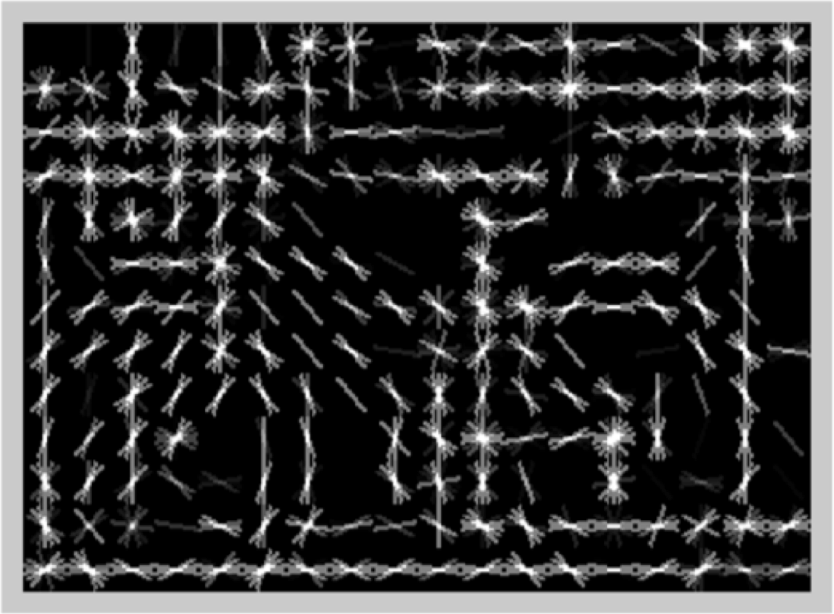
Features and Generalization



[HoG: Dalal and Triggs, 2005]

