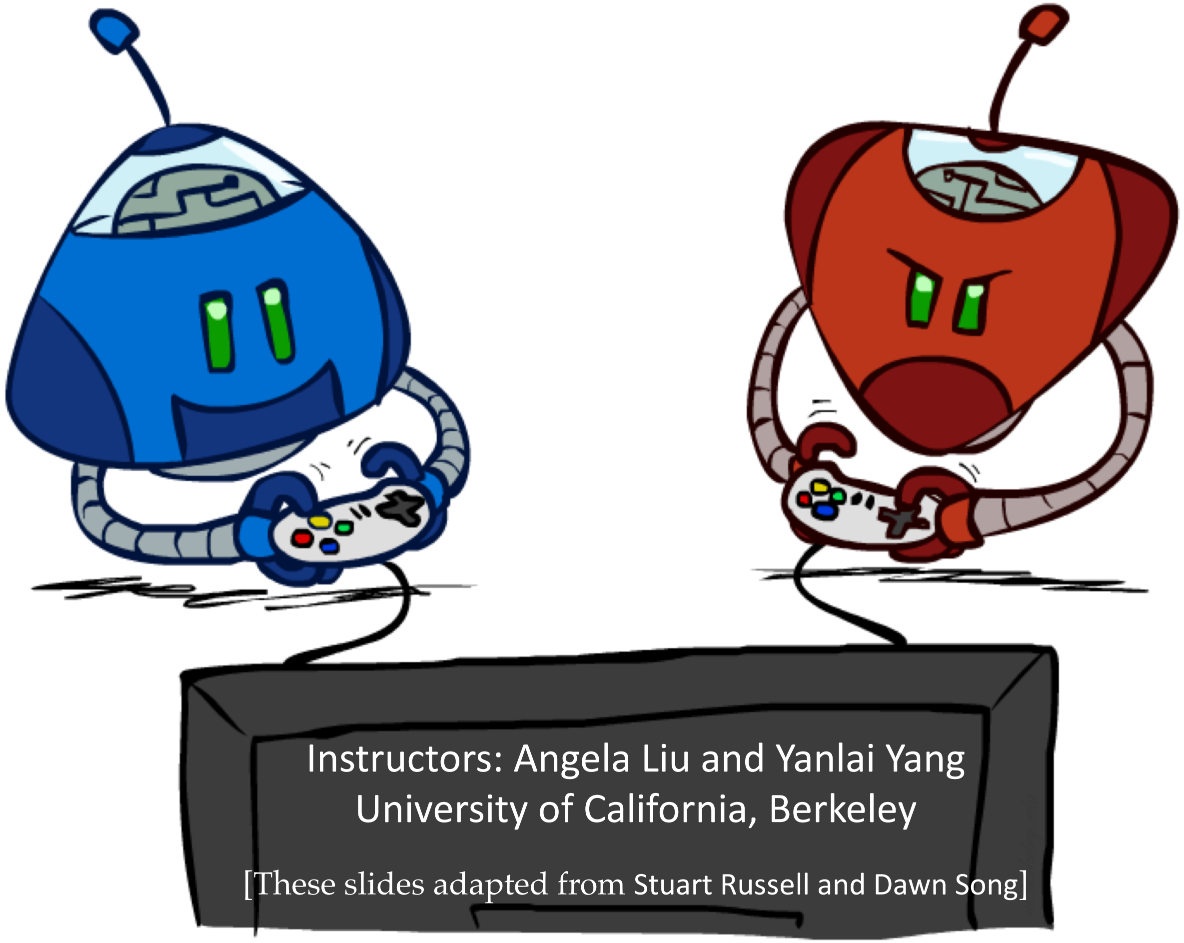
CS 188: Artificial Intelligence

Local Search + Games



Hill Climbing

- Hill-Climbing:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit
- Random-Restart
  - Rerun hill-climbing at different starting states

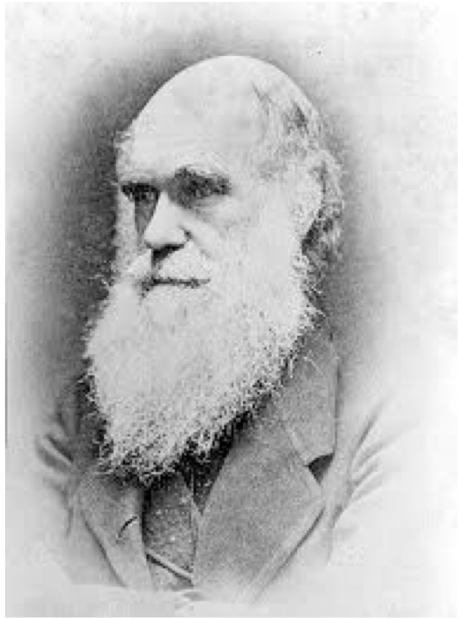
- Simulated Annealing
  - High temperature => more bad moves allowed, shake the system out of its local minimum
  - Gradually reduce temperature according to some schedule to focus later search on (hopefully) the globally optimal region

Local beam search

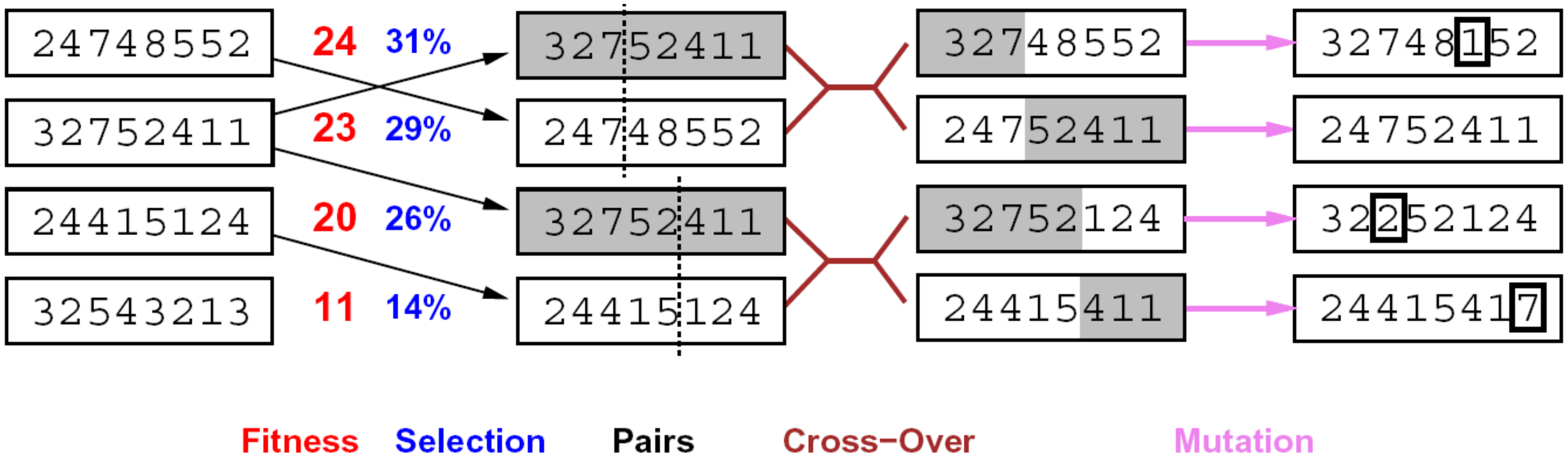
- Basic idea:
  - *K* copies of a local search algorithm, initialized randomly
  - For each iteration
    - Generate ALL successors from *K* current states
    - Choose best *K* of these to be the new current states

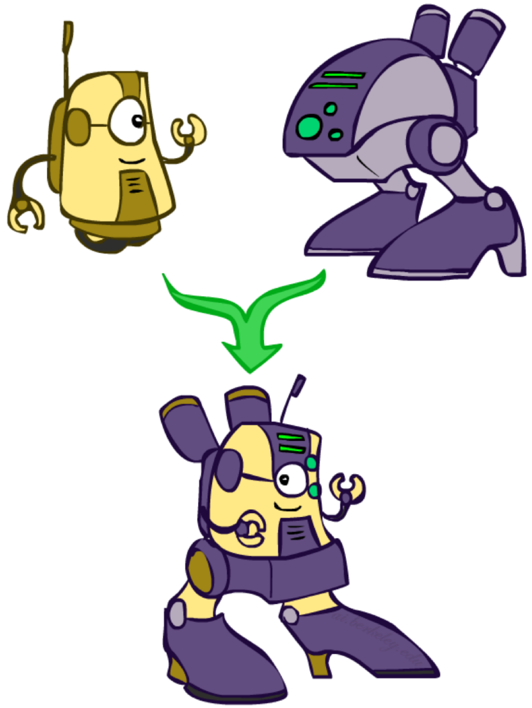
Or, K chosen randomly with a bias towards good ones

- Why is this different from *K* local searches in parallel?
  - The searches ***communicate***! “Come over here, the grass is greener!”
- What other well-known algorithm does this remind you of?
  - Evolution!



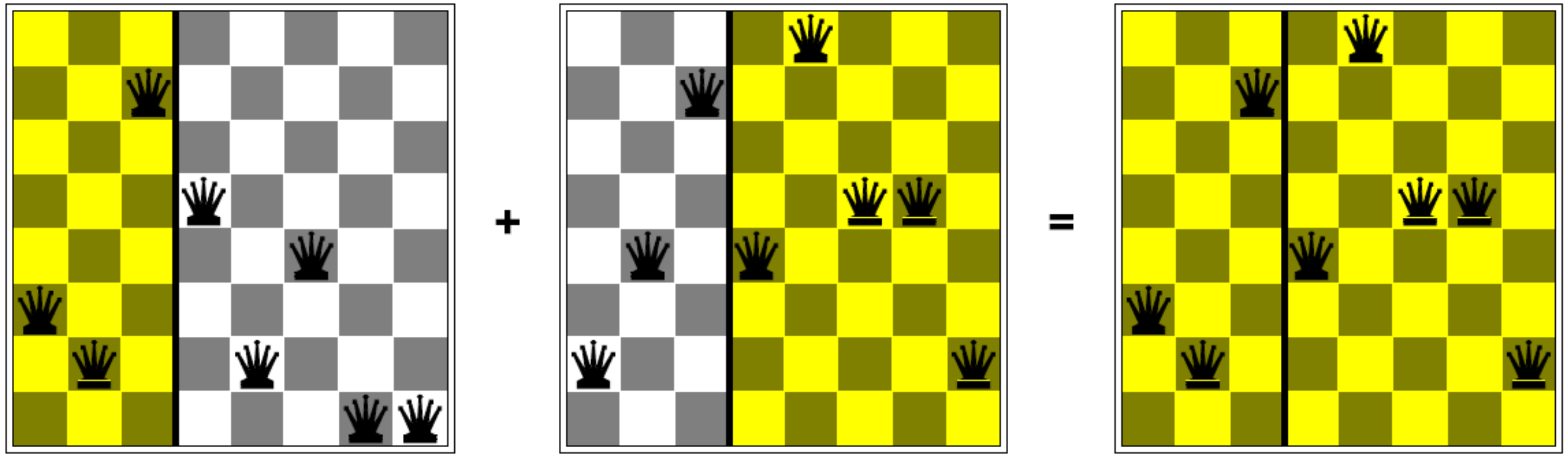
Genetic algorithms





- Genetic algorithms use a natural selection metaphor
  - Resample *K* individuals at each step (selection) weighted by fitness function
  - Combine by pairwise crossover operators, plus mutation to give variety

