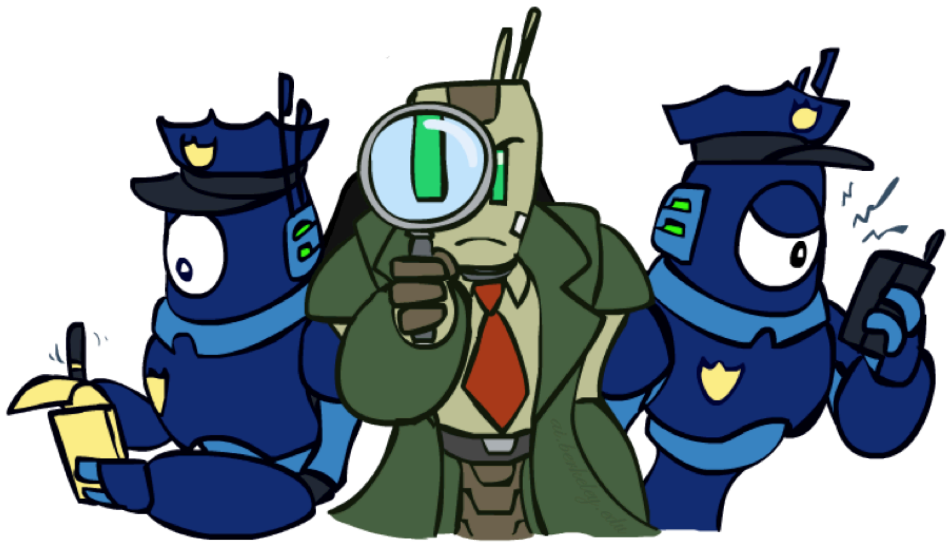
CS 188: Artificial Intelligence

Constraint Satisfaction Problems



Instructor: Nicholas Tomlin

University of California, Berkeley

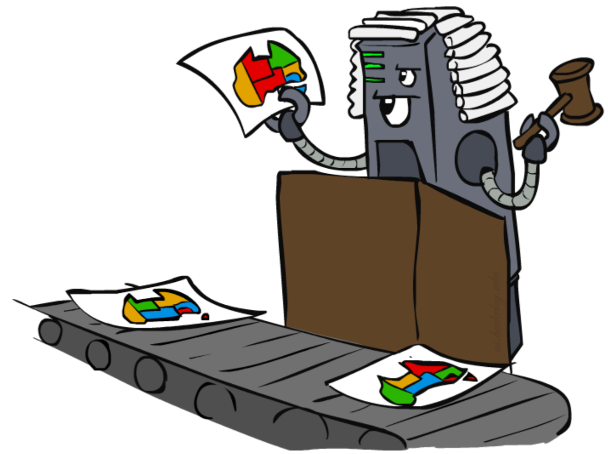
[These slides adapted from Dan Klein, Pieter Abbeel, and Anca Dragan]

Announcements

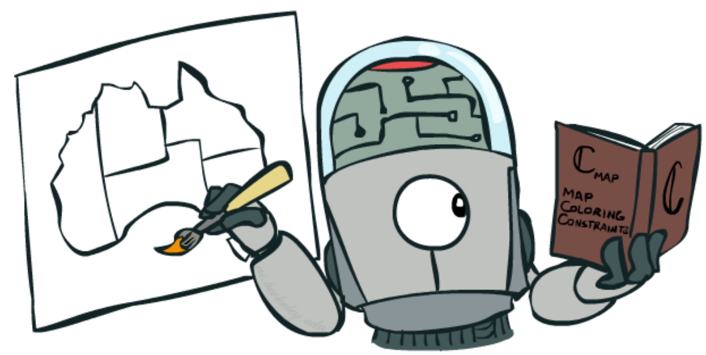
- Extensions
  - Issues with the extension script
  - We’ll be re-opening assignment submission for approved extensions
  - Please don’t abuse the extension process!
  - If you have questions: make a private Ed post (better than email)
- Discussion sections
  - Please make sure you’re going to the right one!
