CS 188: Artificial Intelligence

CSPs Review + Local search

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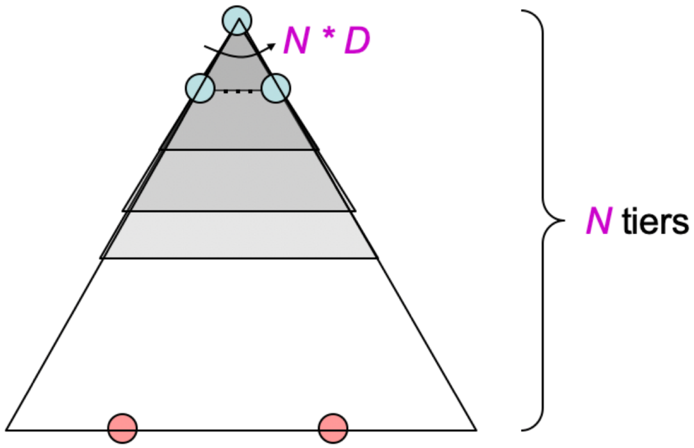
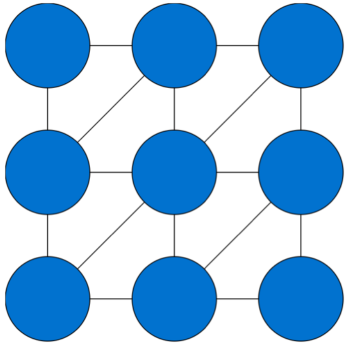
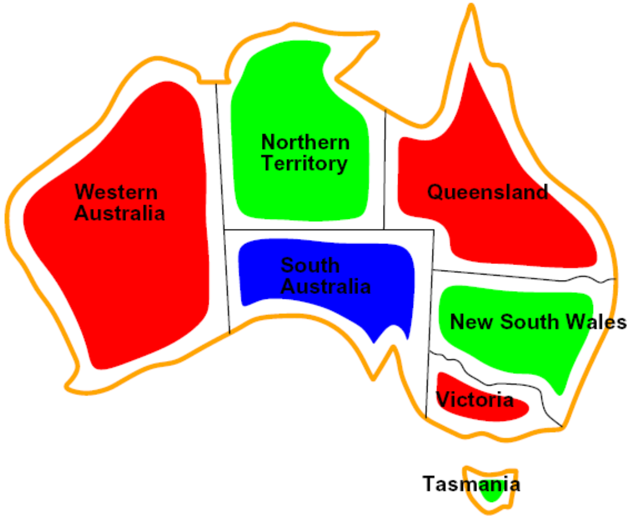
[These slides adapted from Stuart Russell and Dawn Song]

Search Methods

- BFS
  - Tries all possible partial assignments; like brute force search
  - All solutions are on deepest level of tree
- DFS
  - Cannot catch obvious violated constraints in partial assignments due to atomic state representation
- Backtracking Search
  - Fix order of variables
  - Check constraints as you go with “incremental goal test”

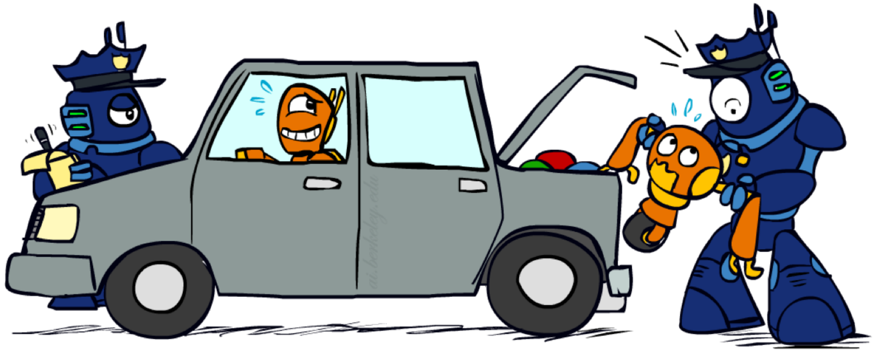






Backtracking Search Improvements: Filtering

- Forward checking: Cross off values that violate a constraint when added to the existing assignment
  - Checks all arcs *into* the assigned variable



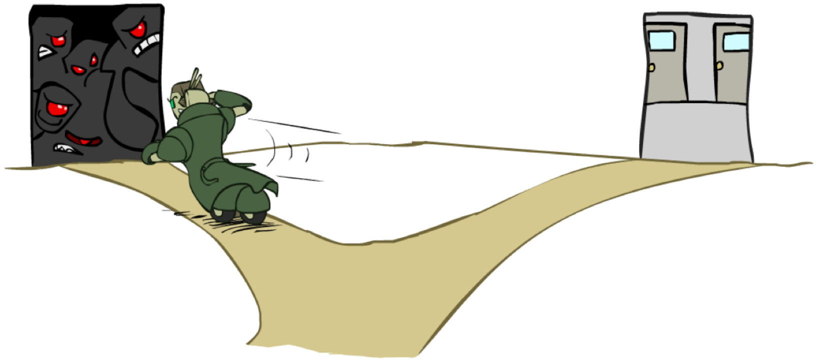
Delete from the tail!