Example: N-Queens

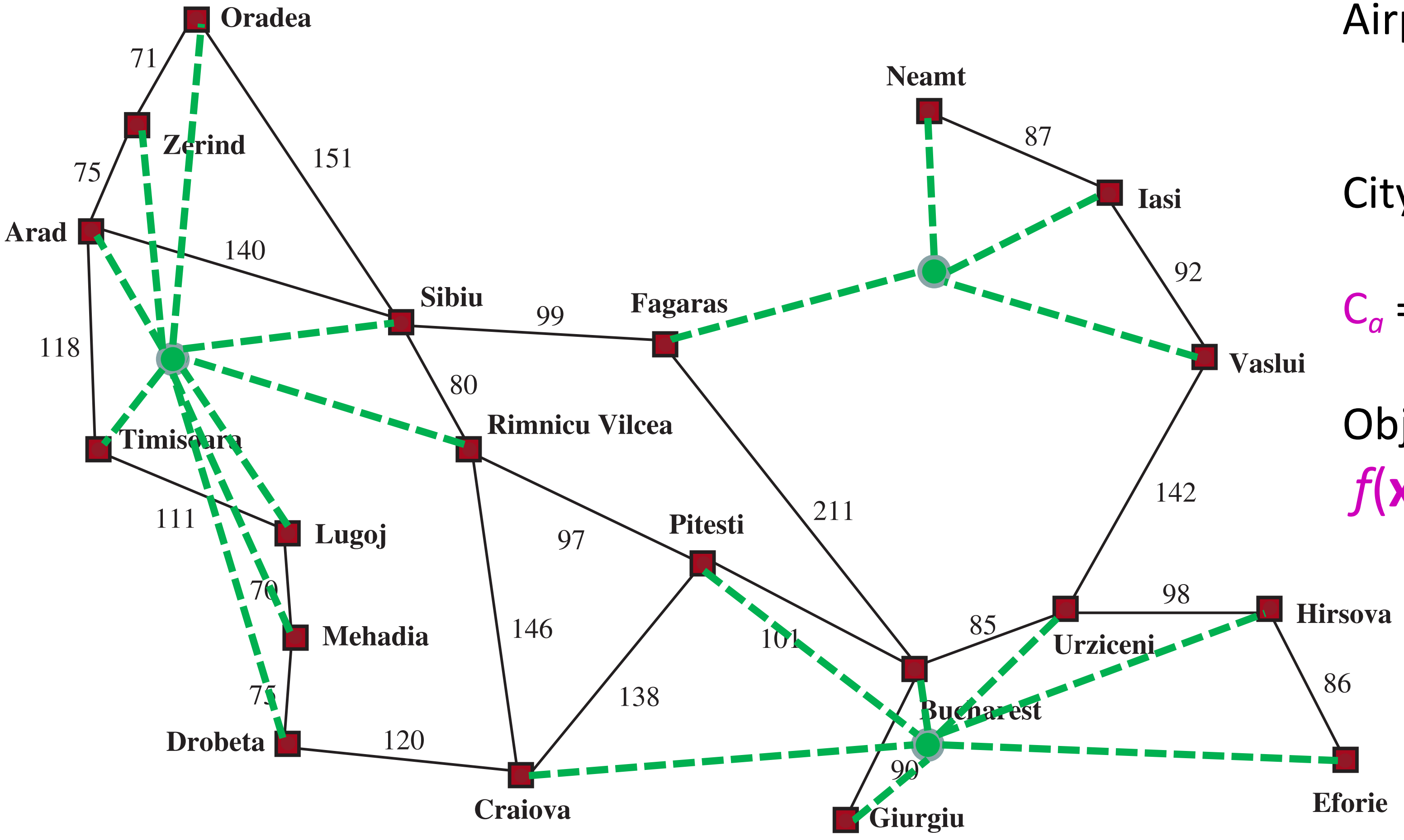


- Does crossover make sense here?
- What would mutation be?
- What would a good fitness function be?

Local search in continuous spaces

Example: Siting airports in Romania

Place 3 airports to minimize the sum of squared distances from each city to its nearest airport



Airport locations

$\mathbf{x} = (x_1,y_1), (x_2,y_2), (x_3,y_3)$

City locations (x_c,y_c)

C_a = cities closest to airport a

Objective: minimize

$$f(\mathbf{x}) = \Sigma_a \Sigma_{c \in C_a} (x_a - x_c)^2 + (y_a - y_c)^2$$

Handling a continuous state/action space

1. Discretize it!
   - Define a grid with increment δ, use any of the discrete algorithms
2. Choose random perturbations to the state
   a. First-choice hill-climbing: keep trying until something improves the state
   b. Simulated annealing
3. Compute gradient of $f(\mathbf{x})$ analytically

Finding extrema in continuous space

- Gradient vector $\nabla f(\mathbf{x}) = (\partial f/\partial x_1, \partial f/\partial y_1, \partial f/\partial x_2, \ldots)^T$
- For the airports, $f(\mathbf{x}) = \Sigma_a \Sigma_{c \in C_a} (x_a - x_c)^2 + (y_a - y_c)^2$
- $\partial f/\partial x_1 = \Sigma_{c \in C_1} 2(x_1 - x_c)$
- At an extremum, $\nabla f(\mathbf{x}) = 0$
- Can sometimes solve in closed form: $x_1 = (\Sigma_{c \in C_1} x_c)/|C_1|$
- Is this a local or global minimum of f?
- Gradient descent: $\mathbf{x} \leftarrow \mathbf{x} - \alpha \nabla f(\mathbf{x})$
  - Huge range of algorithms for finding extrema using gradients

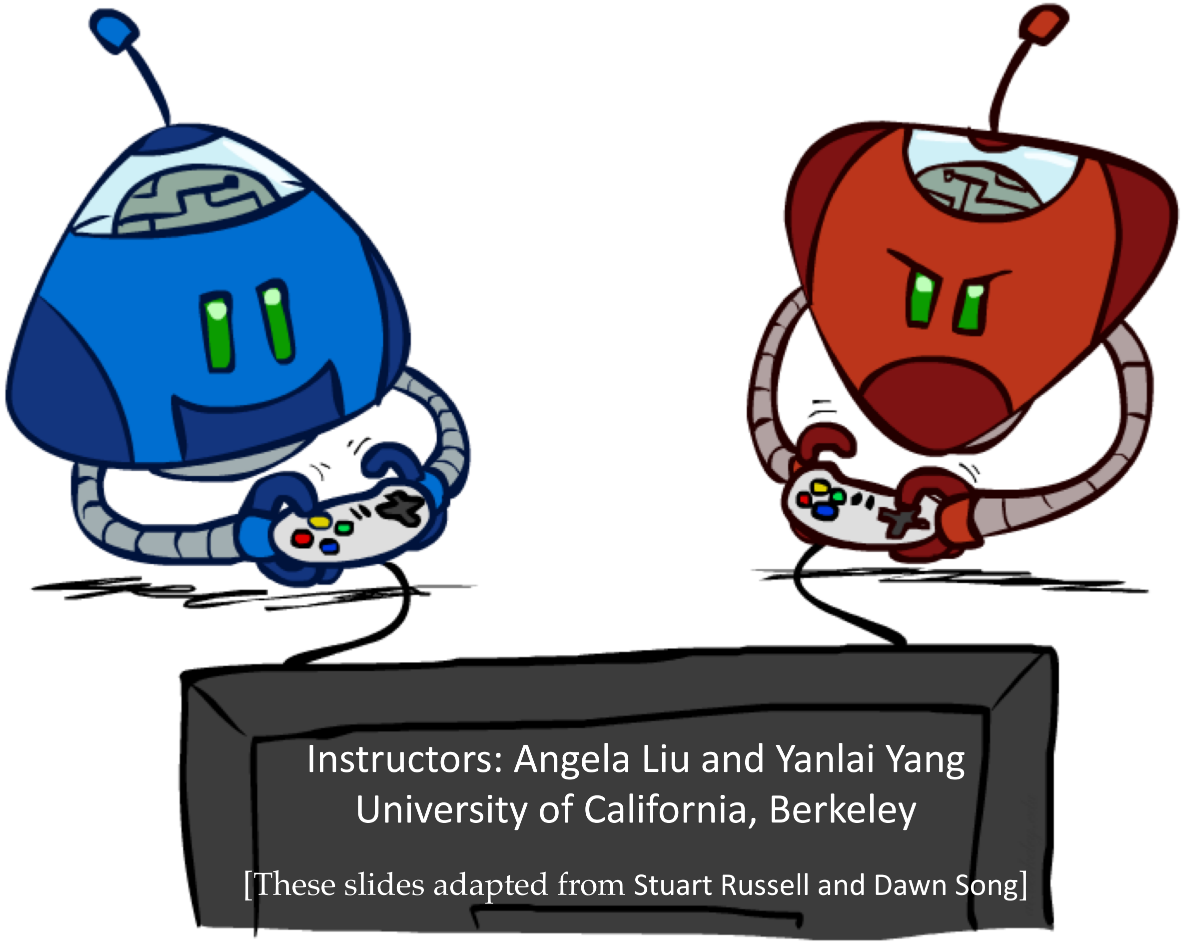
Summary

- Many configuration and optimization problems can be formulated as local search
- General families of algorithms:
  - Hill-climbing, continuous optimization
  - Simulated annealing (and other stochastic methods)
  - Local beam search: multiple interaction searches
  - Genetic algorithms: break and recombine states

Many machine learning algorithms are local searches

CS 188: Artificial Intelligence

Games: Minimax and Alpha-Beta



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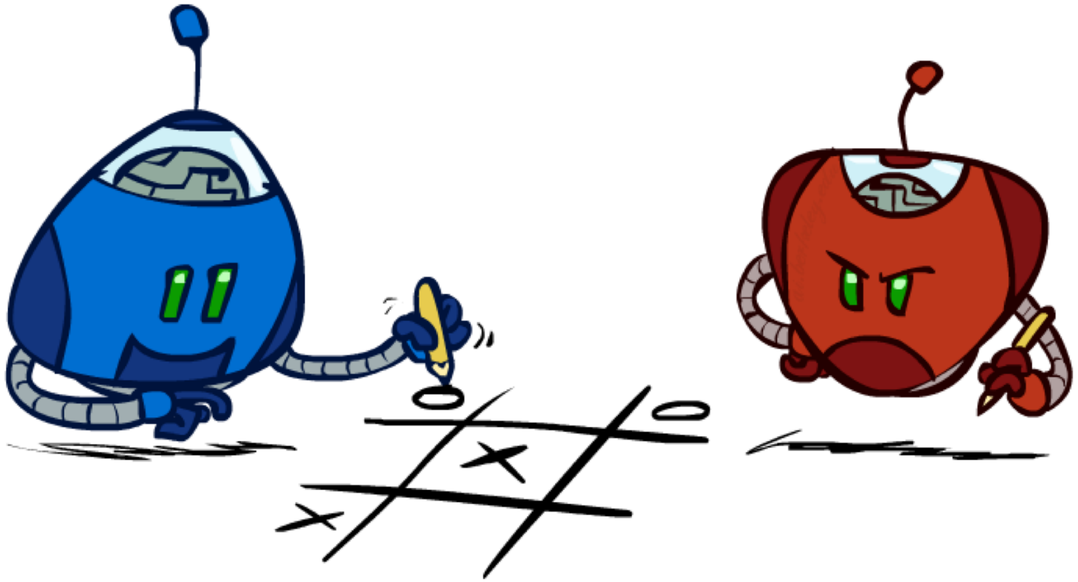
[These slides adapted from Stuart Russell and Dawn Song]