  - Check the SignUpGenius: if you can go to a less popular discussion section and prefer that, get more interaction with the GSIs
- Project 1 due tomorrow, 11PM

Reminder: CSPs

- CSPs:
  - Variables
  - Domains
  - Constraints
    - Implicit (provide code to compute)
    - Explicit (provide a list of all legal tuples)
    - Unary / Binary / N-ary



- Goals:
  - Here: find any solution
  - But also: find all solutions, best solution, etc.



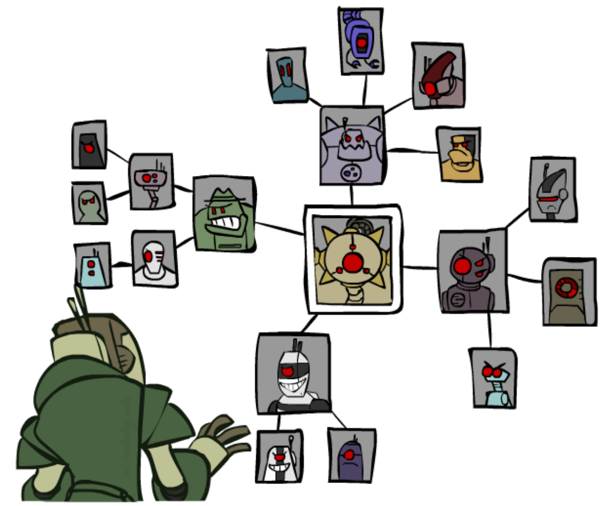
Backtracking Search

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
   return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
      if value is consistent with assignment given CONSTRAINTS[csp] then
         add {var = value} to assignment
         result ← RECURSIVE-BACKTRACKING(assignment, csp)
         if result ≠ failure then return result
         remove {var = value} from assignment
   return failure
```

Improving Backtracking

- General-purpose ideas bring huge gains in speed
  - …but it's all still NP-hard



- Filtering: can we detect inevitable failure early?

- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?

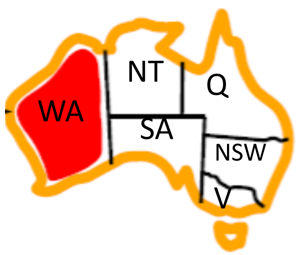
- Structure: can we exploit the structure of the problem/constraint graph?

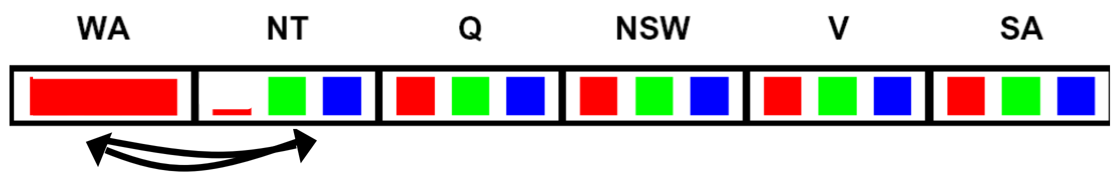
Reminder: Forward Checking

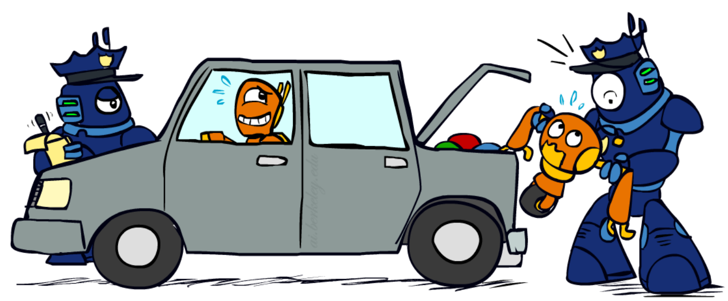
- Filtering: Keep track of domains for unassigned variables and cross off bad options
- Forward checking: Cross off values that violate a constraint when added to the existing assignment

Reminder: Arc Consistency

- An arc X → Y is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint







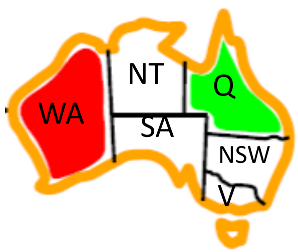
Delete from the tail!