- Arc Consistency: constraint propagation to ensure all arcs are consistent
  - Any time a variable X loses a value, all arcs *into* X need to be rechecked

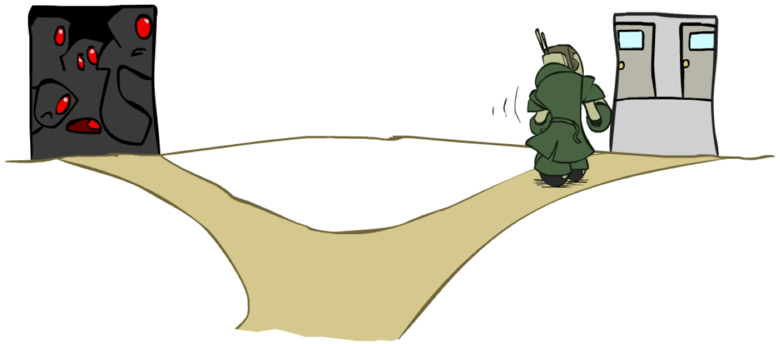
- K-Consistency: any consistent assignment to k-1 variables can extend to kth node
  - Strong K-Consistency: ensuring 1-consistency, 2-consistency, 3-consistency, …, k-consistency

Backtracking Search Improvements: Ordering

- Variable Ordering: Minimum remaining values (MRV)
  - Choose the variable with the fewest legal values left in its domain
  - Tie-break using the variable involved in most constraints
  - “Fail-fast” to prune search tree



- Value Ordering: Least Constraining Value (LCV)
  - Choose the value that rules out the fewest values in the remaining variables
  - May require additional computation (forward-checking/AC3)
  - Leave highest flexibility for later variable assignments



Algorithm Pseudocode

```
function BACKTRACK(csp, assignment) returns a solution or failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(csp, assignment)
    for each value in ORDER-DOMAIN-VALUES(csp, var, assignment) do
        if value is consistent with assignment then
            add {var = value} to assignment
            inferences ← INFERENCE(csp, var, assignment)
            if inferences ≠ failure then
                add inferences to csp
                result ← BACKTRACK(csp, assignment)
                if result ≠ failure then return result
                remove inferences from csp
            remove {var = value} from assignment
    return failure
```

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Local search

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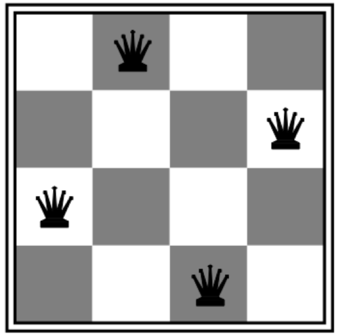
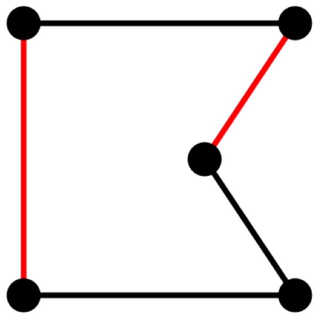
Local search algorithms

- In many optimization problems, ***path*** is irrelevant; the goal state ***is*** the solution
- Then state space = set of "complete" configurations;
  find ***configuration satisfying constraints***, e.g., n-queens problem; or, find ***optimal configuration***, e.g., travelling salesperson problem





