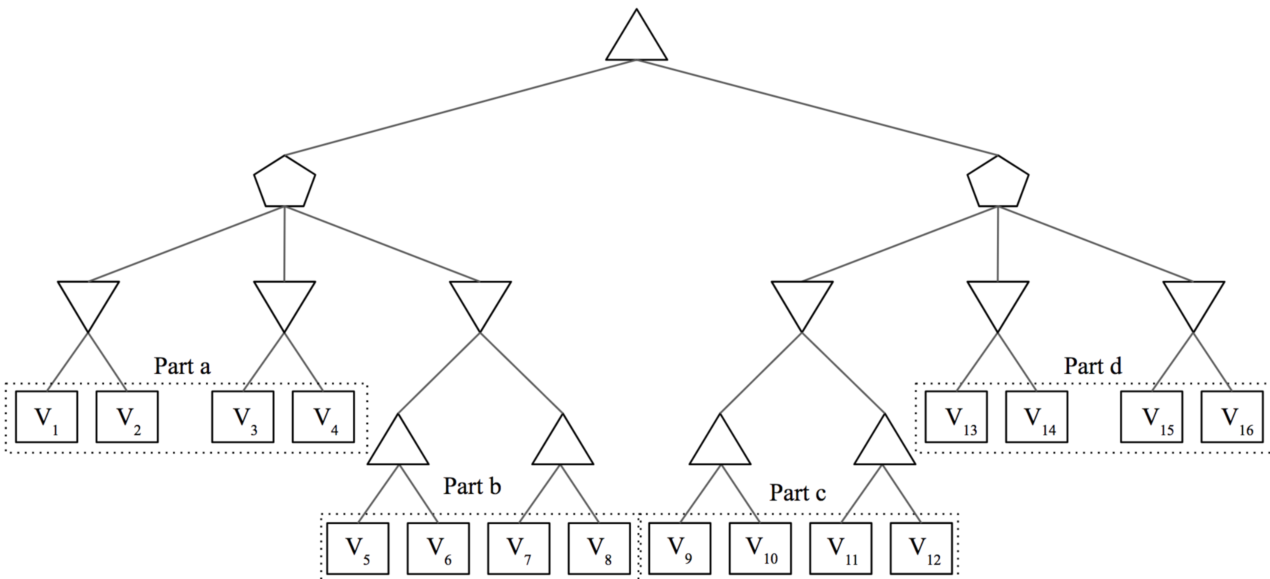
Q1. MedianMiniMax

You're living in utopia! Despite living in utopia, you still believe that you need to maximize your utility in life, other people want to minimize your utility, and the world is a 0 sum game. But because you live in utopia, a benevolent social planner occasionally steps in and chooses an option that is a compromise. Essentially, the social planner (represented as the pentagon) is a median node that chooses the successor with median utility. Your struggle with your fellow citizens can be modelled as follows:



There are some nodes that we are sometimes able to prune. In each part, mark all of the terminal nodes such that **there exists a possible situation** for which the node **can be pruned**. In other words, you must consider **all** possible pruning situations. Assume that evaluation order is **left to right** and all V_i's are **distinct.**

Note that as long as there exists ANY pruning situation (does not have to be the same situation for every node), you should mark the node as prunable. Also, alpha-beta pruning does not apply here, simply prune a sub-tree when you can reason that its value will not affect your final utility.