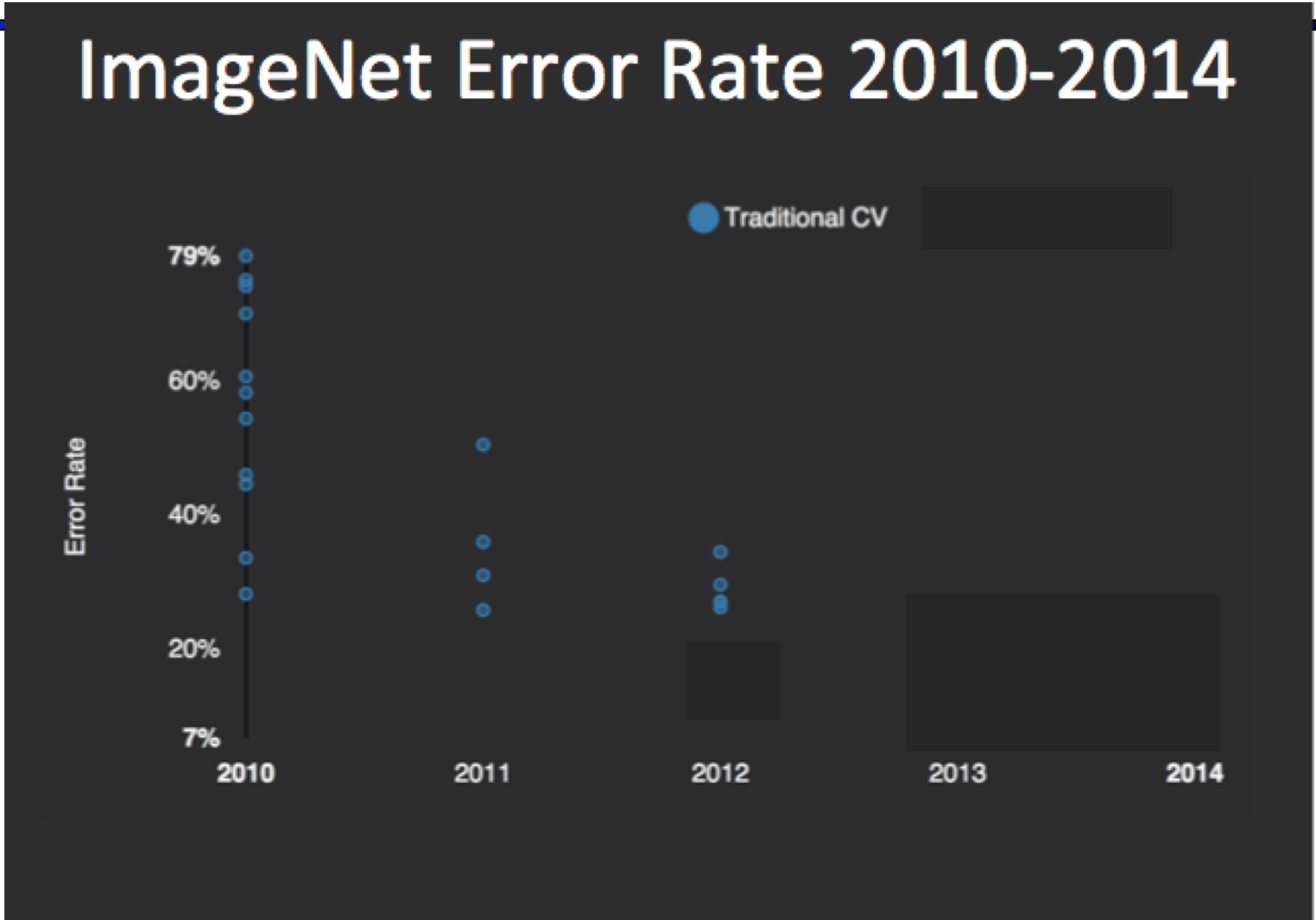
Features and Generalization

Image



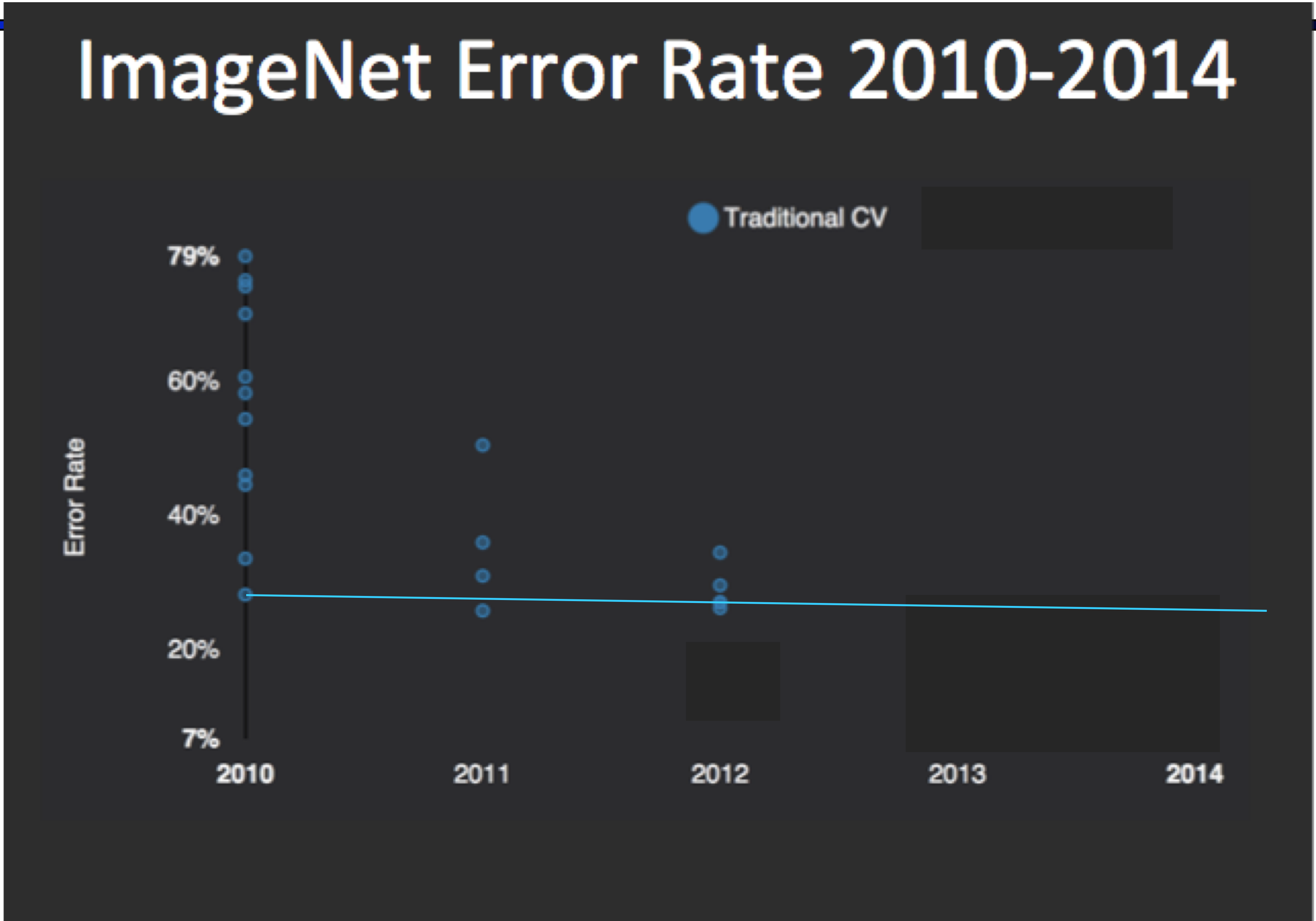
HoG

Performance