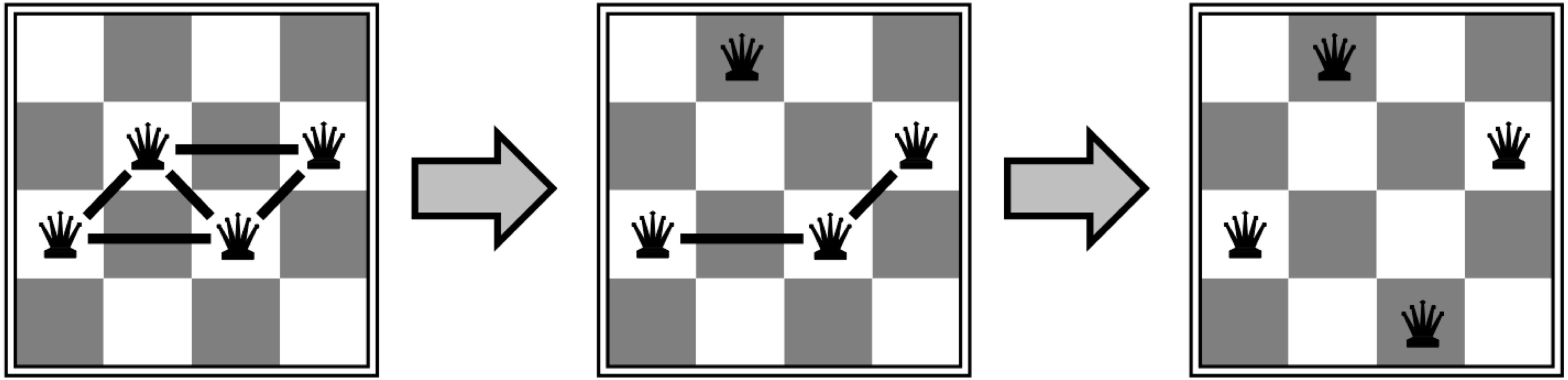
- In such cases, can use ***iterative improvement*** algorithms: keep a single "current" state, try to improve it
- Constant space, suitable for online as well as offline search
- More or less unavoidable if the "state" is yourself (i.e., learning)

Hill Climbing

- Simple, general idea:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit

Heuristic for *n*-queens problem

- Goal: n queens on board with no ***conflicts***, i.e., no queen attacking another
- States: n queens on board, one per column
- Actions: move a queen in its column
- Heuristic value function: number of conflicts