Outline

- History / Overview
- Minimax for Zero-Sum Games
- α-β Pruning
- Finite lookahead and evaluation



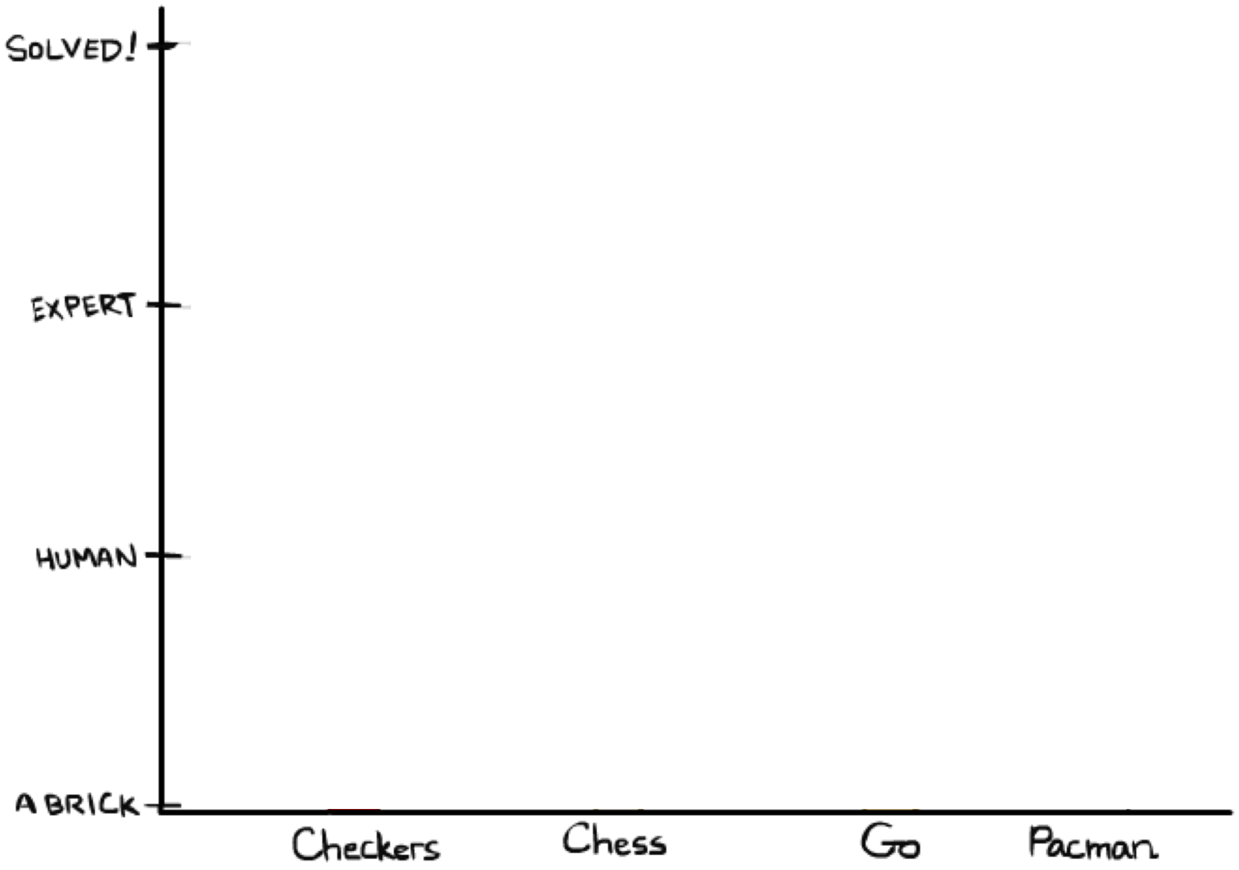
A brief history

- **Checkers:**
  - 1950: First computer player.
  - 1959: Samuel's self-taught program.
  - 1994: First computer world champion: Chinook defeats Tinsley
  - 2007: Checkers solved! Endgame database of 39 trillion states

- **Chess:**
  - 1945-1960: Zuse, Wiener, Shannon, Turing, Newell & Simon, McCarthy.
  - 1960s onward: gradual improvement under "standard model"
  - 1997: Deep Blue defeats human champion Garry Kasparov
  - 2022: Stockfish rating 3541 (vs 2882 for Magnus Carlsen 2015).

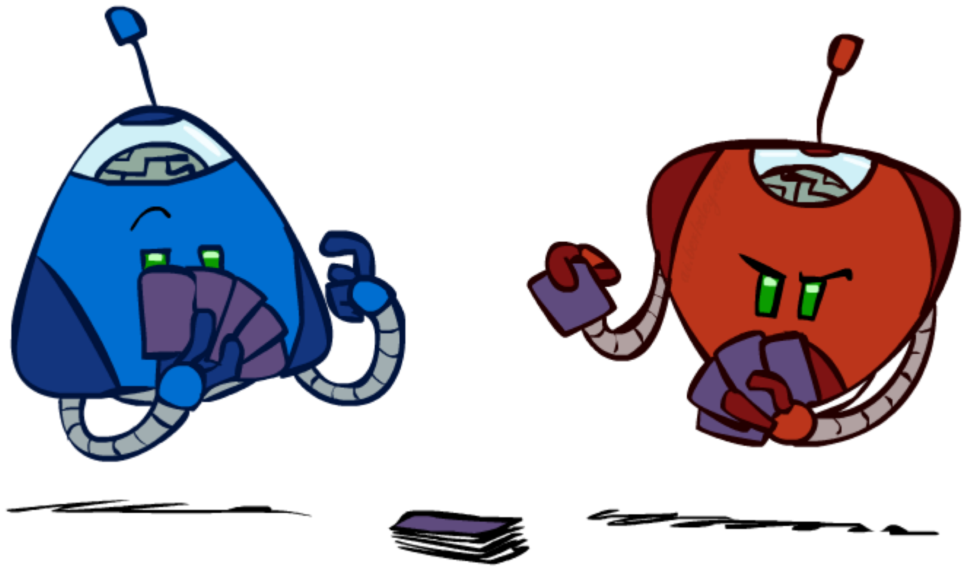
- **Go:**
  - 1968: Zobrist's program plays legal Go, barely (b>300!)
  - 1968-2005: various ad hoc approaches tried, novice level
  - 2005-2014: Monte Carlo tree search -> strong amateur
  - 2016-2017: AlphaGo defeats human world champions

- **Pacman**



Types of Games

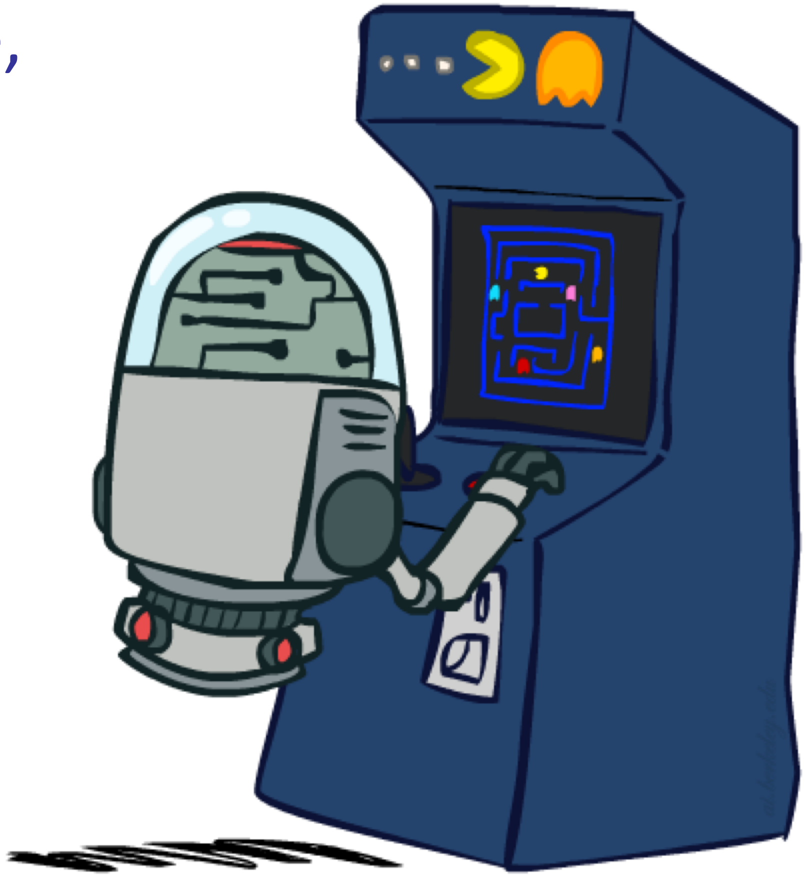
- Game = task environment with > 1 agent
- Axes:
  - Deterministic or stochastic?
  - Perfect information (fully observable)?
  - Two, three, or more players?
  - Teams or individuals?
  - Turn-taking or simultaneous?
  - Zero sum?