graph credit Matt Zeiler, Clarifai

Performance



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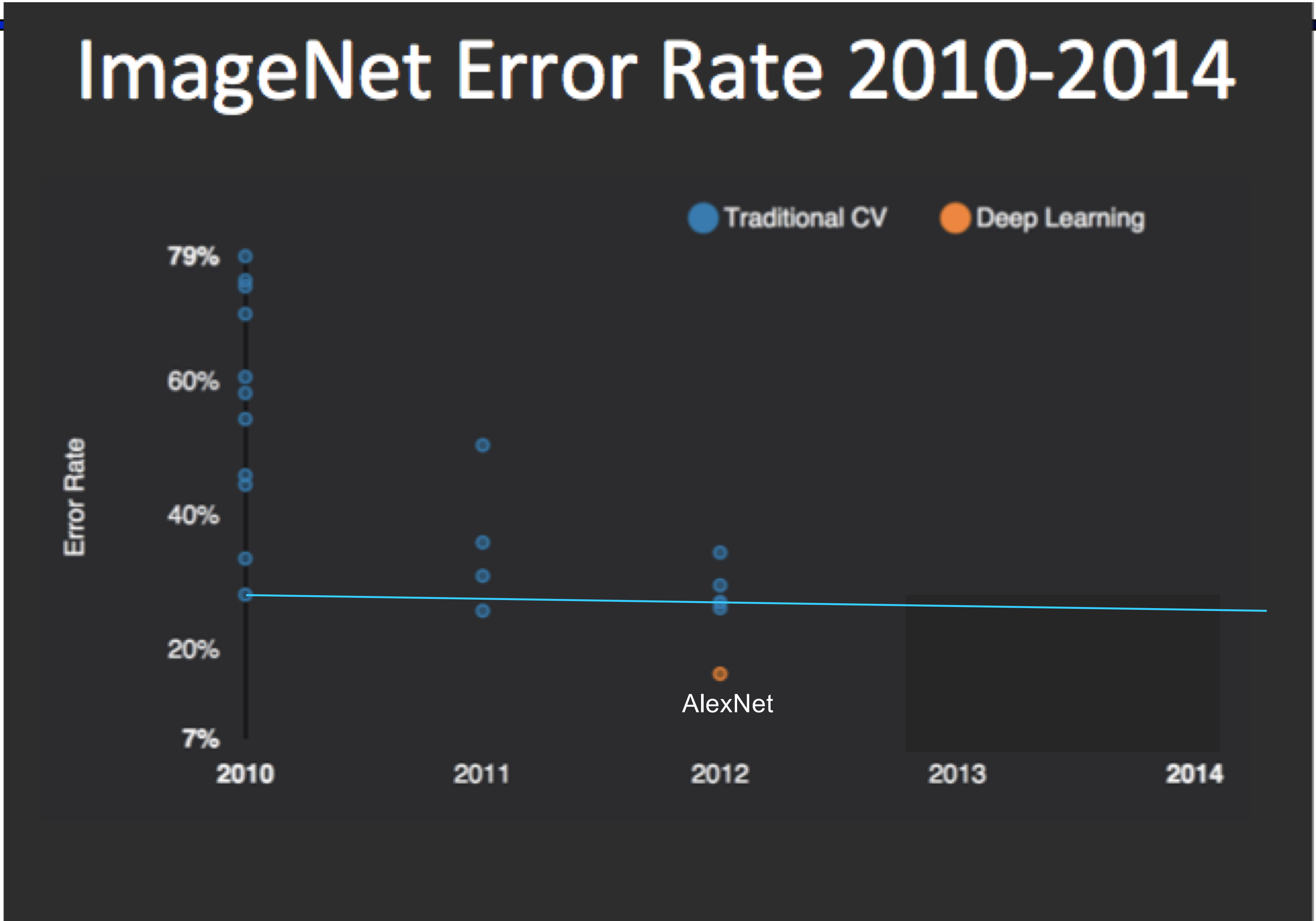