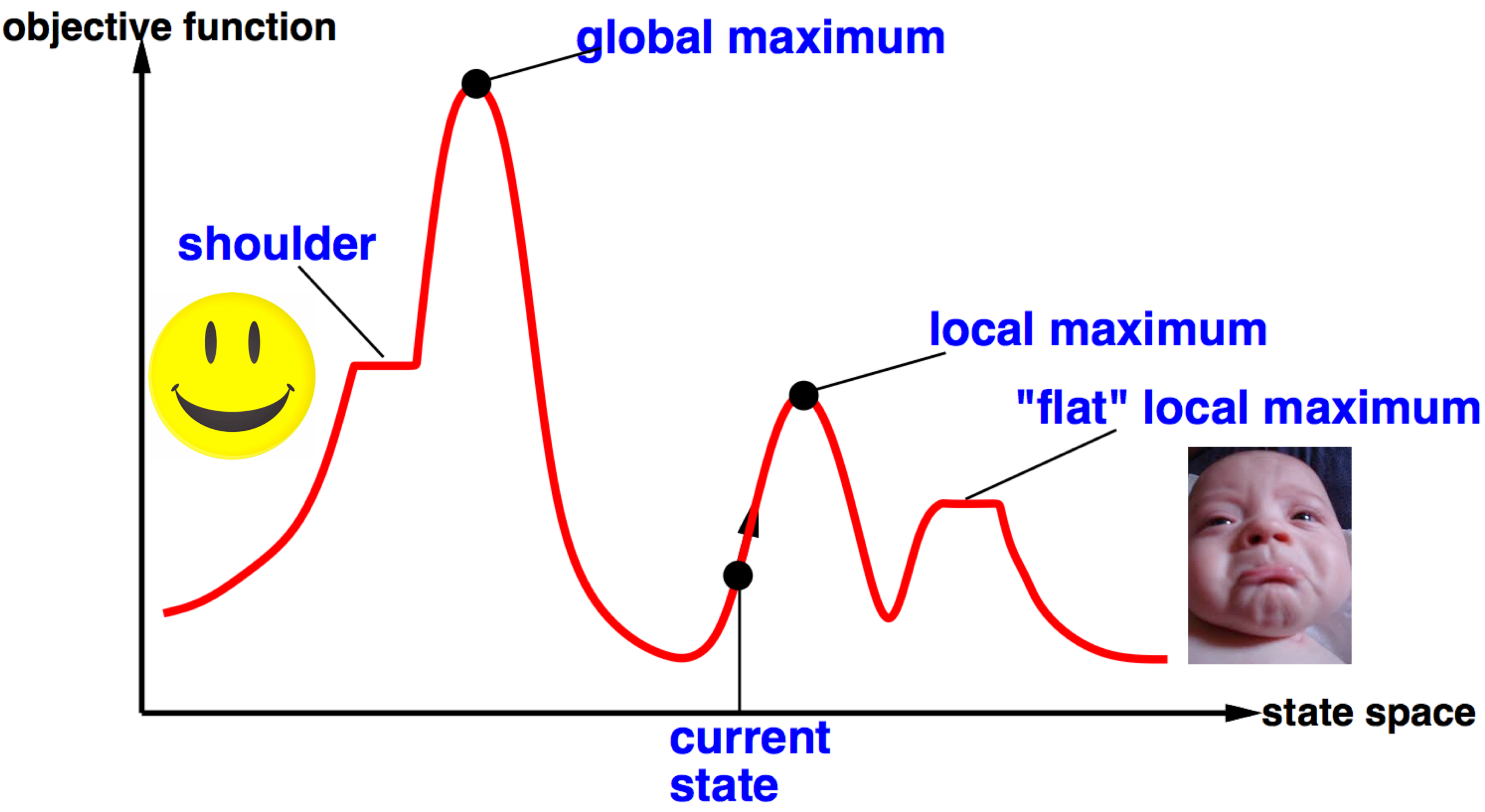
h = 5 **h = 2** **h = 0**

Hill-climbing algorithm

```
function HILL-CLIMBING(problem) returns a state
    current ← make-node(problem.initial-state)
    loop do
        neighbor ← a highest-valued successor of current
        if neighbor.value ≤ current.value then
            return current.state
        current ← neighbor
```

"Like climbing Everest in thick fog with amnesia"

Global and local maxima



Random restarts

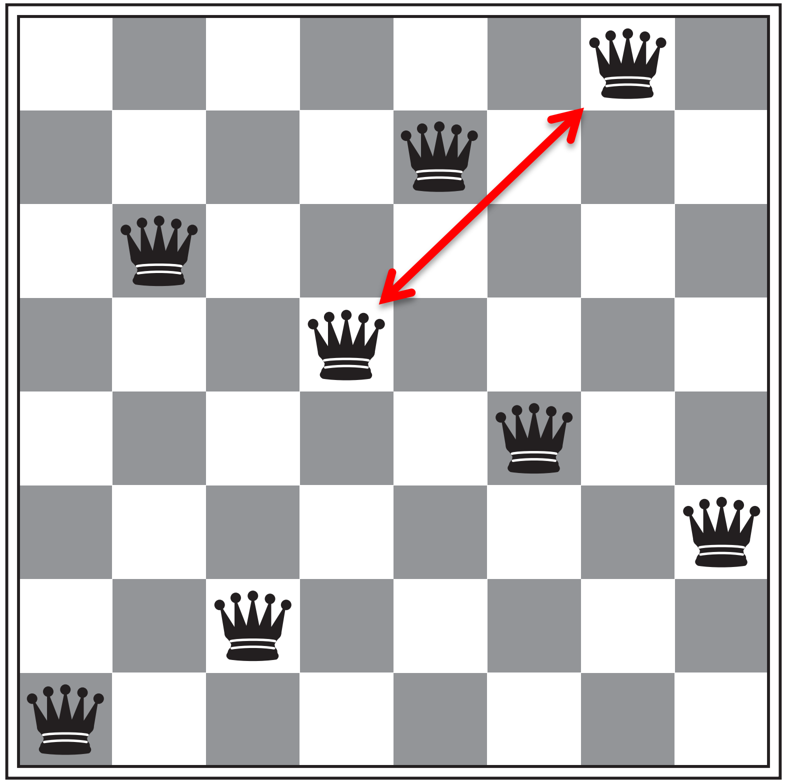
- find global optimum
- duh

Random sideways moves

- Escape from shoulders
- Loop forever on flat local maxima

Hill-climbing on the 8-queens problem

- No sideways moves:
  - Succeeds w/ prob. 0.14
  - Average number of moves per trial:
    - 4 when succeeding, 3 when getting stuck
  - Expected total number of moves needed:
    - 3(1-p)/p + 4 =~ 22 moves
- Allowing 100 sideways moves:
  - Succeeds w/ prob. 0.94
  - Average number of moves per trial:
    - 21 when succeeding, 65 when getting stuck
  - Expected total number of moves needed:
    - 65(1-p)/p + 21 =~ 25 moves



