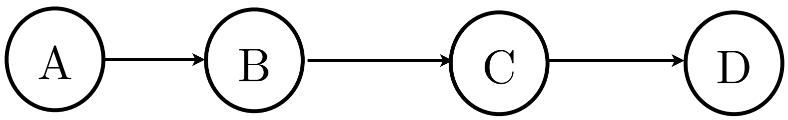
CS 188
Summer 2023

Discussion 3D Solutions

Q1. Bayes' Nets Sampling

Assume the following Bayes' net, and the corresponding distributions over the variables in the Bayes' net:



$P(A)$	
$-a$	3/4
$+a$	1/4

$P(B\|A)$		
$-a$	$-b$	2/3
$-a$	$+b$	1/3
$+a$	$-b$	4/5
$+a$	$+b$	1/5

$P(C\|B)$		
$-b$	$-c$	1/4
$-b$	$+c$	3/4
$+b$	$-c$	1/2
$+b$	$+c$	1/2

$P(D\|C)$		
$-c$	$-d$	1/8
$-c$	$+d$	7/8
$+c$	$-d$	5/6
$+c$	$+d$	1/6

(a) You are given the following samples:

$(+a, +b, -c, -d)$
$(+a, -b, +c, -d)$
$(-a, +b, +c, -d)$
$(-a, -b, +c, -d)$
$(+a, -b, -c, +d)$
$(+a, +b, +c, -d)$
$(-a, +b, -c, +d)$
$(-a, -b, +c, -d)$

(i) If these samples came from doing Prior Sampling, calculate our sample estimate of $P(+c)$. 5/8

(ii) Now we will estimate $P(+c \mid +a, -d)$. Above, clearly cross out the samples that would **not** be used when doing Rejection Sampling for this task, and write down the sample estimate of $P(+c \mid +a, -d)$. 2/3

(b) Using Likelihood Weighting Sampling to estimate $P(-a \mid +b, -d)$, the following samples were obtained. What is the weight of each sample?

Sample				**Weight**
$-a$	$+b$	$+c$	$-d$	$P(+b \mid -a)P(-d \mid +c) = 1/3 * 5/6 = 5/18 = 0.277$
$+a$	$+b$	$+c$	$-d$	$P(+b \mid +a)P(-d \mid +c) = 1/5 * 5/6 = 5/30 = 1/6 = 0.17$
$+a$	$+b$	$-c$	$-d$	$P(+b \mid +a)P(-d \mid -c) = 1/5 * 1/8 = 1/40 = 0.025$
$-a$	$+b$	$-c$	$-d$	$P(+b \mid -a)P(-d \mid -c) = 1/3 * 1/8 = 1/24 = 0.042$

(c) From the weighted samples, estimate $P(-a \mid +b, -d)$.

$\frac{5/18+1/24}{5/18+5/30+1/40+1/24} = 0.625$

(d) Recall that during Gibbs Sampling, samples are generated through an iterative process.

Assume that the only evidence that is available is $A = +a$. Which sequence(s) below could have been generated by Gibbs Sampling?

Sequence 1

1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$+d$
3 :	$+a$	$-b$	$+c$	$+d$

Sequence 2

1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$-d$
3 :	$-a$	$-b$	$-c$	$+d$

Sequence 3

1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$-d$
3 :	$+a$	$+b$	$-c$	$-d$

Sequence 4

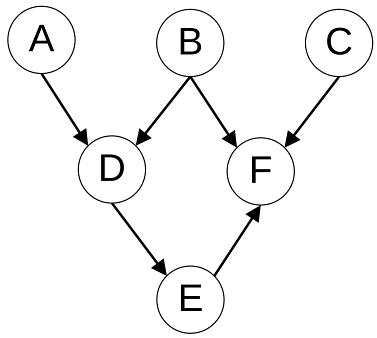
1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$-d$
3 :	$+a$	$+b$	$-c$	$+d$

Sequence 1 and Sequence 3. Gibbs sampling updates one variable at a time and never changes the evidence. The first and third sequences have at most one variable change per row, and hence could have been generated from Gibbs sampling. In sequence 2, the evidence variable is changed. In sequence 4, the second and third samples have both B and D changing.

2 Bayes Nets

(a) For the following graphs, explicitly state the minimum size set of edges that must be removed such that the corresponding independence relations are guaranteed to be true.

Marked the removed edges with an 'X' on the graphs.

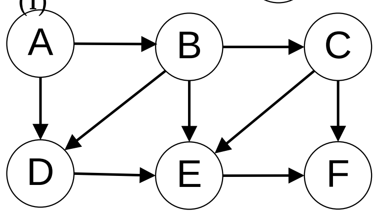


$A \perp\!\!\!\perp B | F$

$A \perp\!\!\!\perp F | D$

$B \perp\!\!\!\perp C$

(i) AD



$A \perp\!\!\!\perp D | B$

$A \perp\!\!\!\perp F | C$

$C \perp\!\!\!\perp D | B$

(ii) AD, (EF OR AB)

(b) You're performing variable elimination over a Bayes Net with variables A, B, C, D, E. So far, you've finished joining over (but not summing out) C, when you realize you've lost the original Bayes Net!

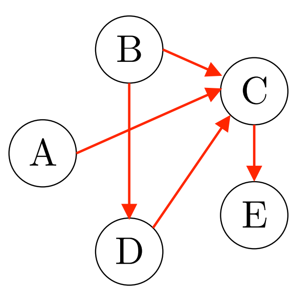
Your current factors are $f(A), f(B), f(B, D), f(A, B, C, D, E)$. Note: these are factors, NOT joint distributions. You don't know which variables are conditioned or unconditioned.

(i) What's the smallest number of edges that could have been in the original Bayes Net? Draw out one such Bayes Net below.

Number of edges = 5

The original Bayes net must have had 5 factors, 1 for each node. f(A) and f(B) must have corresponded to nodes A and B, and indicate that neither A nor B have any parents. f(B, D), then, must correspond to node D, and indicates that D has only B as a parent. Since there is only one factor left, f(A, B, C, D, E), for the nodes C and E, those two nodes must have been joined while you were joining C. This implies two things: 1) E must have had C as a parent, and 2) every other node must have been a parent of either C or E.

The below figure is one possible solution that uses the fewest possible edges to satisfy the above.



(ii) What's the largest number of edges that could have been in the original Bayes Net? Draw out one such Bayes Net below.

Number of edges = 8

The constraints are the same as outlined in part i). To maximize the number of edges, we make each of A, B, and D a parent of both C and E, as opposed to a parent of one of them.

The below figure is the only possible solution.