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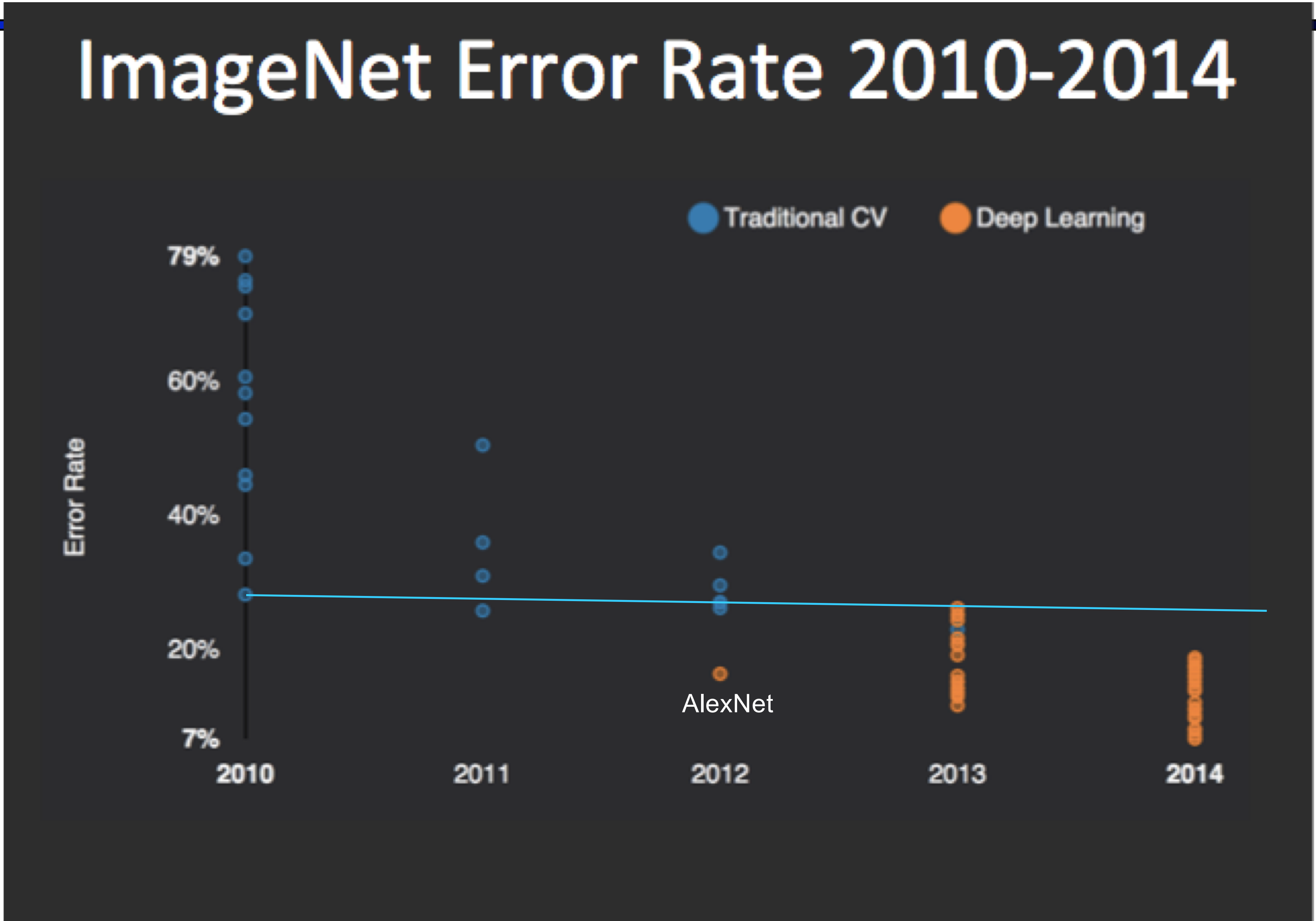

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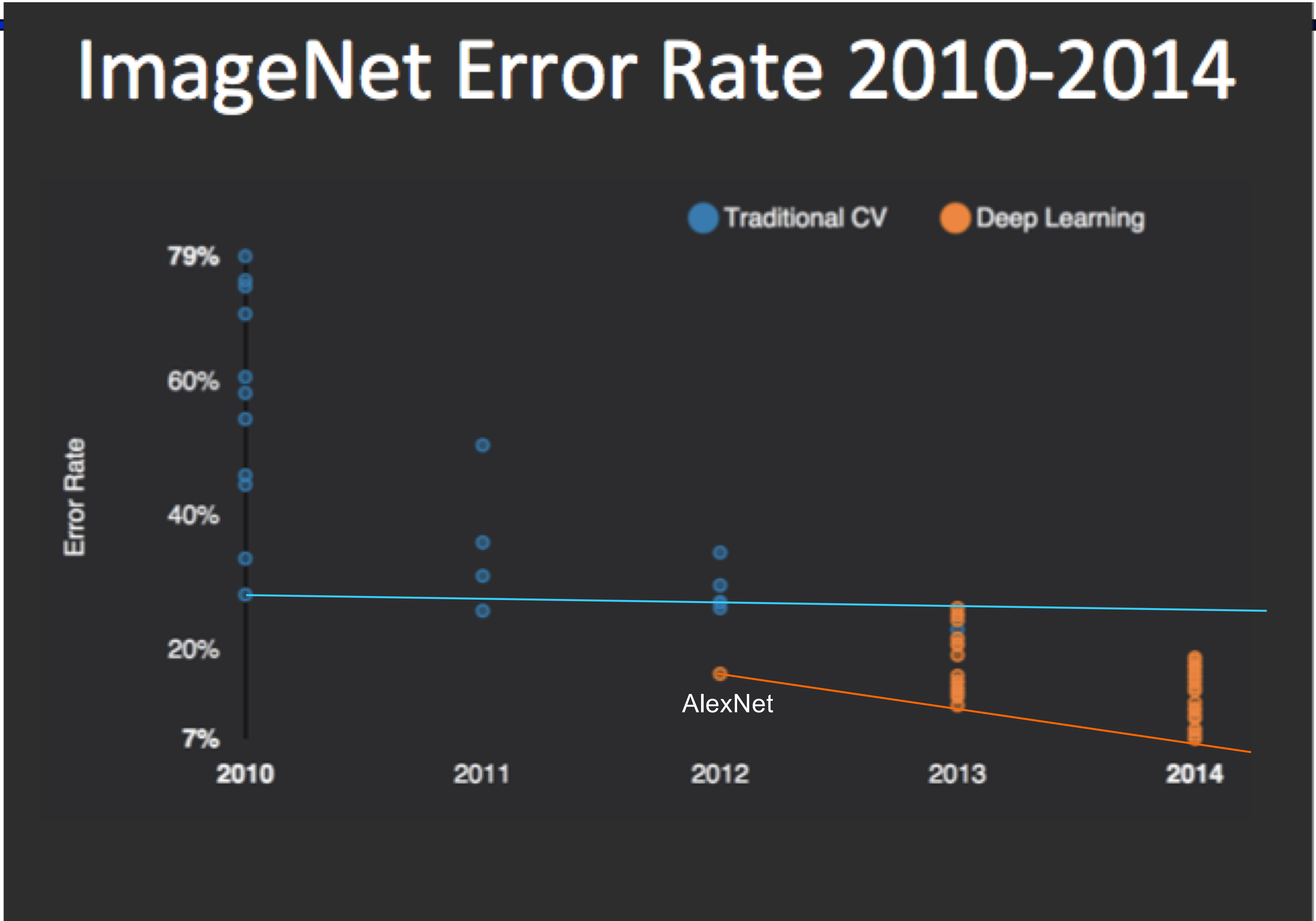