(a)
- ☐ V_1
- ☐ V_2
- ☐ V_3
- ☐ V_4
- ☐ None

(b)
- ☐ V_5
- ☐ V_6
- ☐ V_7
- ☐ V_8
- ☐ None

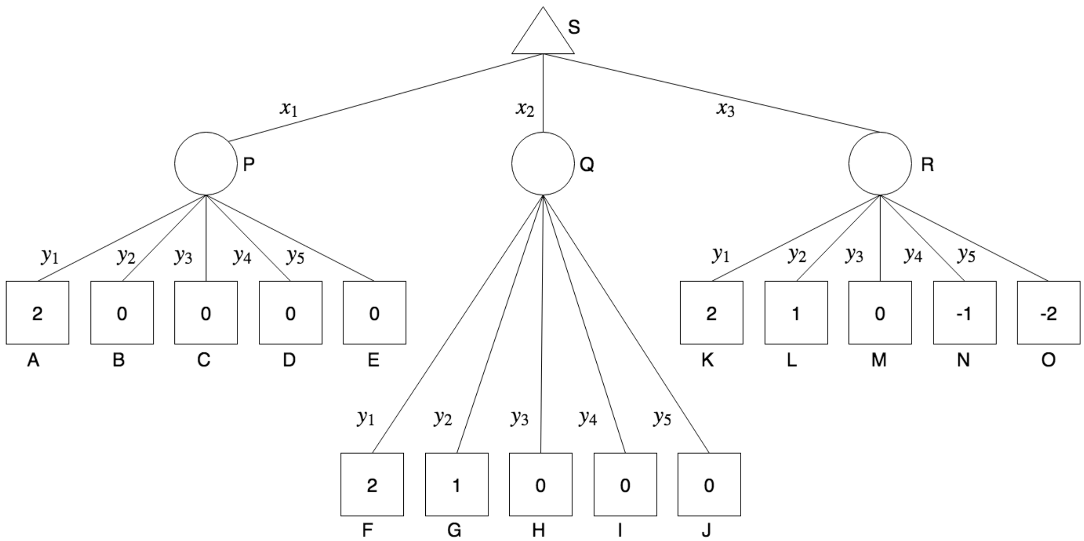
(c)
- ☐ V_9
- ☐ V_{10}
- ☐ V_{11}
- ☐ V_{12}
- ☐ None

(d)
- ☐ V_{13}
- ☐ V_{14}
- ☐ V_{15}
- ☐ V_{16}
- ☐ None

Q2. [Optional] Bike Bidding Battle

Alyssa P. Hacker and Ben Bitdiddle are bidding in an auction at Stanley University for a bike. Alyssa will either bid x_1, x_2, or x_3 for the bike. She knows that Ben will bid y_1, y_2, y_3, y_4, or y_5, but she does not know which. All bids are nonnegative.

(a) Alyssa wants to maximize her payoff given by the expectimax tree below. The leaf nodes show Alyssa's payoff. The nodes are labeled by letters, and the edges are labeled by the bid values x_i and y_i. The maximization node S represents Alyssa, and the branches below it represent each of her bids: x_1, x_2, x_3. The chance nodes P, Q, R represent Ben, and the branches below them represent each of his bids: y_1, y_2, y_3, y_4, y_5.



(i) Suppose that Alyssa believes that Ben would bid any bid with equal probability. What are the values of the chance (circle) and maximization (triangle) nodes?

Node P: Node Q: Node R: Node S:

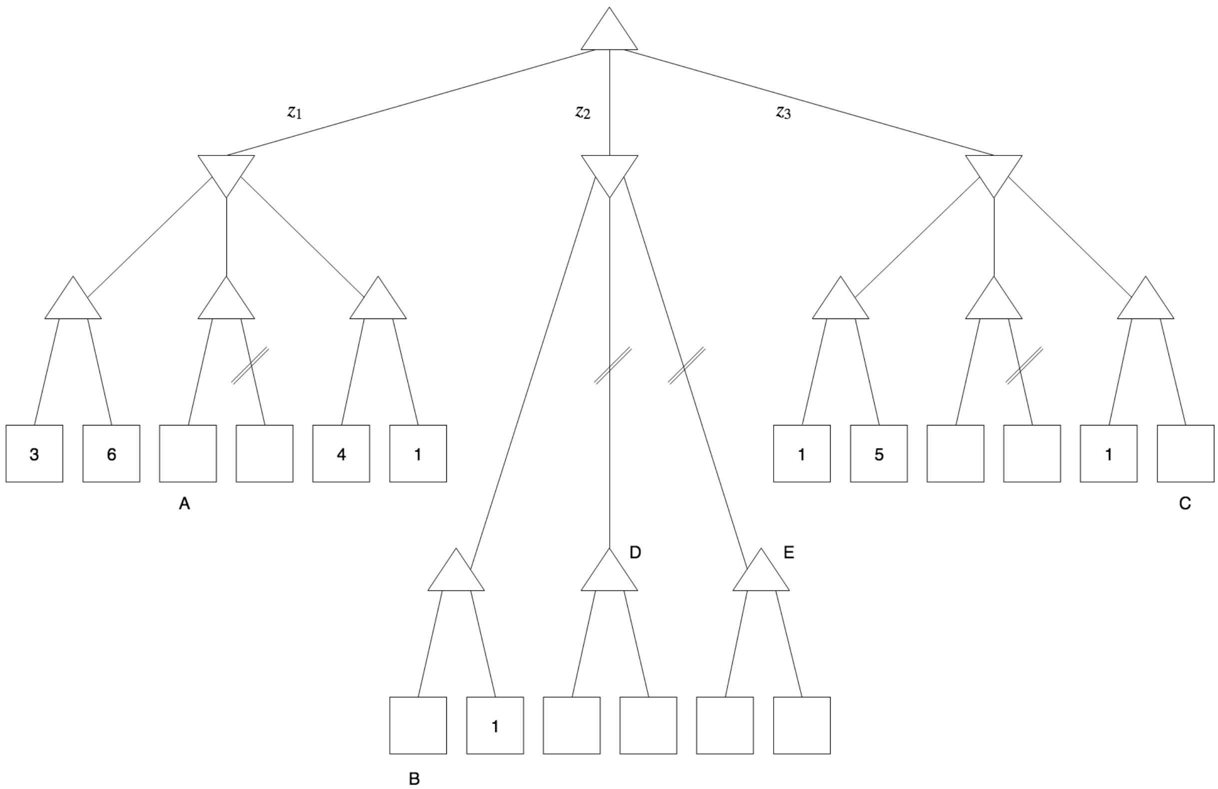
(ii) Based on the information from the above tree, how much should Alyssa bid for the bike?

○ x_1 ○ x_2 ○ x_3

(b) Alyssa does expectimax search by visiting child nodes from left to right. Ordinarily expectimax trees cannot be pruned without some additional information about the tree. Suppose, however, that Alyssa knows that the leaf nodes are ordered such that payoffs are non-increasing from left to right (the leaf nodes of the above diagram is an example of this ordering). Recall that if node X is a child of a maximizer node, a child of node X may be pruned if we know that the value of node X will never be $>$ some threshold (in other words, it is $\leq$ that threshold). Given this information, if it is possible to prune any branches from the tree, mark them below. Otherwise, mark "None of the above."

☐ A ☐ B ☐ C ☐ D ☐ E ☐ F ☐ G ☐ H
☐ I ☐ J ☐ K ☐ L ☐ M ☐ N ☐ O ○ None of the above

(c) Unrelated to parts (a) and (b), consider the minimax tree below. whose leaves represent payoffs for the maximizer. The crossed out edges show the edges that are pruned when doing naive alpha-beta pruning visiting children nodes from left to right. Assume that we prune on equalities (as in, we prune the rest of the children if the current child is $\leq \alpha$ (if the parent is a minimizer) or $\geq \beta$ (if the parent is a maximizer)).



Fill in the inequality expressions for the values of the labeled nodes A and B. Write ∞ and $-\infty$ if there is no upper or lower bound, respectively.

1. $\boxed{} \leq \text{A} \leq \boxed{}$

2. $\boxed{} \leq \text{B} \leq \boxed{}$

(d) Suppose node B took on the largest value it could possibly take on and still be consistent with the pruning scheme above. After running the pruning algorithm, we find that the values of the left and center subtrees have the same minimax value, both 1 greater than the minimax value of the right subtree. Based on this information, what is the numerical value of node C?

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10

(e) For which values of nodes D and E would choosing to take action z_2 be *guaranteed* to yield the same payoff as action z_1? Write ∞ and $-\infty$ if there is no upper or lower bound, respectively (this would correspond to the case where nodes D and E can be any value).

1. $\boxed{} \leq \text{D} \leq \boxed{}$

2. $\boxed{} \leq \text{E} \leq \boxed{}$