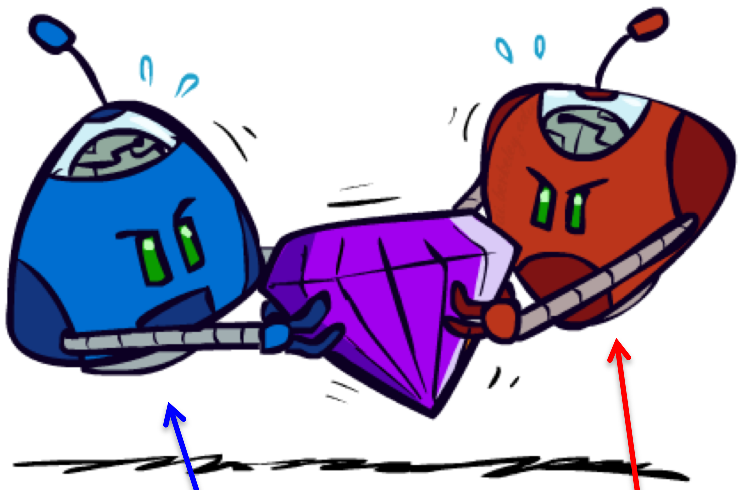
- Want algorithms for calculating a ***contingent plan*** (a.k.a. strategy or policy) which recommends a move for every possible eventuality

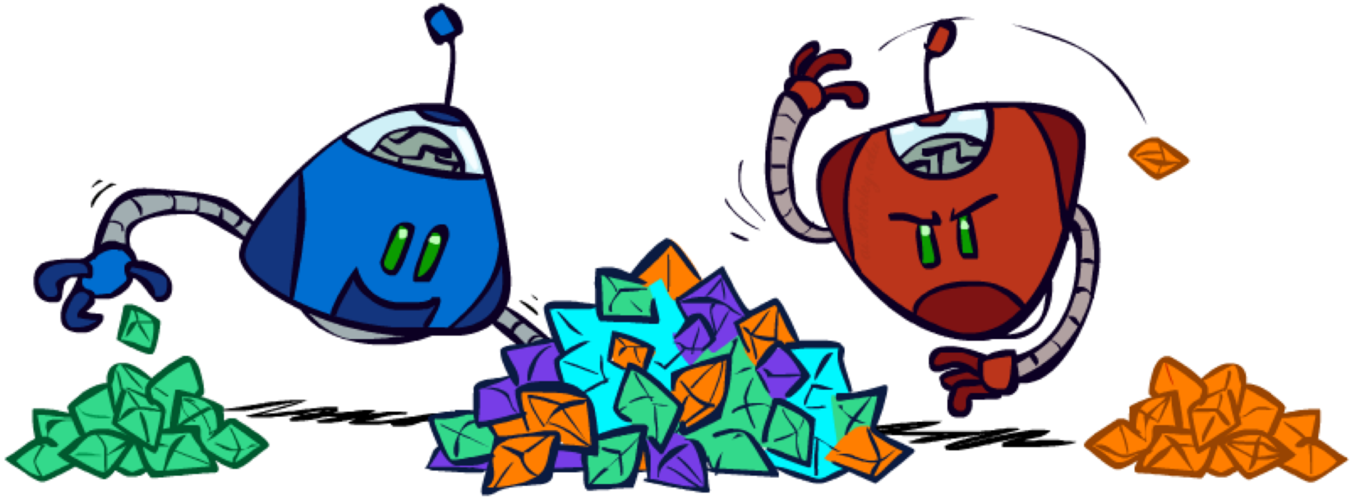
"Standard" Games

- Standard games are deterministic, observable, two-player, turn-taking, zero-sum
- Game formulation:
  - Initial state: s_0
  - Players: Player(s) indicates whose move it is
  - Actions: Actions(s) for player on move
  - Transition model: Result(s,a)
  - Terminal test: Terminal-Test(s)
  - Terminal values: Utility(s,p) for player p
    - Or just Utility(s) for player making the decision at root



Zero-Sum Games

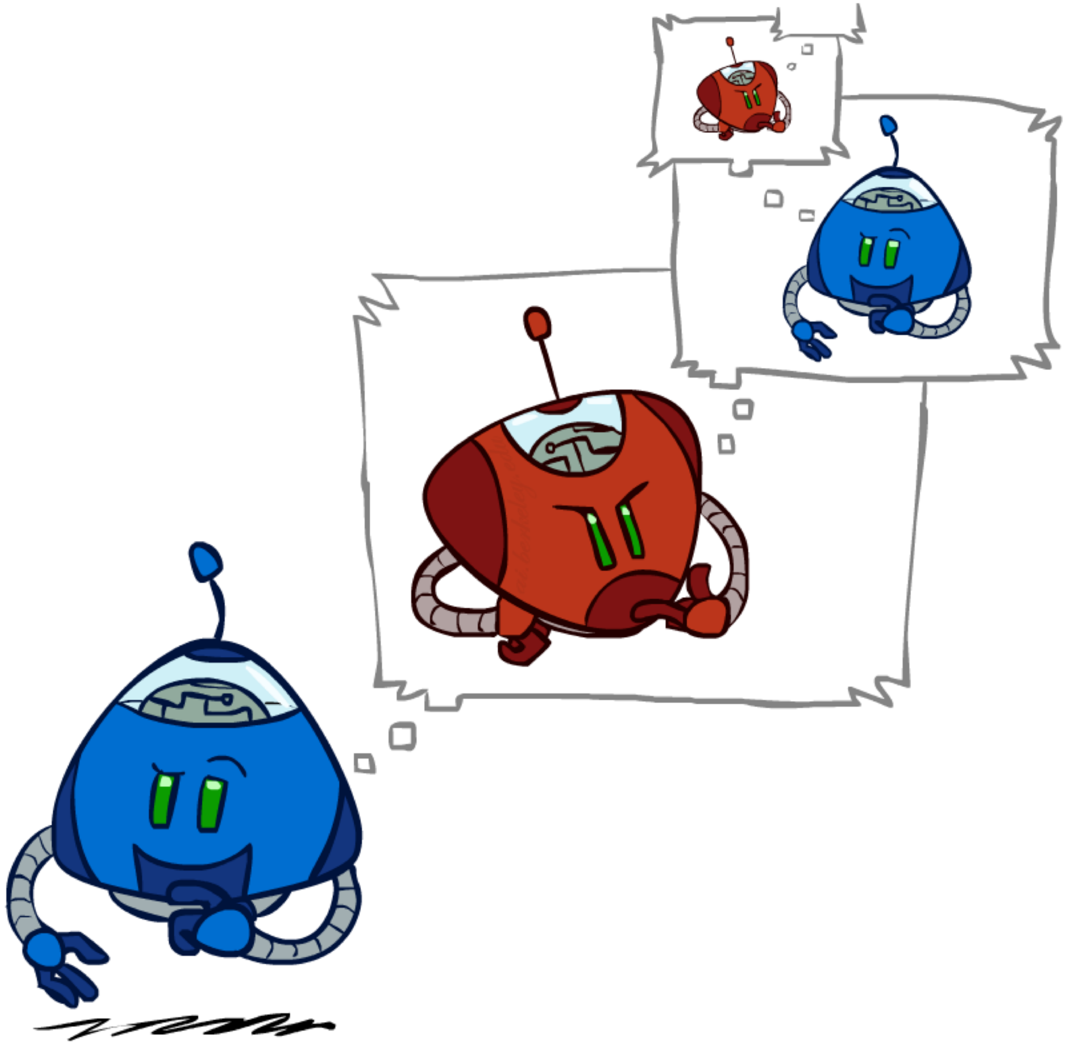




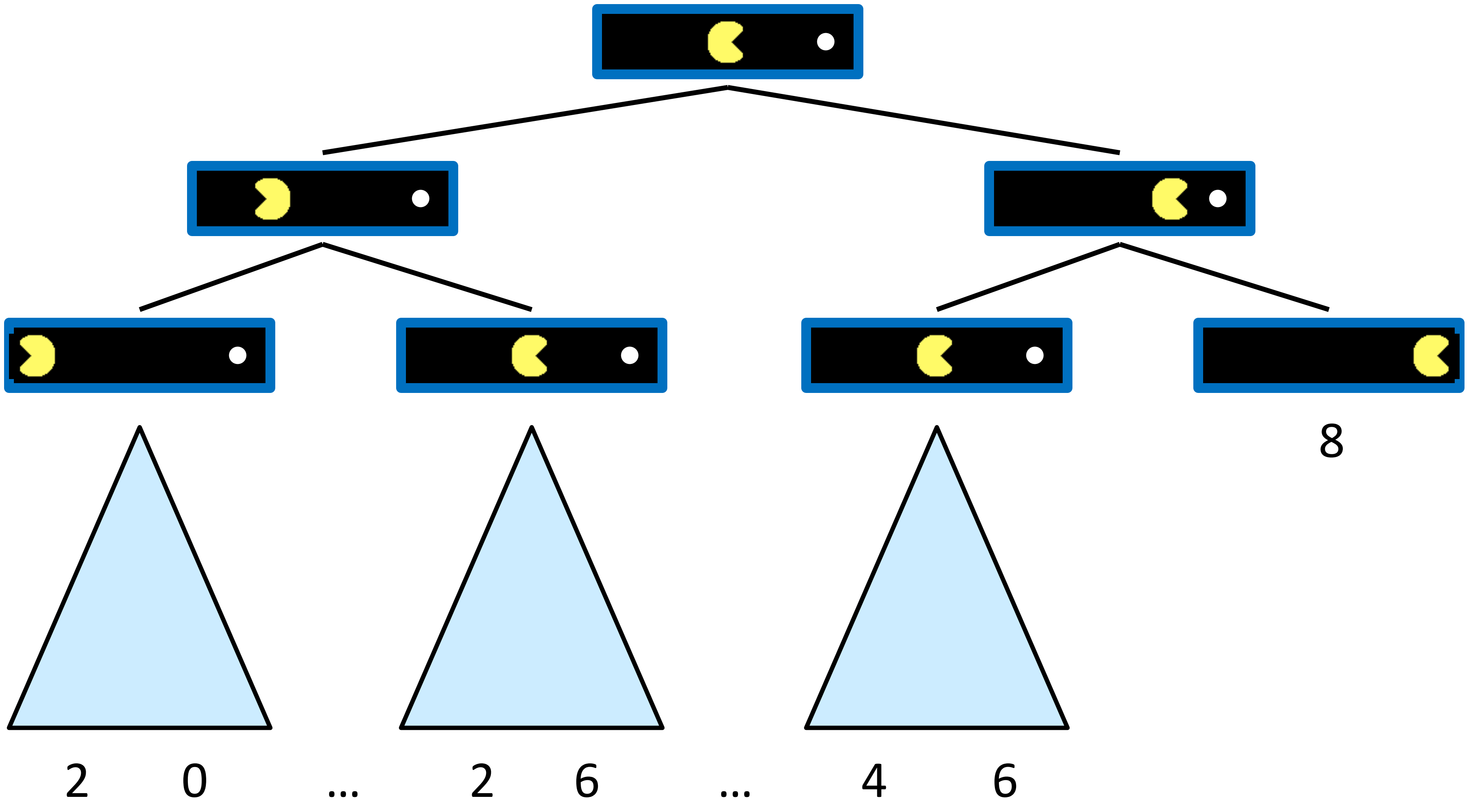
- Zero-Sum Games
  - Agents have ***opposite*** utilities
  - Pure competition:
    - One ***maximizes***, the other ***minimizes***

- General-Sum Games
  - Agents have ***independent*** utilities
  - Cooperation, indifference, competition, shifting alliances, and more are all possible
- Team Games
  - Common payoff for all team members