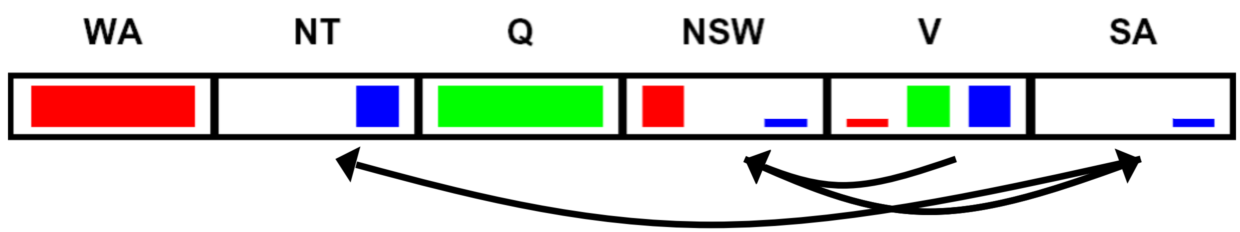
Forward checking?

Enforcing consistency of arcs pointing to each new assignment

Reminder: Arc Consistency

- A simple form of propagation makes sure all arcs are consistent:





- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

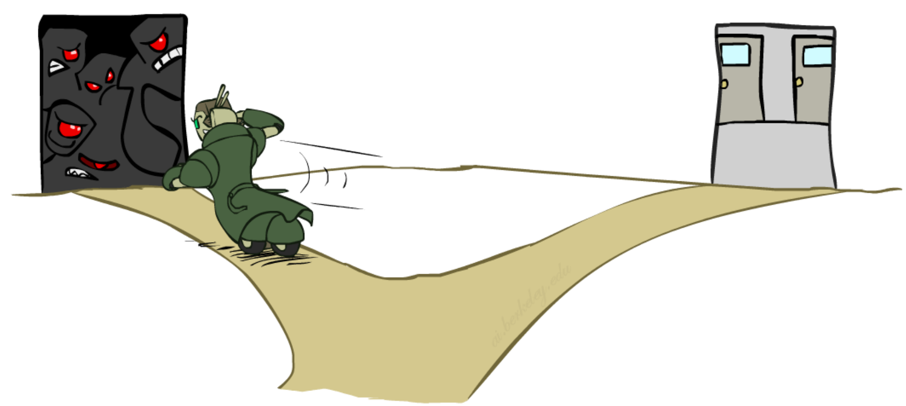
Remember: Delete from the tail!

Ordering

Ordering: Minimum Remaining Values

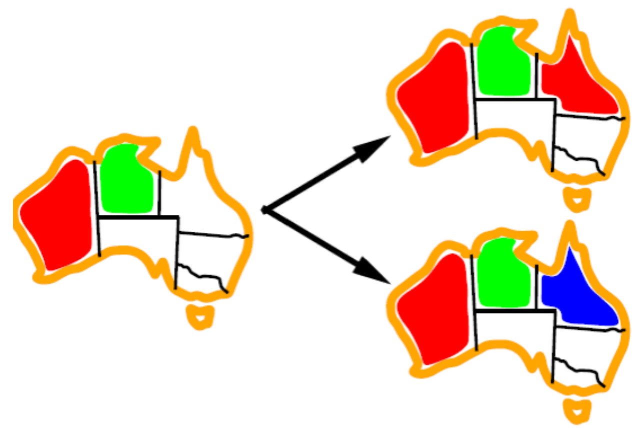
- Variable Ordering: Minimum remaining values (MRV):
  - Choose the variable with the fewest legal values left in its domain

- Why min rather than max?