graph credit Matt Zeiler, Clarifai

MS COCO Image Captioning Challenge

"man in black shirt is playing guitar."

"construction worker in orange safety vest is working on road."

"two young girls are playing with lego toy."

"boy is doing backflip on wakeboard."

"girl in pink dress is jumping in air."

"black and white dog jumps over bar."

"young girl in pink shirt is swinging on swing."

"man in blue wetsuit is surfing on wave."

Karpathy & Fei-Fei, 2015; Donahue et al., 2015; Xu et al, 2015; many more

Visual QA Challenge

Stanislaw Antol, Aishwarya Agrawal, Jiasen Lu, Margaret Mitchell, Dhruv Batra, C. Lawrence Zitnick, Devi Parikh

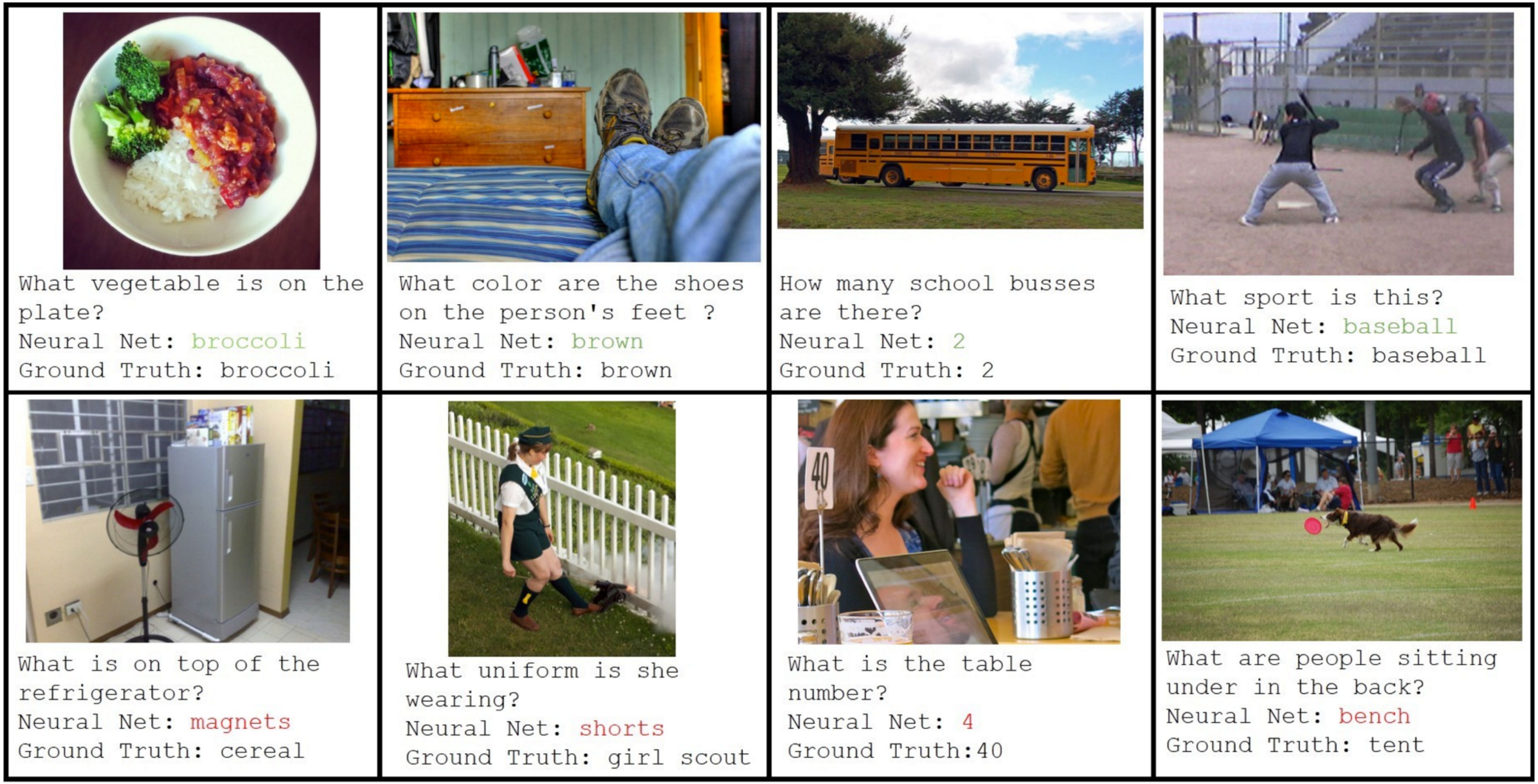
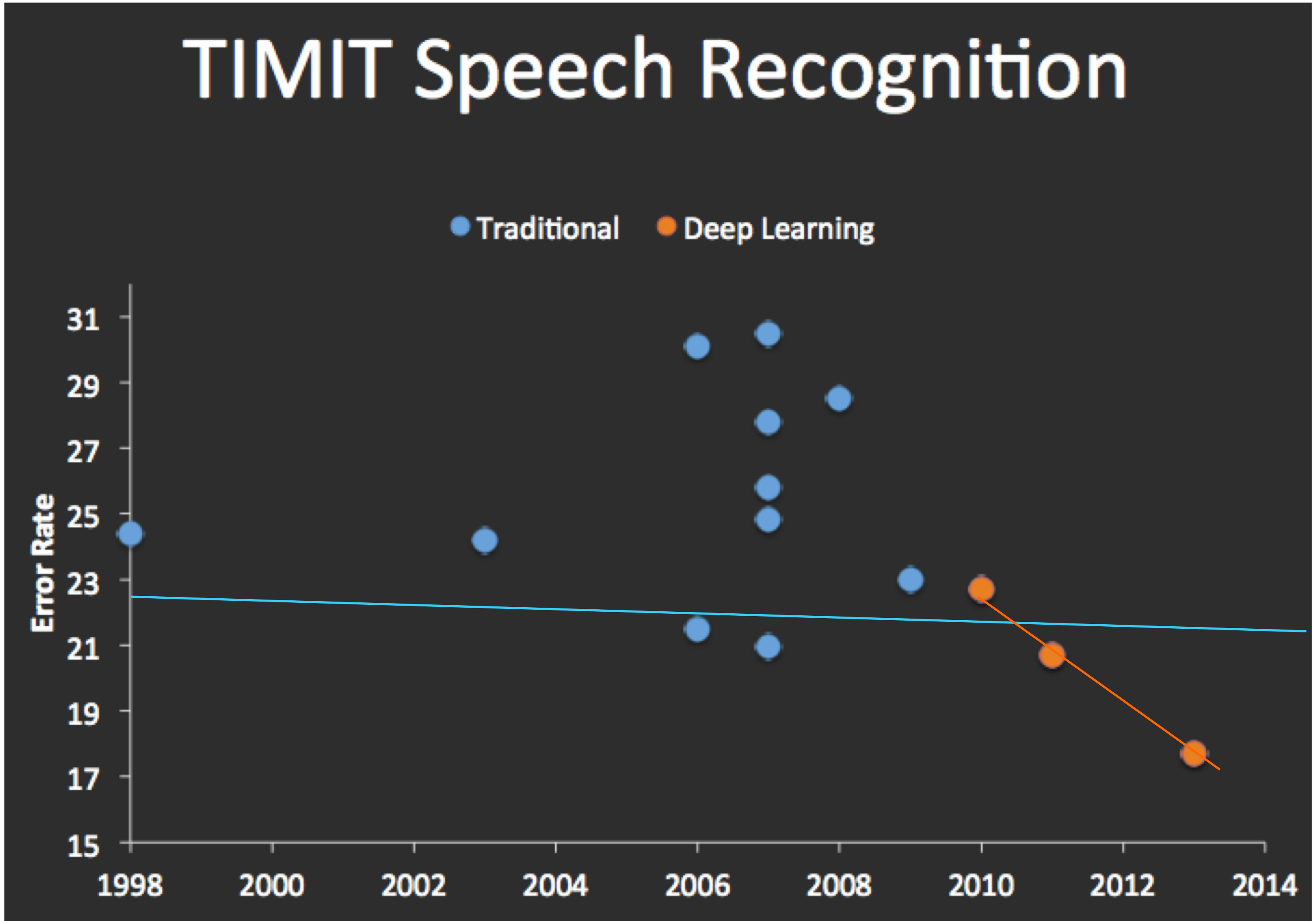
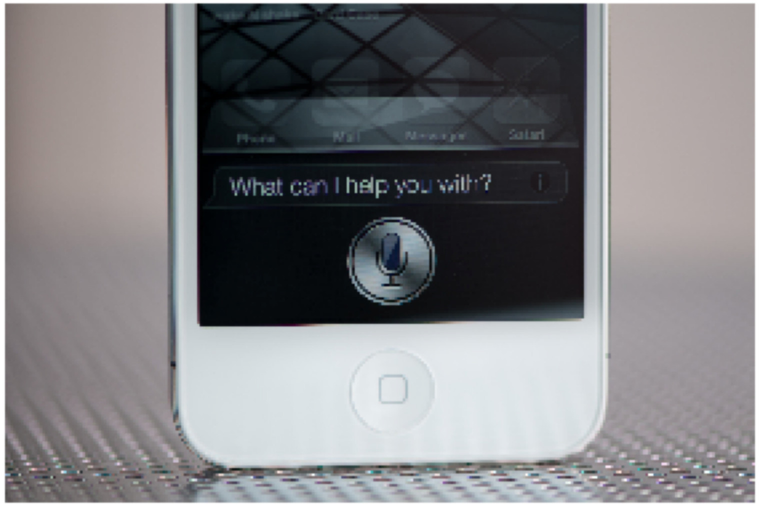
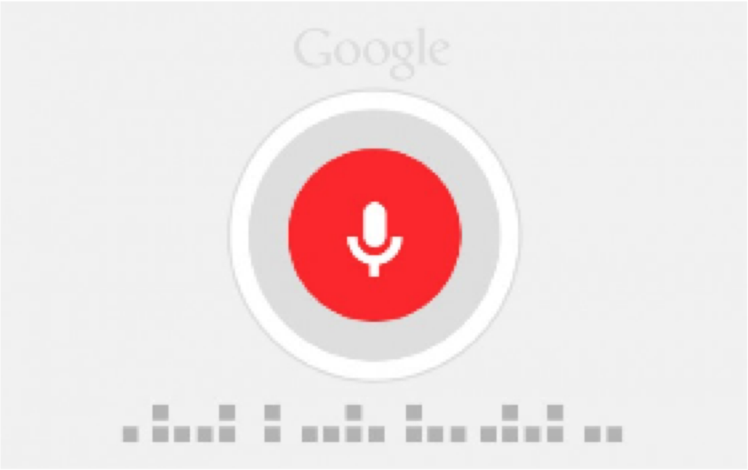