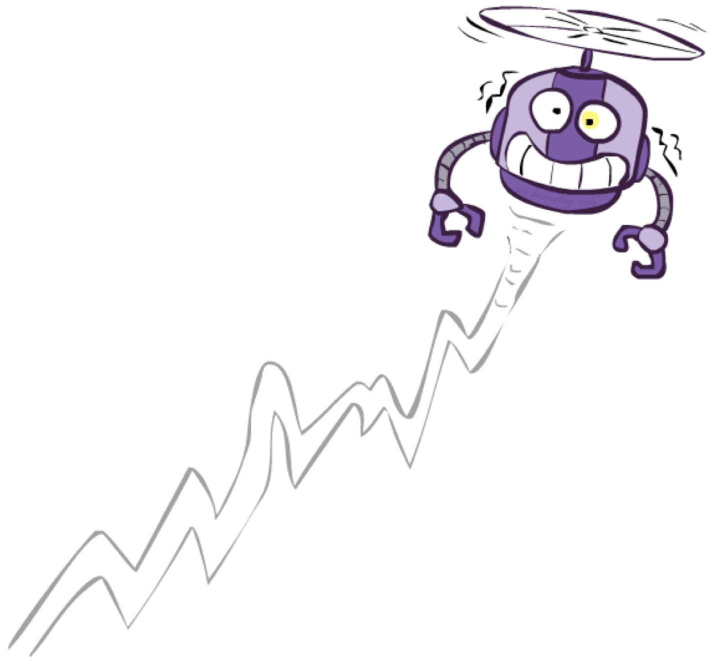
Moral: algorithms with knobs to twiddle are irritating

Simulated annealing

- Resembles the annealing process used to cool metals slowly to reach an ordered (low-energy) state
- Basic idea:
  - Allow “bad” moves occasionally, depending on “temperature”
  - High temperature => more bad moves allowed, shake the system out of its local minimum
  - Gradually reduce temperature according to some schedule
  - Sounds pretty flaky, doesn’t it?

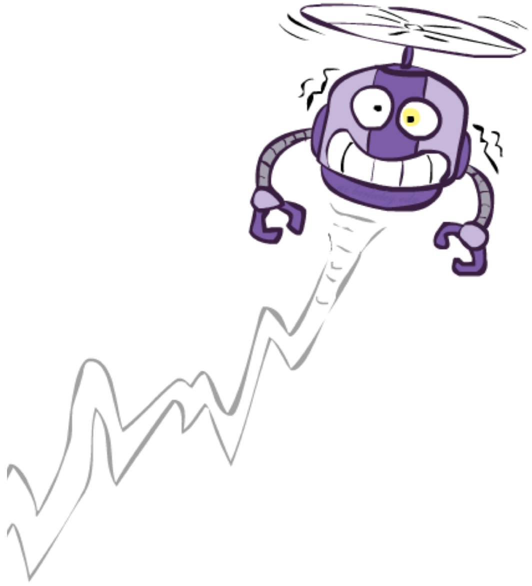
Simulated annealing algorithm

```
function SIMULATED-ANNEALING(problem,schedule) returns a  state
current ← problem.initial-state
for t = 1 to ∞ do
        T ←schedule(t)
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← next.value – current.value
        if ΔE > 0 then current ← next
                 else current ← next only with probability e^(ΔE/T)
```



Simulated Annealing

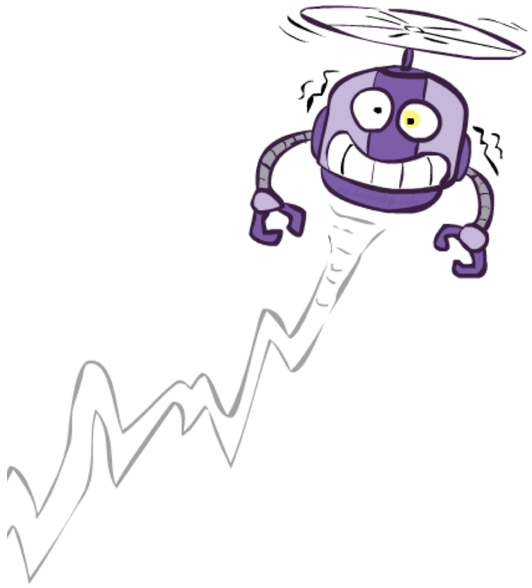
- Theoretical guarantee:
  - Stationary distribution (Boltzmann): $P(x) \propto e^{E(x)/T}$
  - If T decreased slowly enough, will converge to optimal state!
- Proof sketch
  - Consider two adjacent states x, y with $E(y) > E(x)$ [high is good]
  - Assume $x \rightarrow y$ and $y \rightarrow x$ and outdegrees $D(x) = D(y) = D$
  - Let $P(x)$, $P(y)$ be the equilibrium occupancy probabilities at T
  - Let $P(x \rightarrow y)$ be the probability that state x transitions to state y



Simulated Annealing

- Is this convergence an interesting guarantee?

- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape a local optimum, the less likely you are to ever make them all in a row
  - “Slowly enough” may mean exponentially slowly
  - Random restart hillclimbing also converges to optimal state…



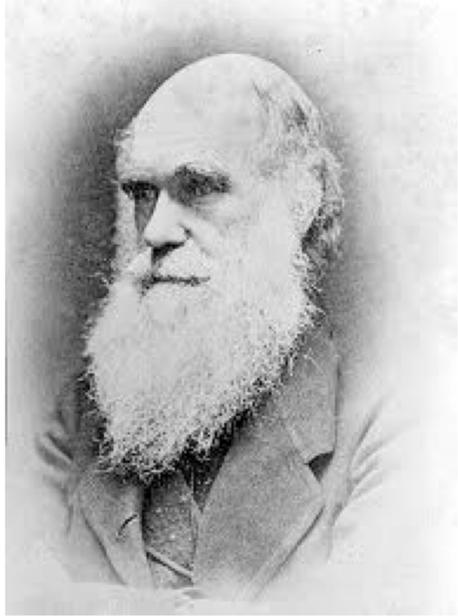
- Simulated annealing and its relatives are a key workhorse in VLSI layout and other optimal configuration problems

Local beam search

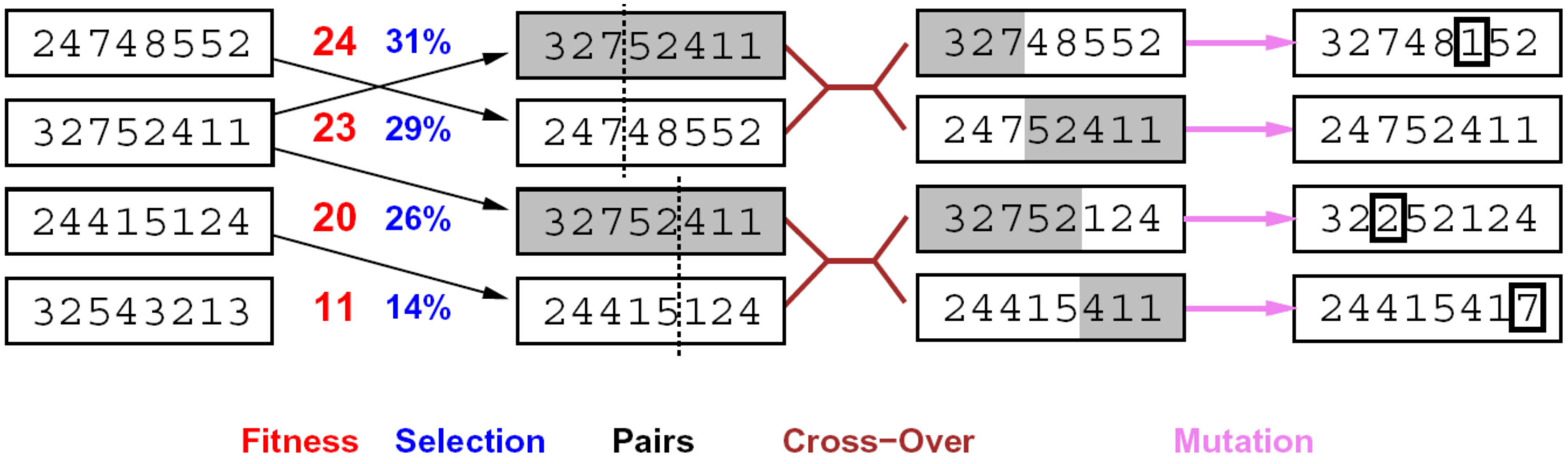
- Basic idea:
  - *K* copies of a local search algorithm, initialized randomly
  - For each iteration
    - Generate ALL successors from *K* current states
    - Choose best *K* of these to be the new current states

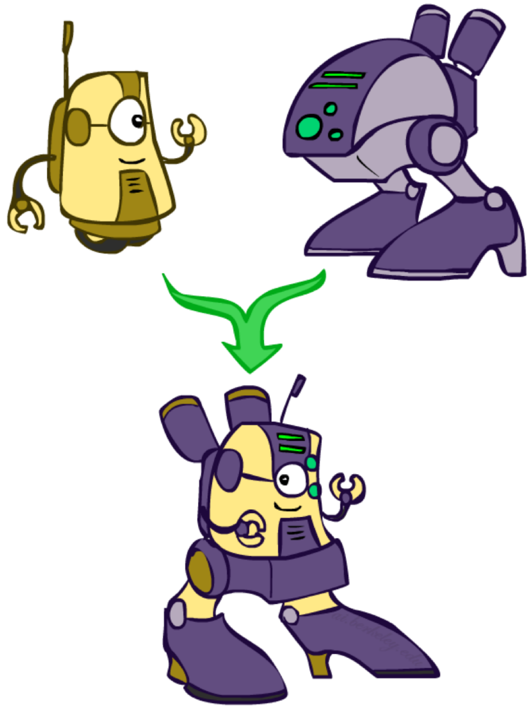
Or, K chosen randomly with a bias towards good ones

- Why is this different from *K* local searches in parallel?
  - The searches ***communicate***! "Come over here, the grass is greener!"
- What other well-known algorithm does this remind you of?
  - Evolution!



