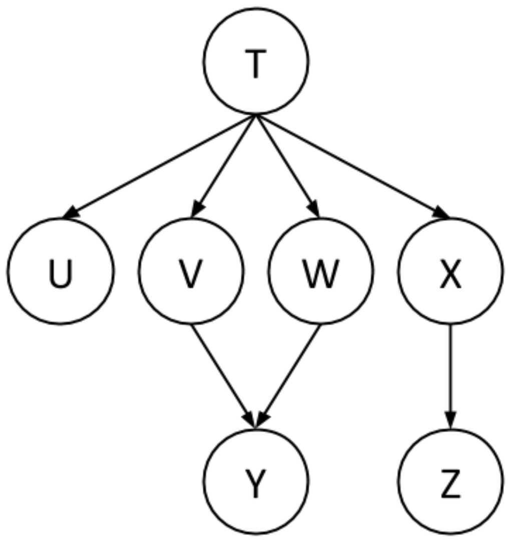
CS 188
Summer 2023

Discussion 3C Solutions

1 D-Separation

Indicate whether each of the following conditional independence relationships is guaranteed to be true in the Bayes Net below. If the independence relationship does not hold, identify all active (d-connected) paths in the graph.



1. $U \perp\!\!\!\perp X$

   Not guaranteed, path U-T-X is active

2. $U \perp\!\!\!\perp X|T$

   Guaranteed

3. $V \perp\!\!\!\perp W|Y$

   Not guaranteed, paths V-T-W and V-Y-W are both active

4. $V \perp\!\!\!\perp W|T$

   Guaranteed

5. $T \perp\!\!\!\perp Y|V$

   Not guaranteed, path T-W-Y is active

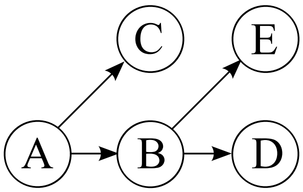
6. $Y \perp\!\!\!\perp Z|W$

   Not guaranteed, path Y-V-T-X-Z is active

7. $Y \perp\!\!\!\perp Z|T$

   Guaranteed, with T being observed, there are no active paths from Y to Z

2 Bayes Nets: Variable Elimination



	$P(A)$
$+a$	0.25
$-a$	0.75

$P(B\|A)$	$+b$	$-b$
$+a$	0.5	0.5
$-a$	0.25	0.75

$P(C\|A)$	$+c$	$-c$
$+a$	0.2	0.8
$-a$	0.6	0.4

$P(D\|B)$	$+d$	$-d$
$+b$	0.6	0.4
$-b$	0.8	0.2

$P(E\|B)$	$+e$	$-e$
$+b$	0.25	0.75
$-b$	0.1	0.9

(a) Using the Bayes' Net and conditional probability tables above, calculate the following quantities:

(i) $P(+b|+a) =$ 0.5

(ii) $P(+a, +b) =$

$0.25 * 0.5 = 0.125 = \frac{1}{8}$

(iii) $P(+a|+b) =$

$\frac{0.25*0.5}{0.25*0.5+0.25*0.75} = 0.4 = \frac{2}{5}$

(b) Now we are going to consider variable elimination in the Bayes' Net above.

(i) Assume we have the evidence $+c$ and wish to calculate $P(E \mid +\text{c})$. What factors do we have initially?

$P(A)$, $P(B \mid A)$, $P(+\text{c} \mid A)$, $P(D \mid B)$, $P(E \mid B)$

(ii) If we eliminate variable B, we create a new factor. What probability does that factor correspond to?

$P(D, E \mid A)$

(iii) What is the equation to calculate the factor we create when eliminating variable B?

$f(A, D, E) = \sum_B P(B \mid A) * P(D \mid B) * P(E \mid B)$

(iv) After eliminating variable B, what are the new set of factors? As in (ii), write the probabilities that the factors represent. For each factor, also provide its size.

Factor	Size after elimination
$P(A)$	2
$P(+\text{c} \mid A)$	2
$P(D, E \mid A)$	2^3

(v) Now assume we have the evidence $-c$ and are trying to calculate $P(A|-c)$. What is the most efficient elimination ordering? If more than one ordering is most efficient, provide any one of them. E, D, B or D, E, B

(vi) Once we have run variable elimination and have $f(A, -c)$ how do we calculate $P(+a \mid -c)$? $\frac{f(+a,-c)}{f(+a,-c)+f(-a,-c)}$

or note that elimination is unnecessary - just use Bayes' rule