- Also called "most constrained variable"
- "Fail-fast" ordering



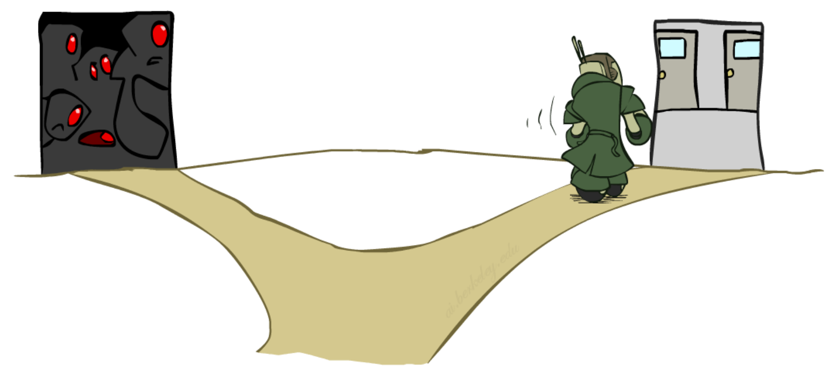
Ordering: Least Constraining Value

- Value Ordering: Least Constraining Value
  - Given a choice of variable, choose the *least constraining value*
  - I.e., the one that rules out the fewest values in the remaining variables
  - Note that it may take some computation to determine this! (E.g., rerunning filtering)

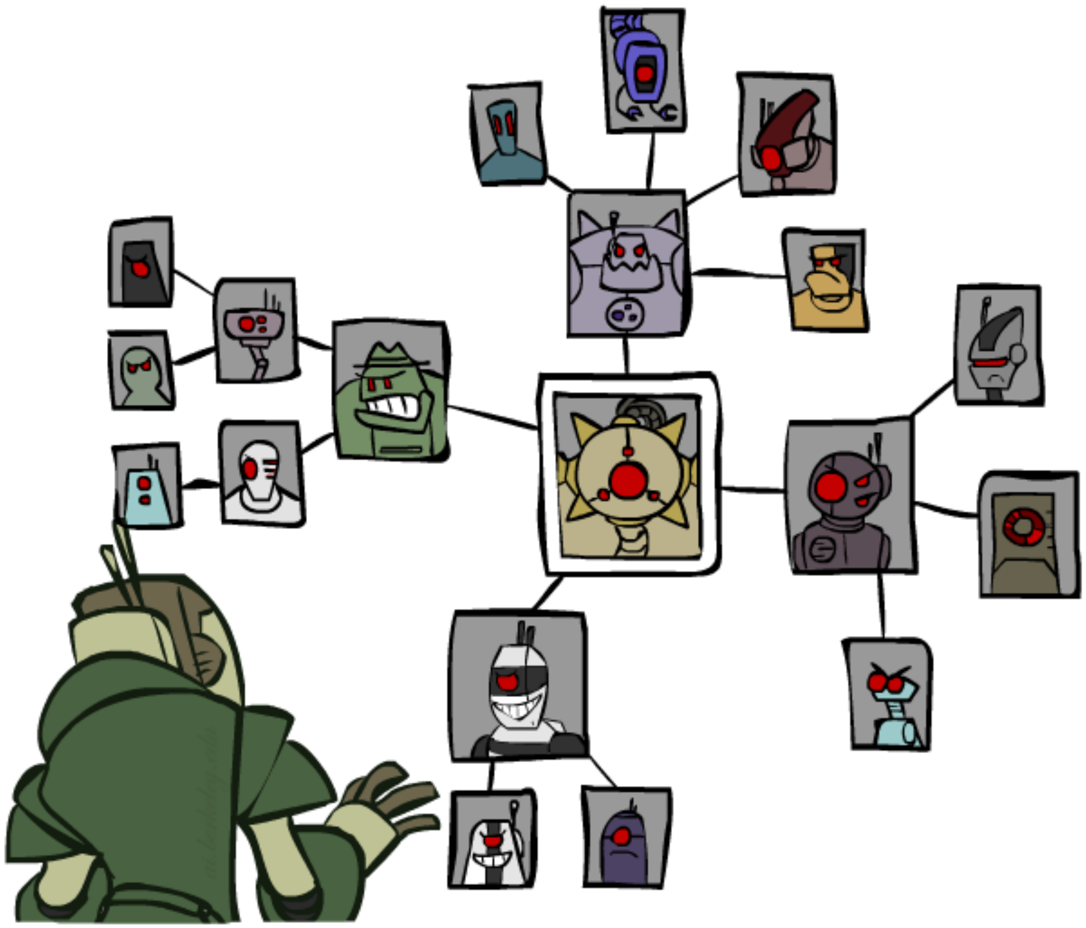


- Why least rather than most?