Adversarial Search



Single-Agent Trees

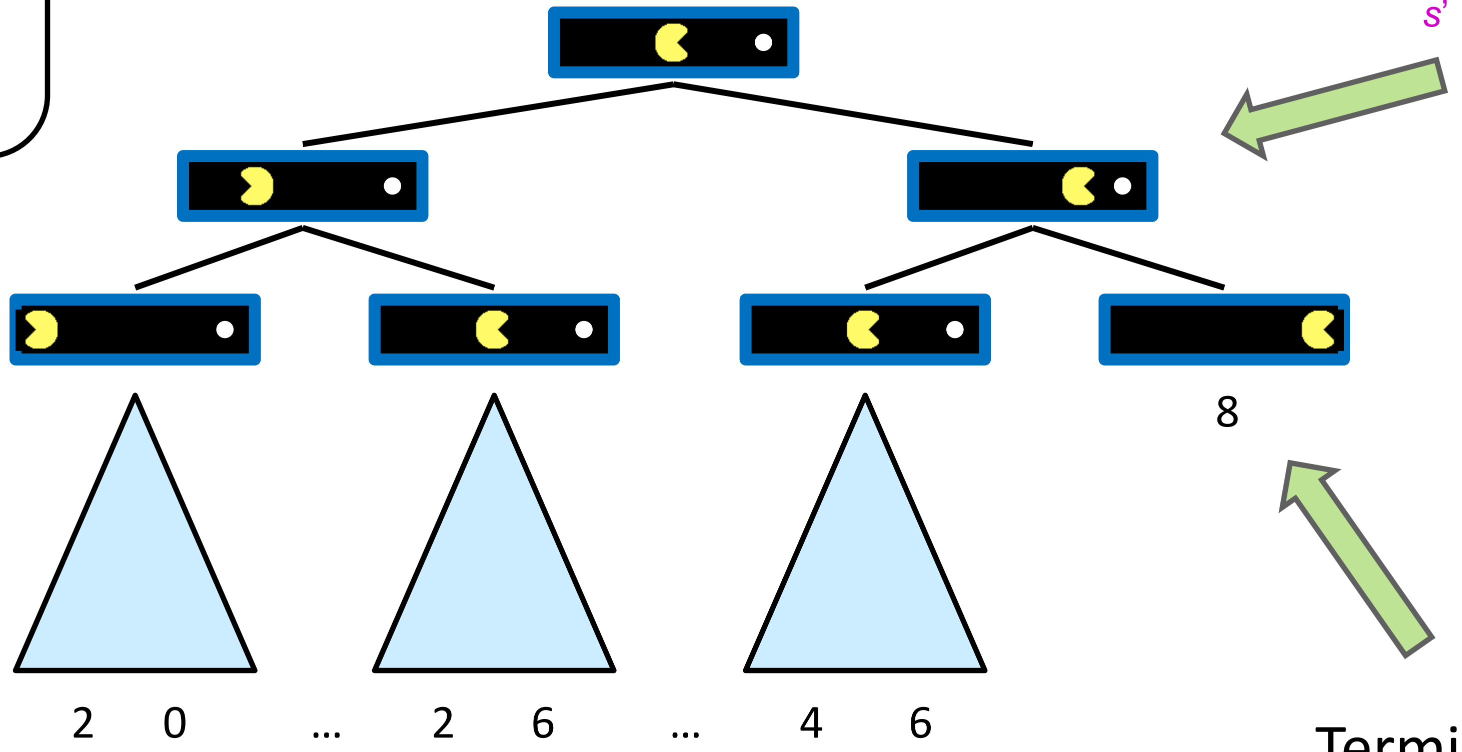


Value of a State

Value of a state:
The best achievable outcome (utility) from that state

Non-Terminal States:

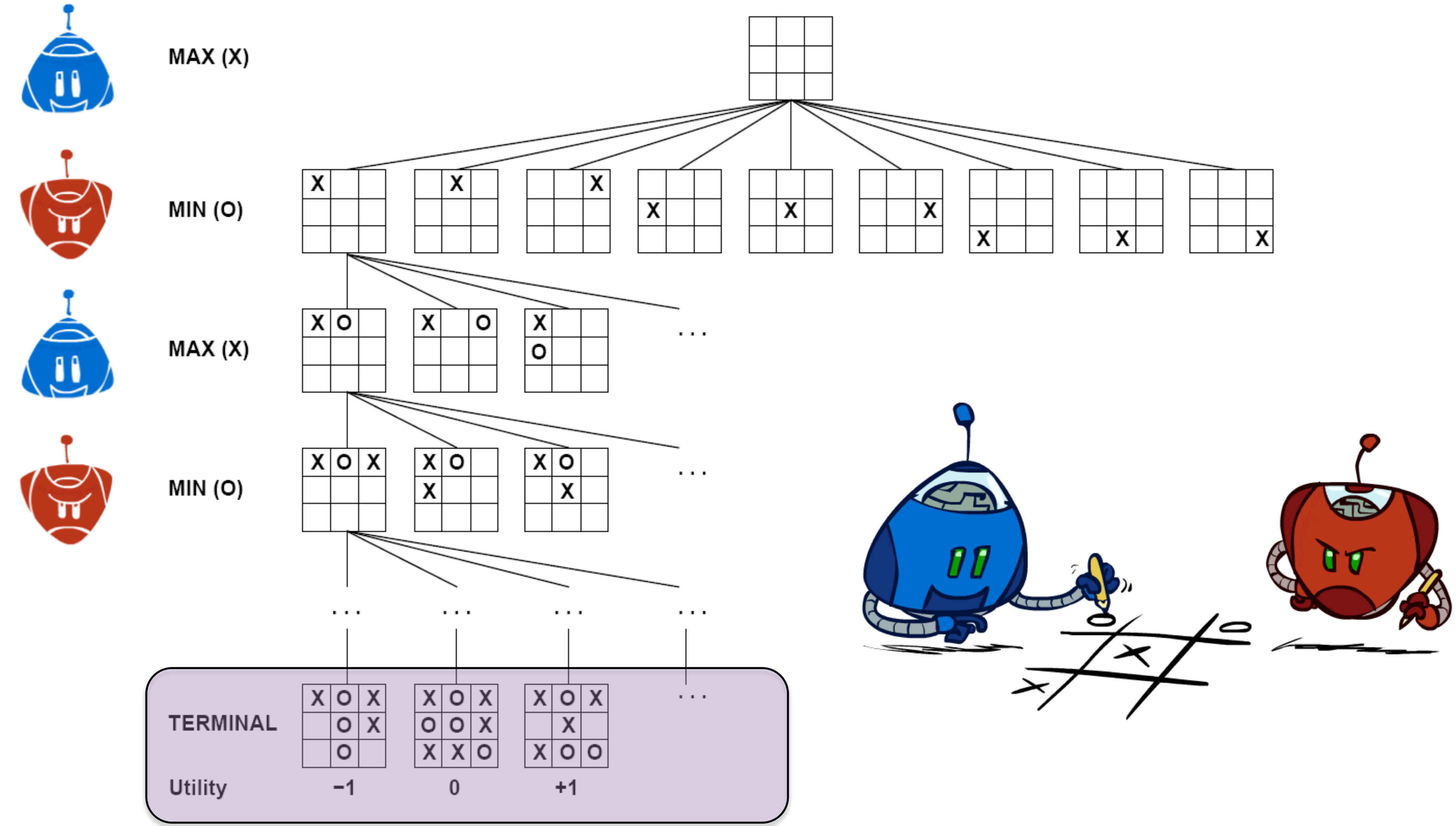
$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$



Terminal States:

$V(s)$ = known

Tic-Tac-Toe Game Tree



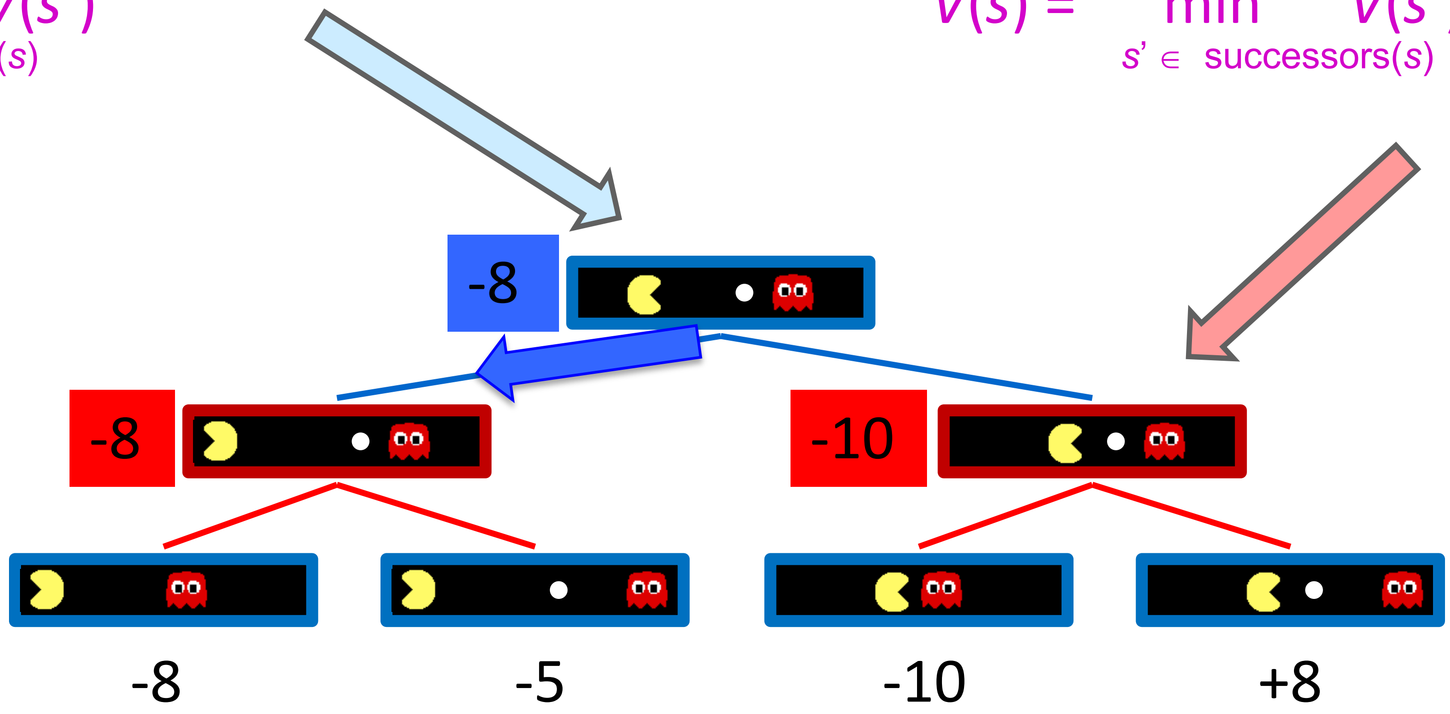
Minimax Values

MAX nodes: under Agent's control

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

MIN nodes: under Opponent's control

$$V(s) = \min_{s' \in \text{successors}(s)} V(s')$$



Terminal States:

$V(s)$ = known

Minimax algorithm

- Choose action leading to state with best ***minimax value***
- Assumes all future moves will be optimal
- => rational against a rational player

Implementation

function minimax-decision(s) returns an action

return the action a in Actions(s) with the highest minimax_value(Result(s,a))

function minimax_value(s) returns a value

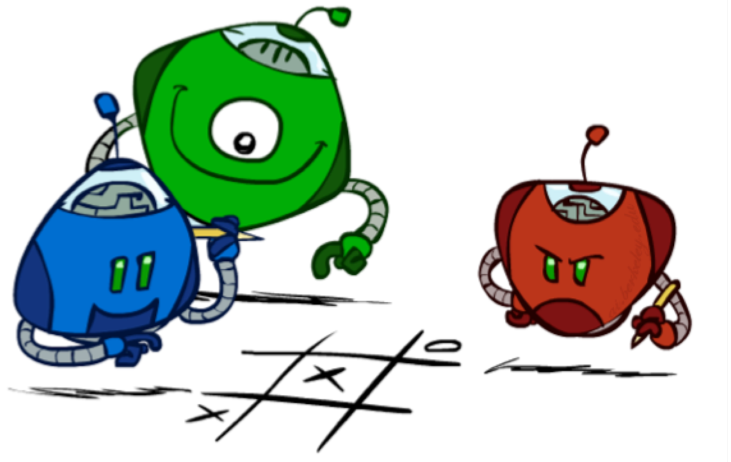
if Terminal-Test(s) then return Utility(s)

