1 Particle Filtering

Let’s use Particle Filtering to estimate the distribution of \( P(W_2|O_1 = a, O_2 = b) \). Here’s the HMM again.

We start with two particles representing our distribution for \( W_1 \).

\[ P_1 : W_1 = 0 \]
\[ P_2 : W_1 = 1 \]

Use the following random numbers to run particle filtering:

\[ [0.22, 0.05, 0.33, 0.20, 0.84, 0.54, 0.79, 0.66, 0.14, 0.96] \]

(a) **Observe**: Compute the weight of the two particles after evidence \( O_1 = a \).

(b) **Resample**: Using the random numbers, resample \( P_1 \) and \( P_2 \) based on the weights.

(c) **Predict**: Sample \( P_1 \) and \( P_2 \) from applying the time update.

(d) **Update**: Compute the weight of the two particles after evidence \( O_2 = b \).

(e) **Resample**: Using the random numbers, resample \( P_1 \) and \( P_2 \) based on the weights.

(f) What is our estimated distribution for \( P(W_2|O_1 = a, O_2 = b) \)?

2 MangoBot Human Detector

Your startup company MangoBot wants to build robots that deliver packages on the road. One core module of the robot’s software is to detect whether a human is standing in front of it. We model the presence of humans with a Markov model:

\[
\begin{array}{c}
H_0 \\
\text{P(H_{t+1}|H_t)} \\
H_1 \\
\text{P(H_{t+1}|H_t)} \\
H_2 \\
\text{P(H_{t+1}|H_t)} \\
\end{array}
\]

where \( H_t \in \{0, 1\} \) corresponds to a human being absent or present respectively. The initial distribution and the transition probabilities are given as follows:
(a) Express the following quantities in terms of $p$:

(i) $P(H_1|1) = \ldots$

(ii) $\lim_{t \to \infty} P(H_t = 0) = \ldots$

To make things simple, we stick to the original first-order Markov chain formulation. To make the detection more accurate, the company built a sensor that returns an observation $O_t$ each time step as a noisy measurement of the unknown $H_t$. The new model is illustrated in the figure, and the relationship between $H_t$ and $O_t$ is provided in the table below.

(b) Based on the observed sensor values $o_0, o_1, \ldots, o_t$, we now want the robot to find the most likely sequence $H_0, H_1, \ldots, H_t$ indicating the presence/absence of a human up to the current time.

(i) Suppose that $[o_0, o_1, o_2] = [0, 1, 1]$ are observed. The following "trellis diagram" shows the possible state transitions. Fill in the values for the arcs labeled A, B, C, and D with the product of the transition probability and the observation likelihood for the destination state. The values may depend on $p$.

(ii) There are two possible most likely state sequences, depending on the value of $p$. Complete the following (Write the sequence as "x,y,z" (without quotes), where x, y, z are either 0 or 1):

- When $p < \ldots$, the most likely sequence $H_0, H_1, H_2$ is \( \ldots \)

- Otherwise, the most likely sequence $H_0, H_1, H_2$ is \( \ldots \)

(c) True or False: For a fixed $p$ value and observations $\{o_0, o_1, o_2\}$ in general, $H_1^*$, the most likely value for $H_1$, is always the same as the value of $H_1$ in the most likely sequence $H_0, H_1, H_2$.  \( \bigcirc \) True  \( \bigcirc \) False