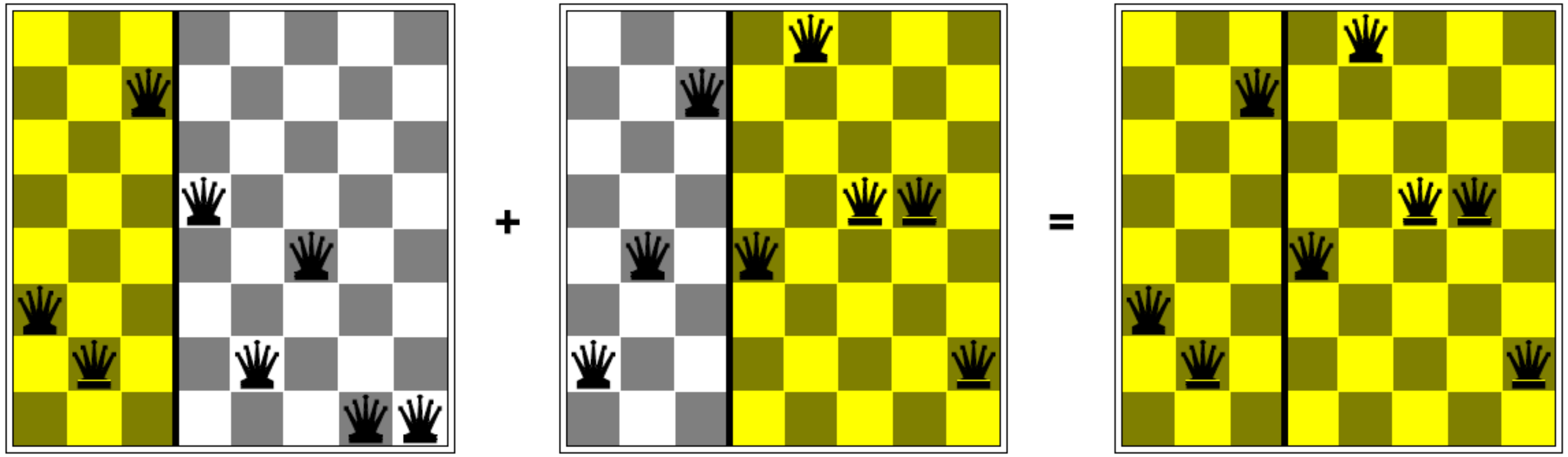
Genetic algorithms





- Genetic algorithms use a natural selection metaphor
  - Resample *K* individuals at each step (selection) weighted by fitness function
  - Combine by pairwise crossover operators, plus mutation to give variety