Image Segmentation

Speech Recognition



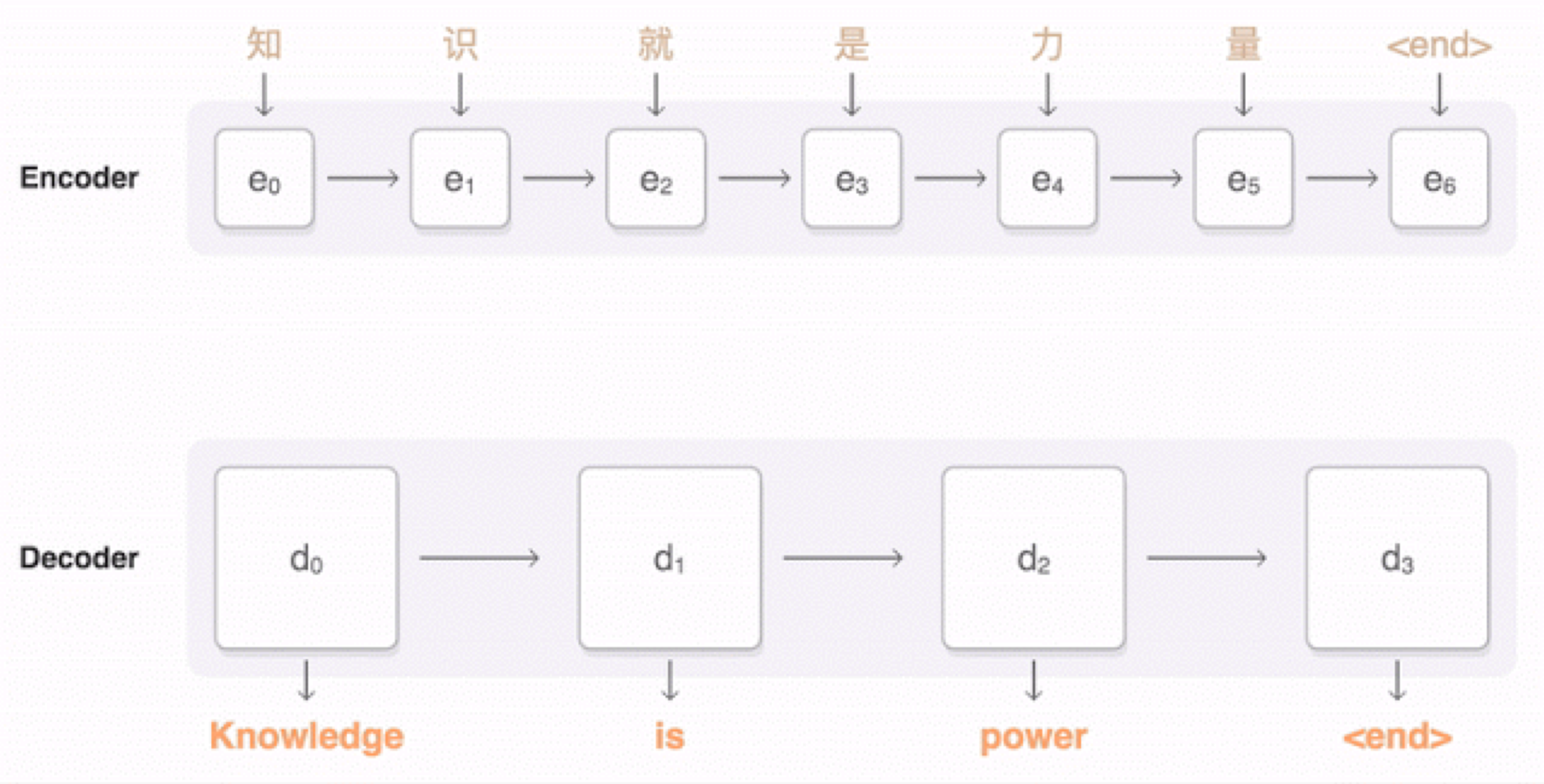




graph credit Matt Zeiler, Clarifai

Machine Translation

Google Neural Machine Translation (in production)



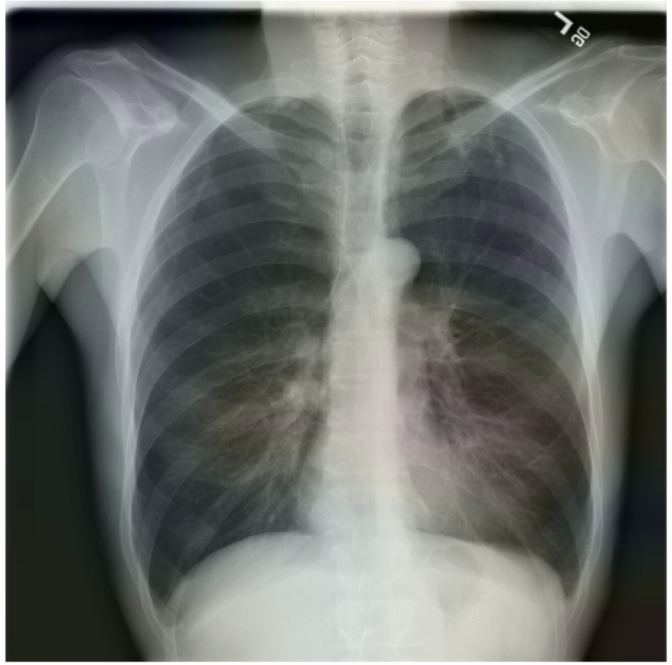
CheXNet: Radiologist-Level Pneumonia Detection on Chest X-Rays with Deep Learning

Pranav Rajpurkar*, Jeremy Irvin*, Kaylie Zhu, Brandon Yang, Hershel Mehta, Tony Duan, Daisy Ding, Aarti Bagul, Curtis Langlotz, Katie Shpanskaya, Matthew P. Lungren, Andrew Y. Ng

We develop an algorithm that can detect pneumonia from chest X-rays at a level exceeding practicing radiologists.

Chest X-rays are currently the best available method for diagnosing pneumonia, playing a crucial role in clinical care and epidemiological studies. Pneumonia is responsible for more than 1 million hospitalizations and 50,000 deaths per year in the US alone.

READ OUR PAPER



Google and DeepMind are using AI to predict the energy output of wind farms

To help make that energy more valuable to the power grid