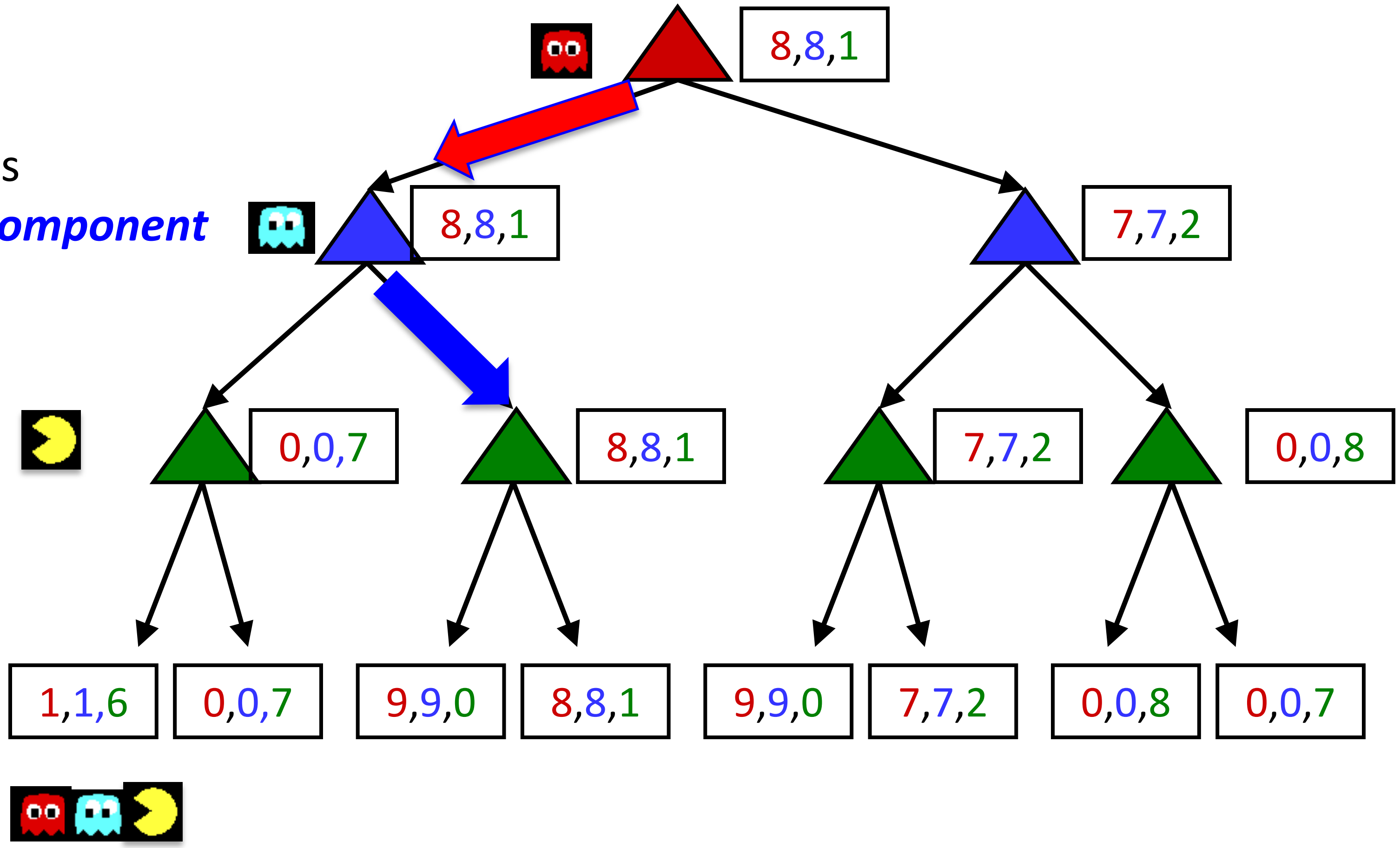
if Player(s) = MAX then return $\max_{a \text{ in Actions}(s)}$ minimax_value(Result(s,a))

if Player(s) = MIN then return $\min_{a \text{ in Actions}(s)}$ minimax_value(Result(s,a))

Generalized minimax

- What if the game is not zero-sum, or has multiple players?
- Generalization of minimax:
  - Terminals have ***utility tuples***
  - Node values are also utility tuples
  - ***Each player maximizes its own component***
  - Can give rise to cooperation and competition dynamically…

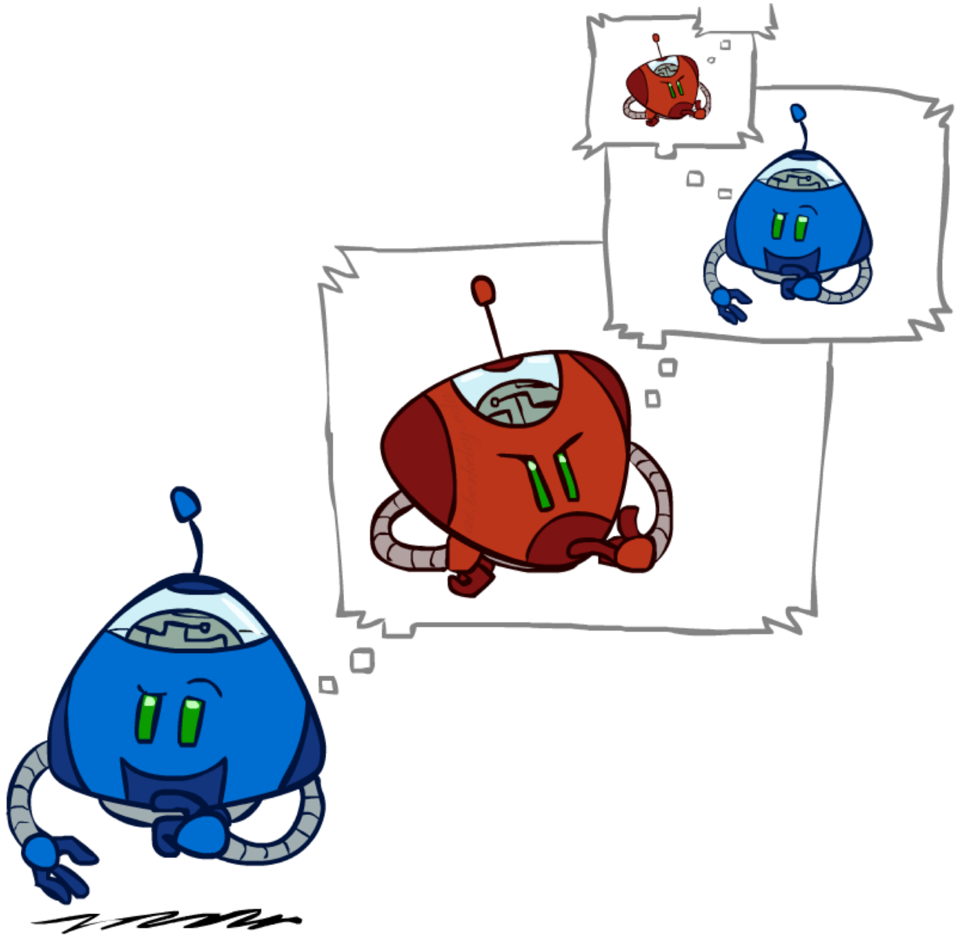




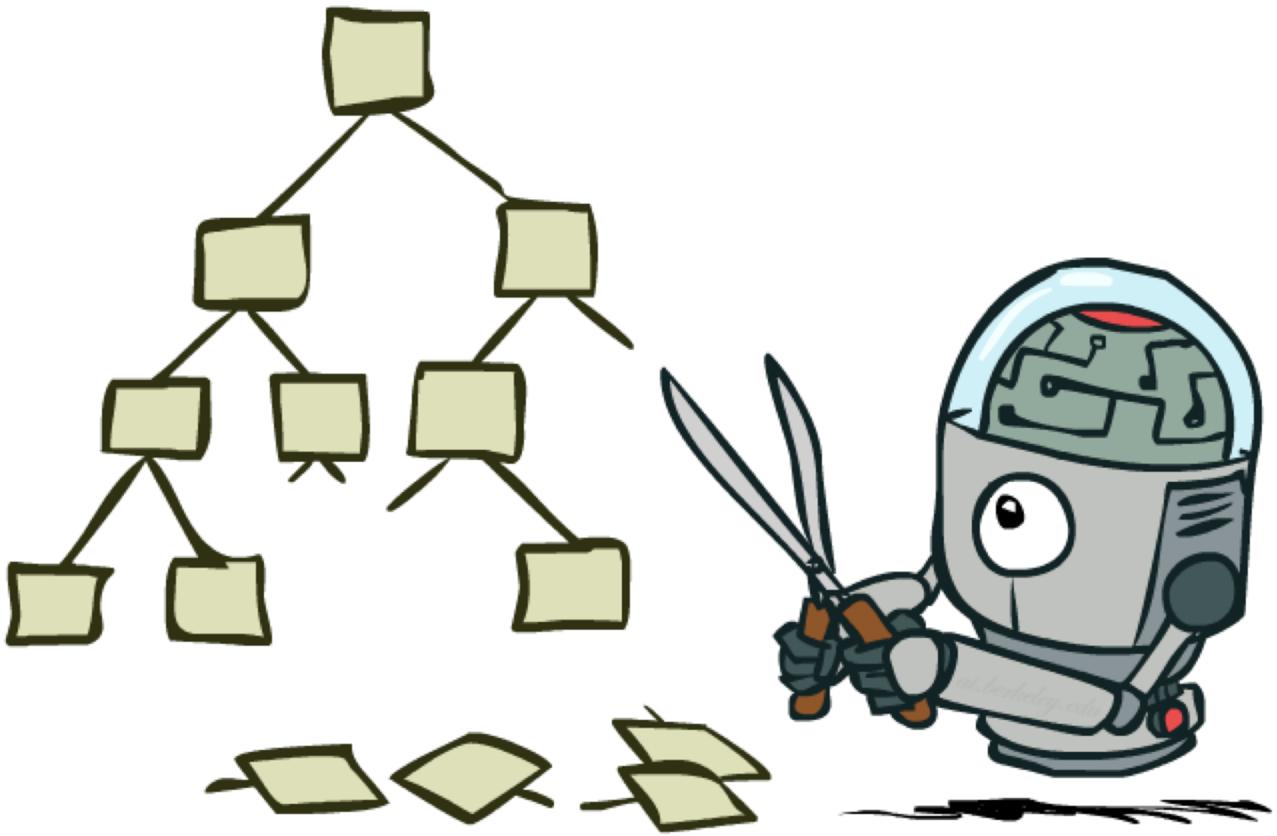
Minimax Efficiency

- How efficient is minimax?
  - Just like (exhaustive) DFS
  - Time: $O(b^m)$
  - Space: $O(bm)$