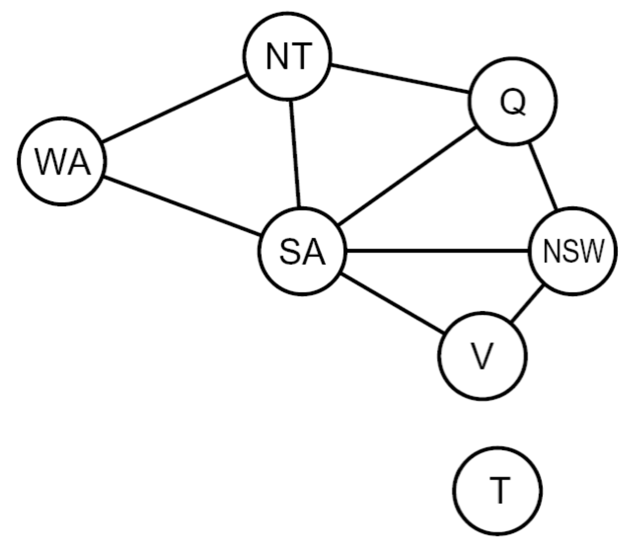
- Combining these ordering ideas makes 1000 queens feasible



Structure

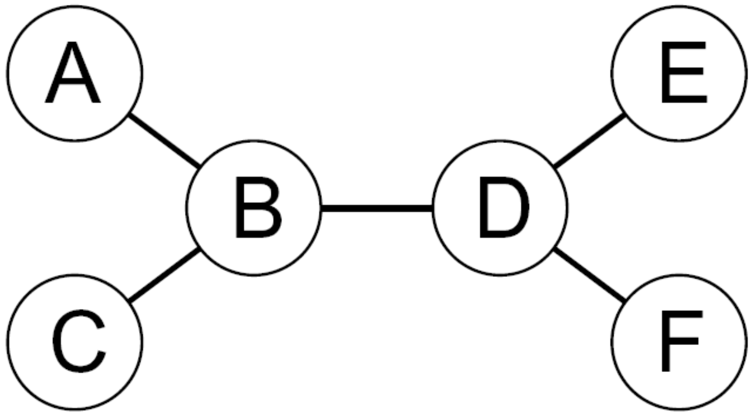


Problem Structure



- Extreme case: independent subproblems
  - Example: Tasmania and mainland do not interact
- Independent subproblems are identifiable as connected components of constraint graph
- Suppose a graph of n variables can be broken into subproblems of only c variables:
  - Worst-case solution cost is $O((n/c)(d^c))$, linear in n
  - E.g., n = 80, d = 2, c =20
  - 2^{80} = 4 billion years at 10 million nodes/sec