By Nick Statt | @nickstatt | Feb 26, 2019, 2:42pm EST

SHARE

Google announced today that it has made energy produced by wind farms more viable using the artificial intelligence software of its London-based subsidiary DeepMind. By using DeepMind's machine learning algorithms to predict the wind output from the farms Google uses for its green energy initiatives, the company says it can now schedule set deliveries of energy output, which are more valuable to the grid than standard, non-time-based deliveries.

Change in Programming Paradigm!

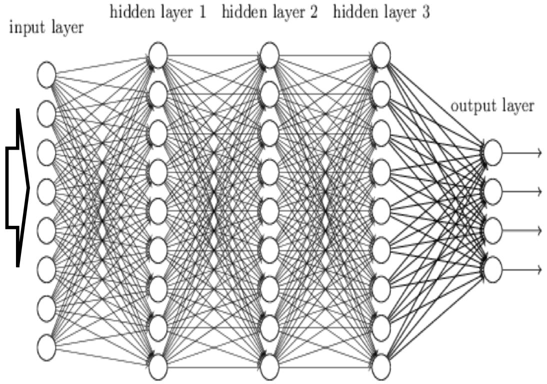
Traditional Programming: program by writing lines of code

```
If Number = 1 Then
    Count1 = Count1 + 1
Else
    If Number = 2 Then
        Count2 = Count2 + 1
    Else
        If Number = 3 then
            Count3 = Count3 + 1
        Else
            CountX = CountX + 1
        End if
    End If
End If
```

?

Poor performance on AI problems

Deep Learning ("Software 2.0"): program by providing data



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Success!