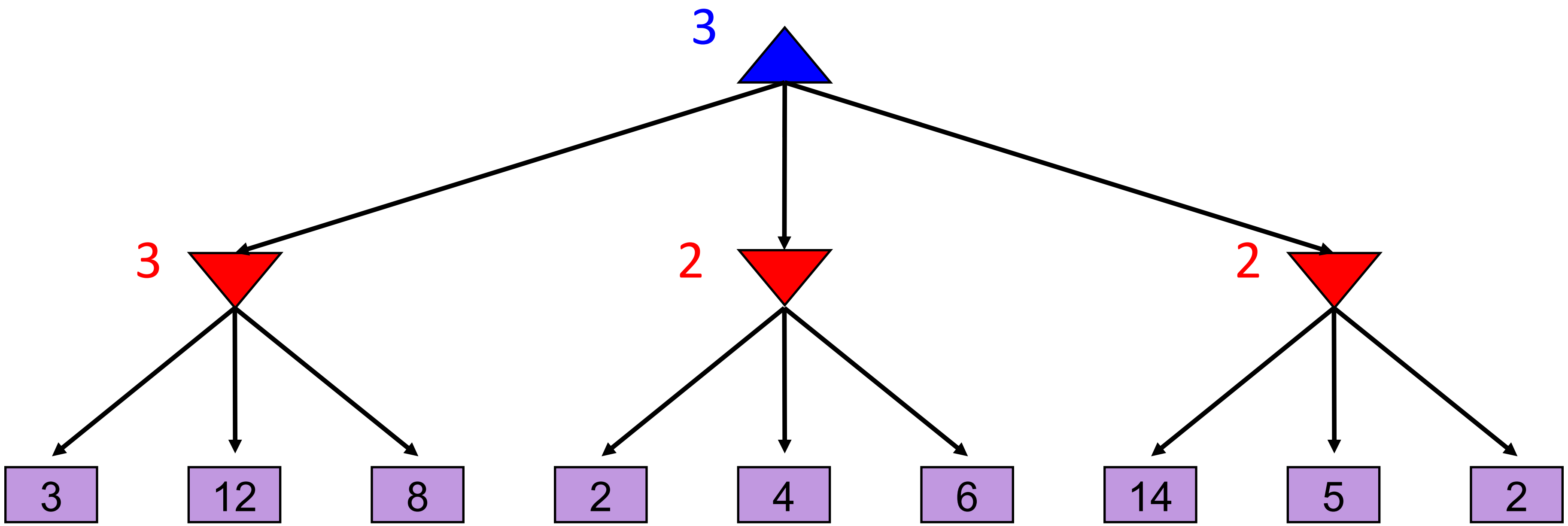
- Example: For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - Humans can't do this either, so how do we play chess?