  - $(4)(2^{20})$ = 0.4 seconds at 10 million nodes/sec

Tree-Structured CSPs

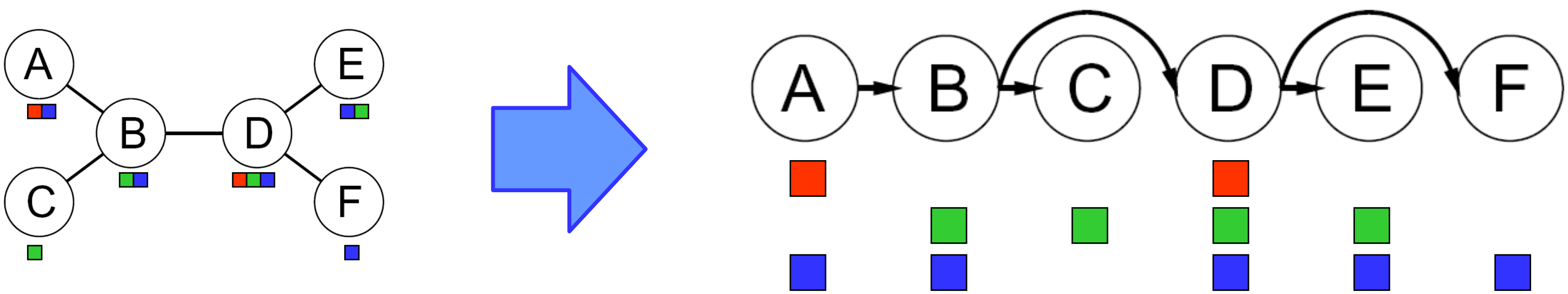


- Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\ d^2)$ time
  - Compare to general CSPs, where worst-case time is $O(d^n)$

- This property also applies to probabilistic reasoning (later): an example of the relation between syntactic restrictions and the complexity of reasoning

Tree-Structured CSPs

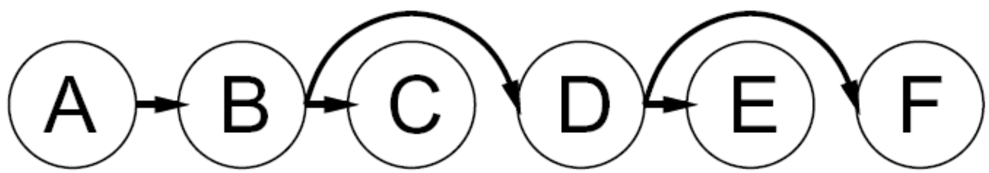
- Algorithm for tree-structured CSPs:
  - Order: Choose a root variable, order variables so that parents precede children



  - Remove backward: For $i = n : 2$, apply RemoveInconsistent(Parent(X_i),X_i)
  - Assign forward: For $i = 1 : n$, assign X_i consistently with Parent(X_i)
- Runtime: $O(n\ d^2)$ (why?)

Tree-Structured CSPs

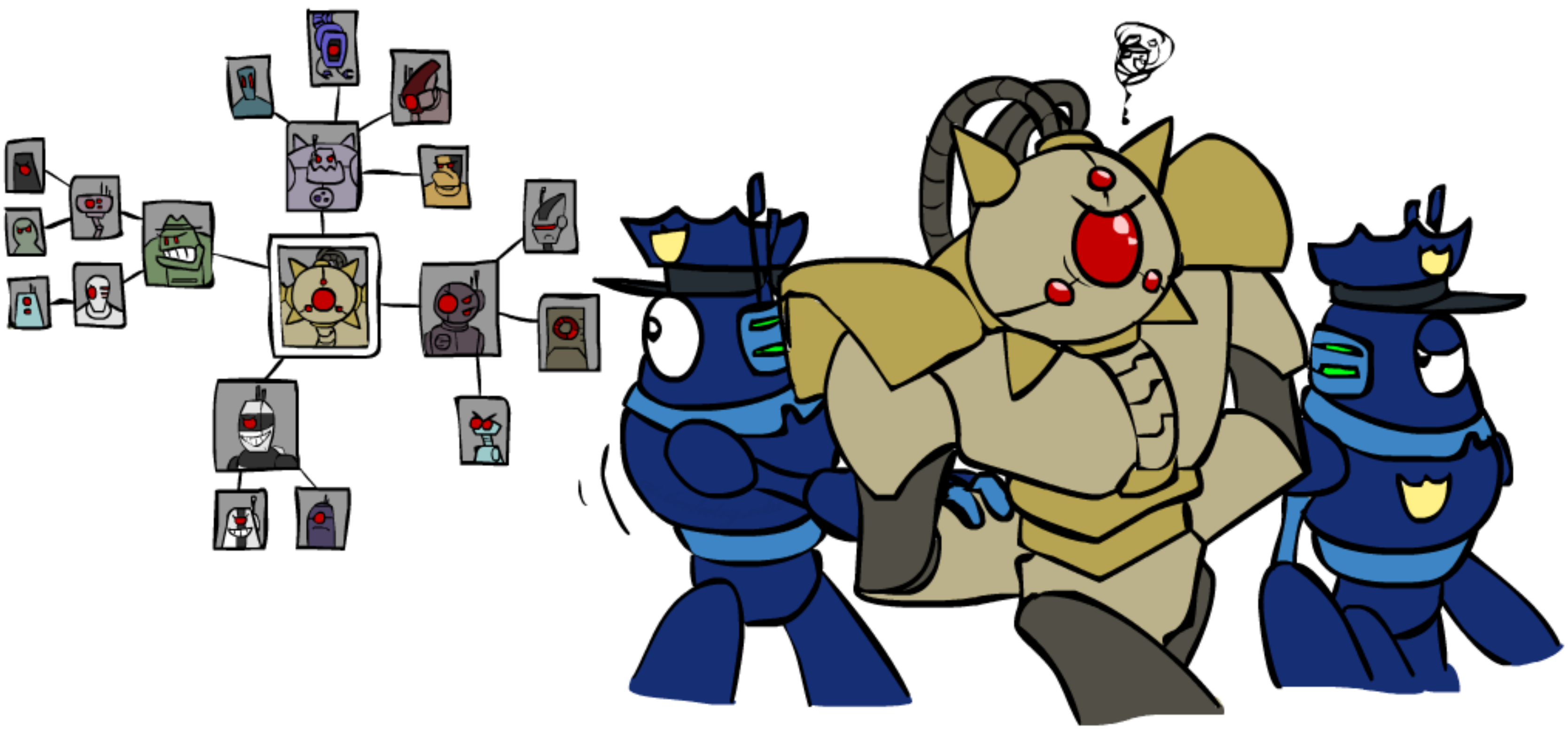
- Claim 1: After backward pass, all root-to-leaf arcs are consistent
- Proof: Each X→Y was made consistent at one point and Y's domain could not have been reduced thereafter (because Y's children were processed before Y)



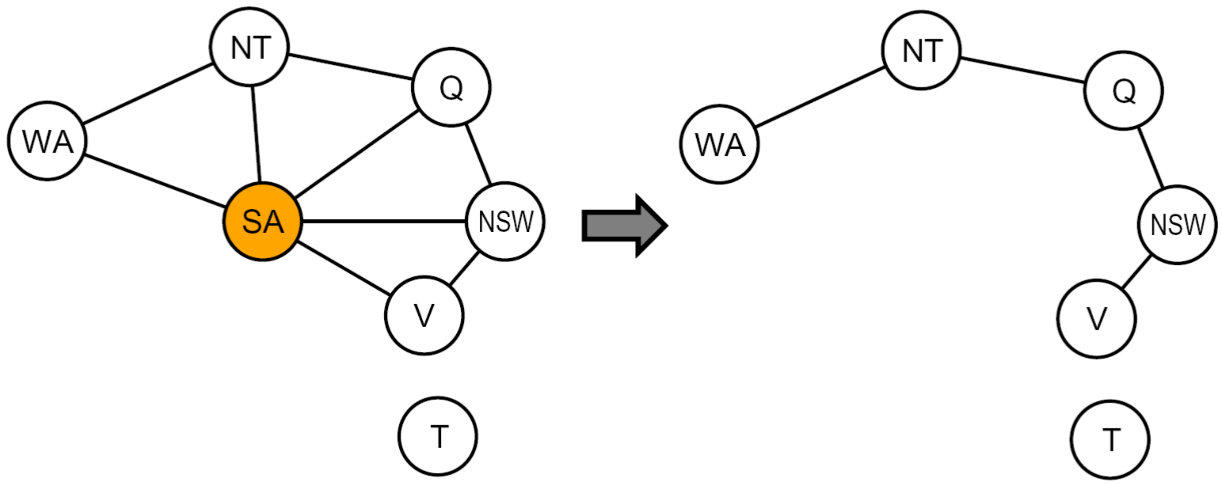