Example: N-Queens

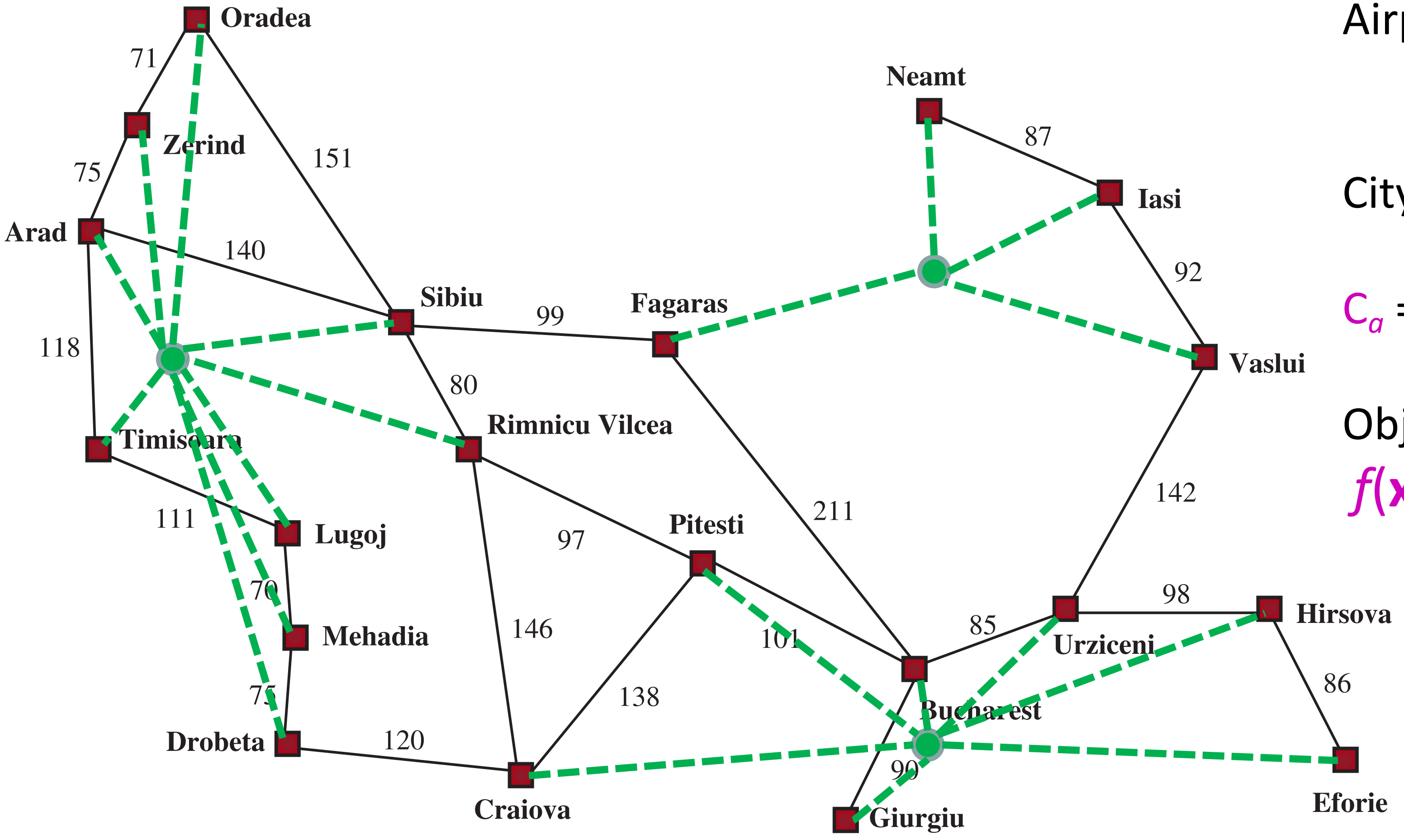


- Does crossover make sense here?
- What would mutation be?
- What would a good fitness function be?

Local search in continuous spaces

Example: Siting airports in Romania

Place 3 airports to minimize the sum of squared distances from each city to its nearest airport



Airport locations

$\mathbf{x} = (x_1,y_1), (x_2,y_2), (x_3,y_3)$

City locations (x_c,y_c)

C_a = cities closest to airport a

Objective: minimize

$f(\mathbf{x}) = \Sigma_a \Sigma_{c \in C_a} (x_a - x_c)^2 + (y_a - y_c)^2$

Handling a continuous state/action space

1. Discretize it!
   - Define a grid with increment δ, use any of the discrete algorithms
2. Choose random perturbations to the state
   a. First-choice hill-climbing: keep trying until something improves the state
   b. Simulated annealing
3. Compute gradient of $f(\mathbf{x})$ analytically

Finding extrema in continuous space

- Gradient vector $\nabla f(\mathbf{x}) = (\partial f/\partial x_1, \partial f/\partial y_1, \partial f/\partial x_2, \ldots)^T$
- For the airports, $f(\mathbf{x}) = \Sigma_a \Sigma_{c\in C_a} (x_a - x_c)^2 + (y_a - y_c)^2$
- $\partial f/\partial x_1 = \Sigma_{c\in C_1} 2(x_1 - x_c)$
- At an extremum, $\nabla f(\mathbf{x}) = 0$
- Can sometimes solve in closed form: $x_1 = (\Sigma_{c\in C_1} x_c)/|C_1|$
- Is this a local or global minimum of f?
- Gradient descent: $\mathbf{x} \leftarrow \mathbf{x} - \alpha\nabla f(\mathbf{x})$
  - Huge range of algorithms for finding extrema using gradients

Summary

- Many configuration and optimization problems can be formulated as local search
- General families of algorithms:
  - Hill-climbing, continuous optimization
  - Simulated annealing (and other stochastic methods)
  - Local beam search: multiple interaction searches
  - Genetic algorithms: break and recombine states

Many machine learning algorithms are local searches