Game Tree Pruning

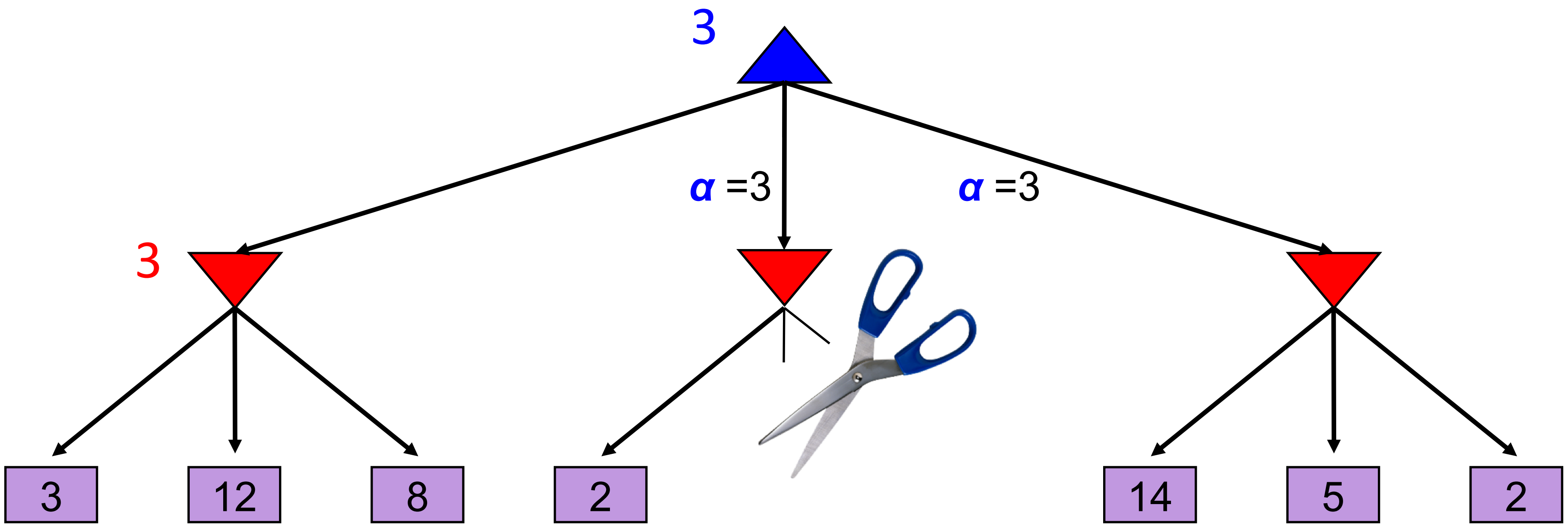


Minimax Example



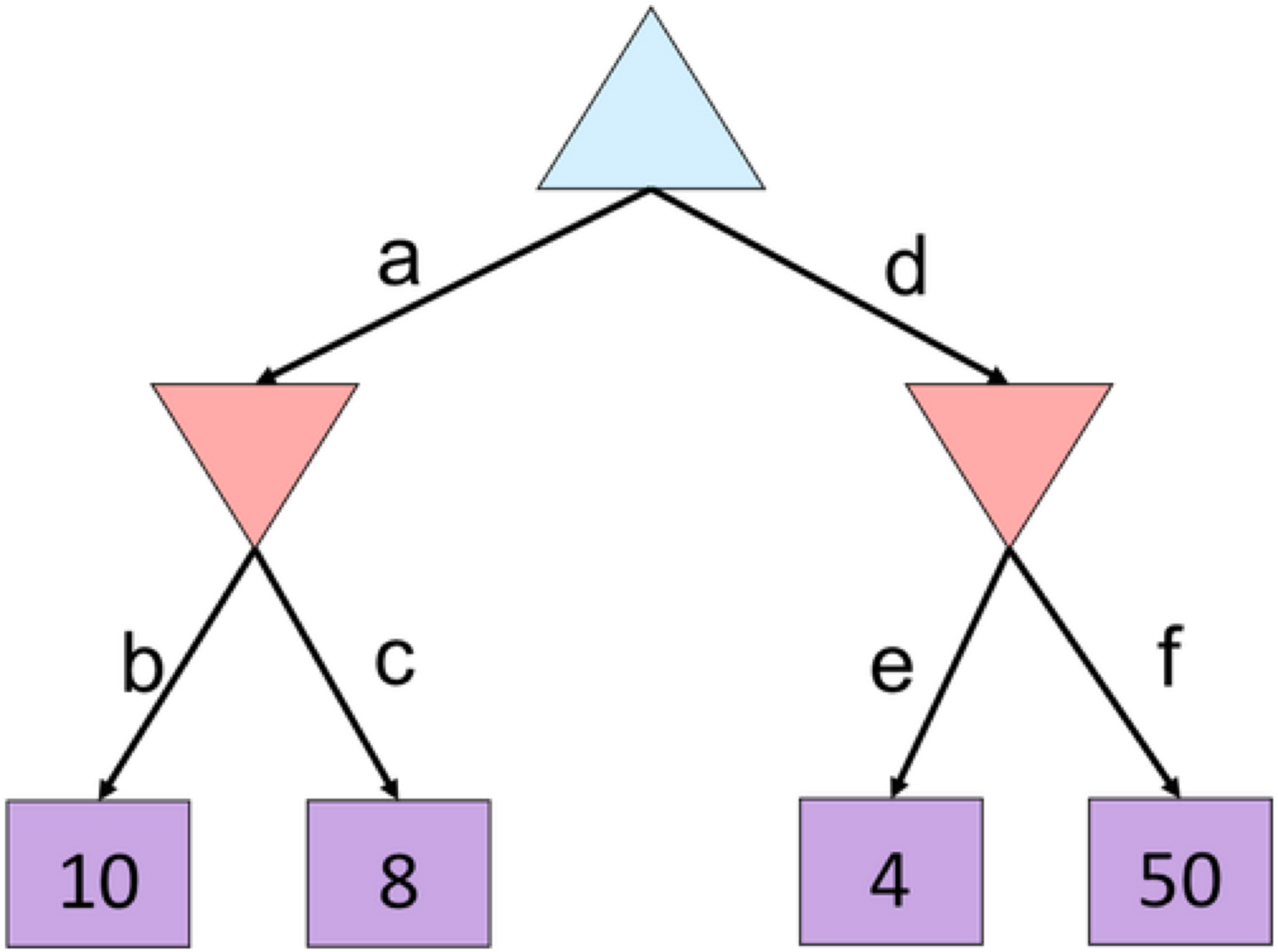
Alpha-Beta Example

α = best option so far from any MAX node on this path

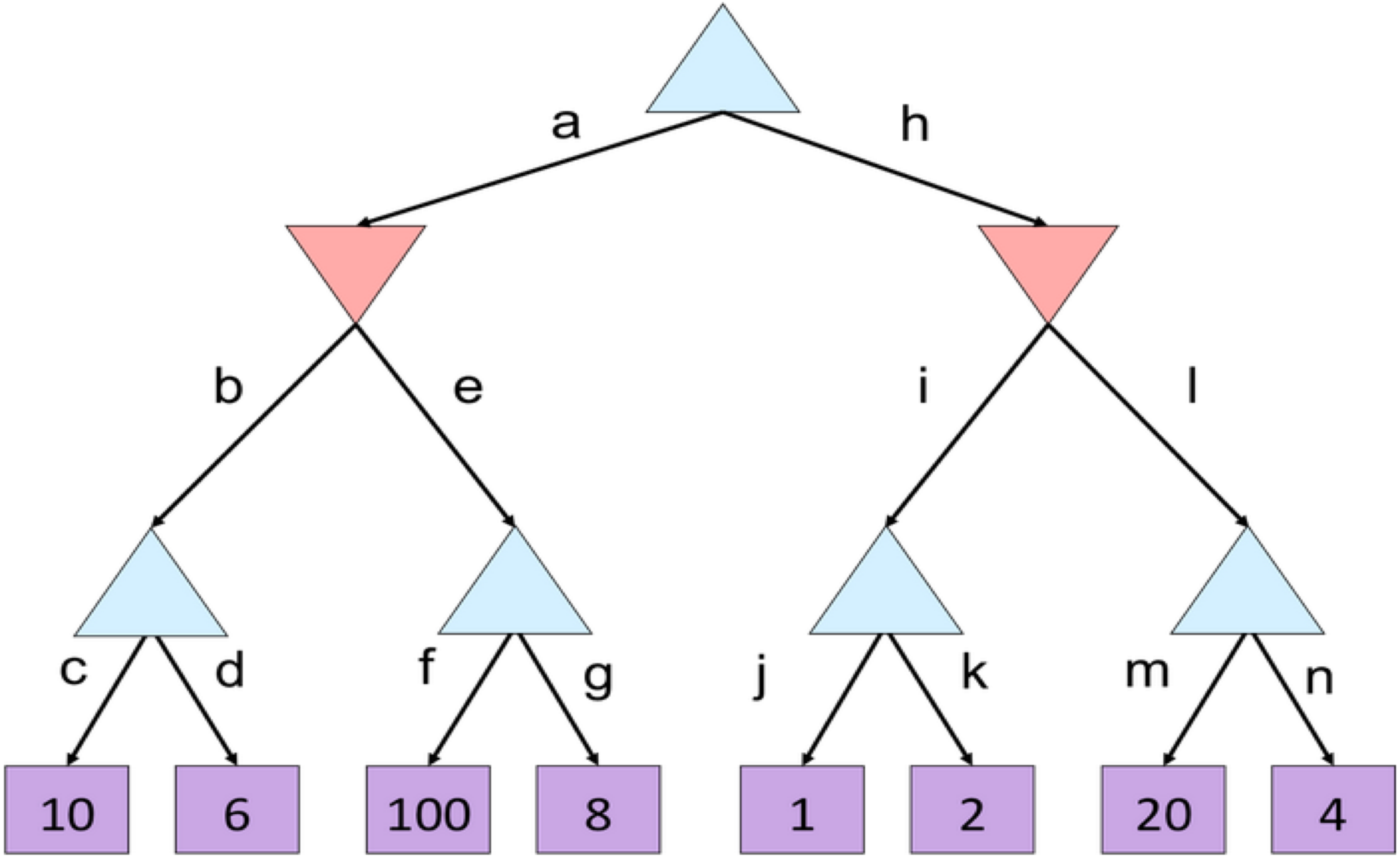


- ***The order of generation matters***: more pruning is possible if good moves come first

Alpha-Beta Quiz

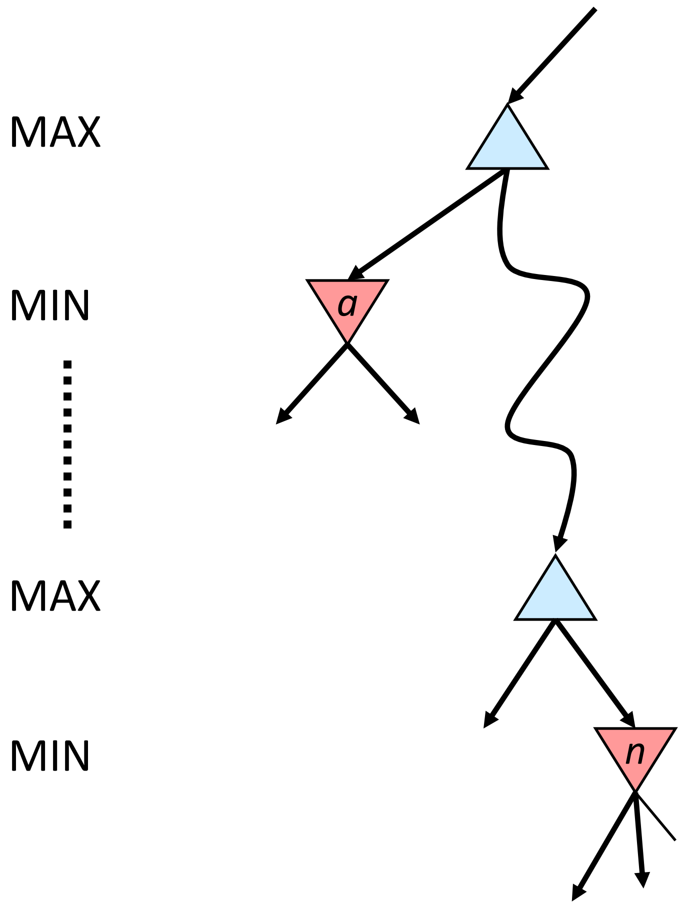


Alpha-Beta Quiz 2



Alpha-Beta Pruning

- General case (pruning children of MIN node)
  - We're computing the MIN-VALUE at some node *n*
  - We're looping over *n*'s children
  - *n*'s estimate of the childrens' min is dropping
  - Who cares about *n*'s value? MAX
  - Let ***α*** be the best value that MAX can get so far at any choice point along the current path from the root
  - If *n* becomes worse than ***α***, MAX will avoid it, so we can prune *n*'s other children (it's already bad enough that it won't be played)
- Pruning children of MAX node is symmetric
  - Let **β** be the best value that MIN can get so far at any choice point along the current path from the root



Alpha-Beta Implementation

α: MAX's best option on path to root
β: MIN's best option on path to root

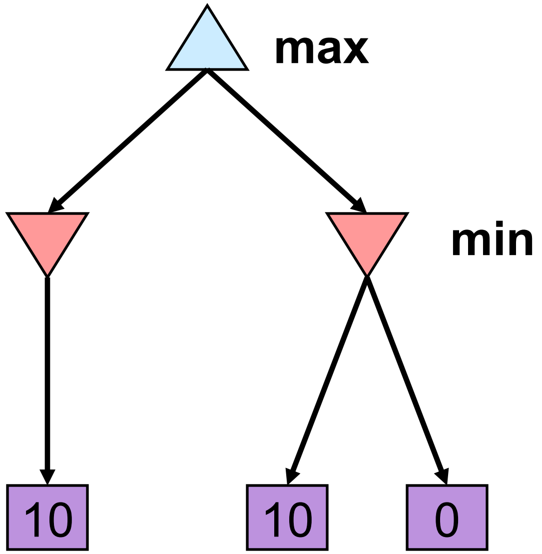
```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor, α,β))
        if v ≥ β
            return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, max-value(successor, α, β))
        if v ≤ α
            return v
        β = min(β, v)
    return v
```

Alpha-Beta Pruning Properties

- Theorem: This pruning has ***no effect*** on minimax value computed for the root!

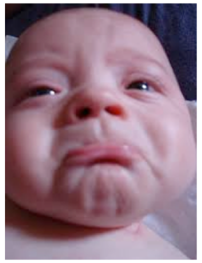
- Good child ordering improves effectiveness of pruning
  - Iterative deepening helps with this



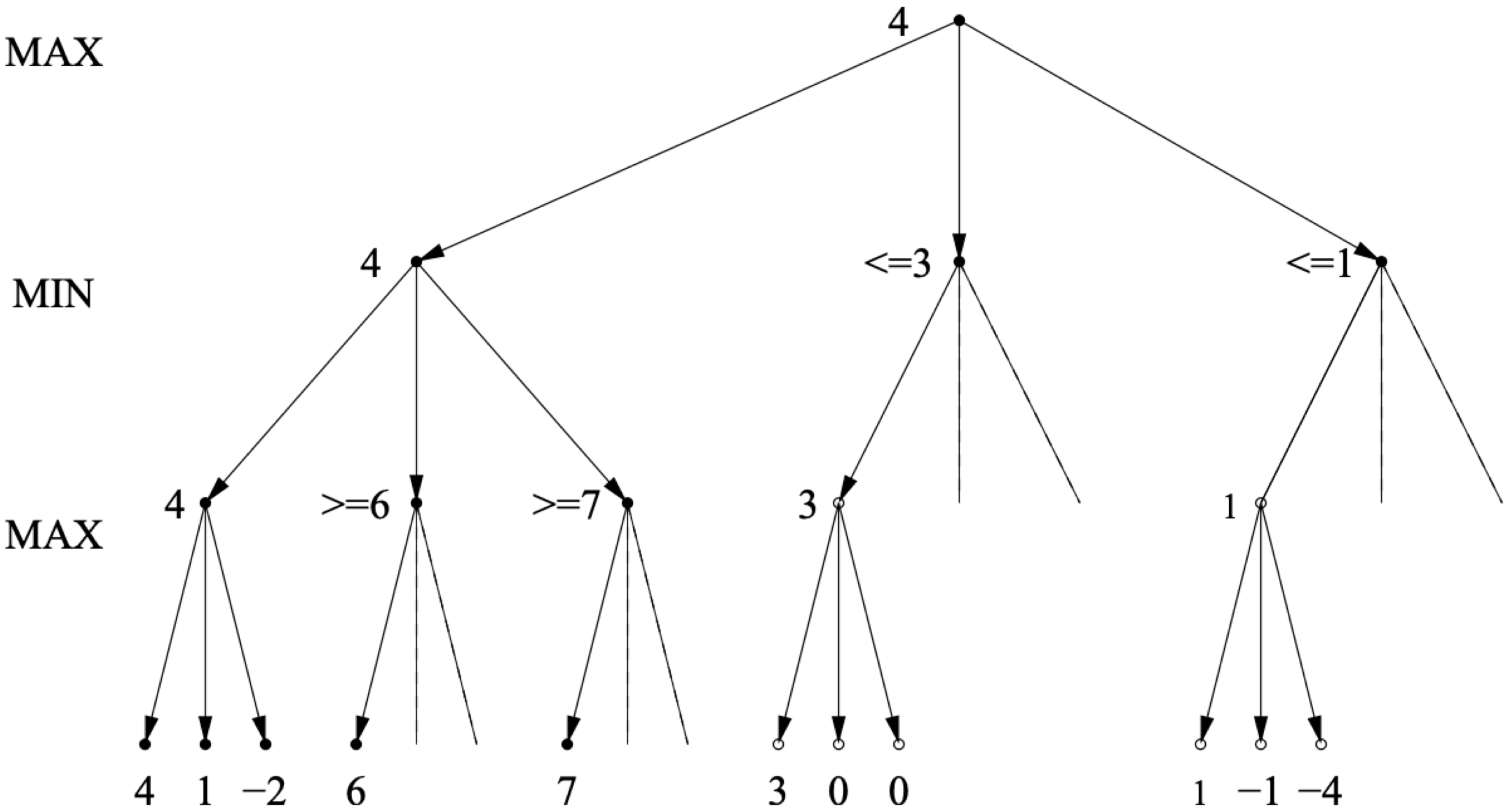
- With "perfect ordering":
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!

- This is a simple example of metareasoning (reasoning about reasoning)

- For chess: only 35^{50} instead of 35^{100}!! Yaaay!!!!!



Alpha-Beta Best-Case Analysis



http://www.cs.utsa.edu/~bylander/cs5233/a-b-analysis.pdf

Summary

- Games are decision problems with $\geq$ 2 agents
  - Huge variety of issues and phenomena depending on details of interactions and payoffs
- For zero-sum games, optimal decisions defined by minimax
  - Simple extension to n-player “rotating” max with vectors of utilities
  - Implementable as a depth-first traversal of the game tree
  - Time complexity $O(b^m)$, space complexity $O(bm)$
- Alpha-beta pruning
  - Preserves optimal choice at the root
  - Alpha/beta values keep track of best obtainable values from any max/min nodes on path from root to current node
  - Time complexity drops to $O(b^{m/2})$ with ideal node ordering
- Exact solution is impossible even for “small” games like chess