- Claim 2: If root-to-leaf arcs are consistent, forward assignment will not backtrack
- Proof: Induction on position

- Note: we'll see this basic idea again with Bayes' nets

Improving Structure



Nearly Tree-Structured CSPs



- Conditioning: instantiate a variable, prune its neighbors' domains
- Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree
- Cutset size c gives runtime $O((d^c) (n-c) d^2)$, very fast for small c

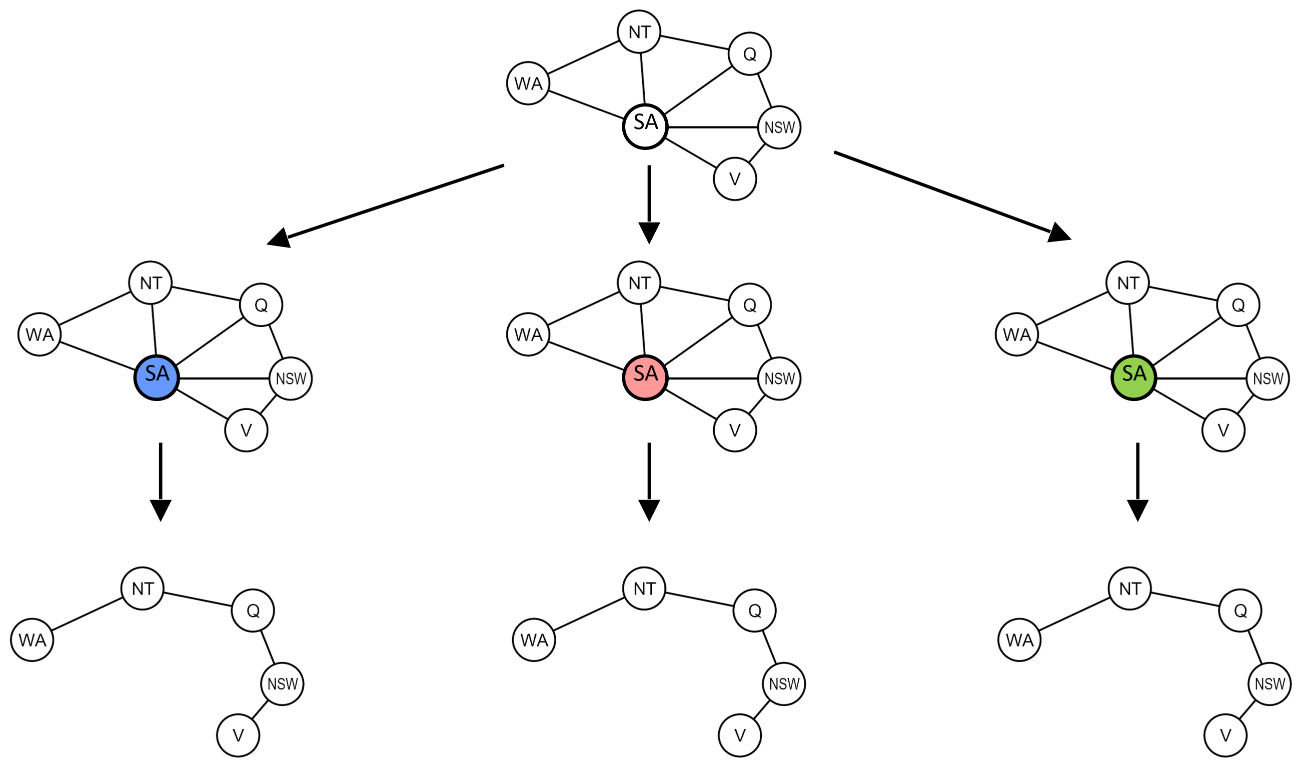
Cutset Conditioning

Choose a cutset

Instantiate the cutset (all possible ways)

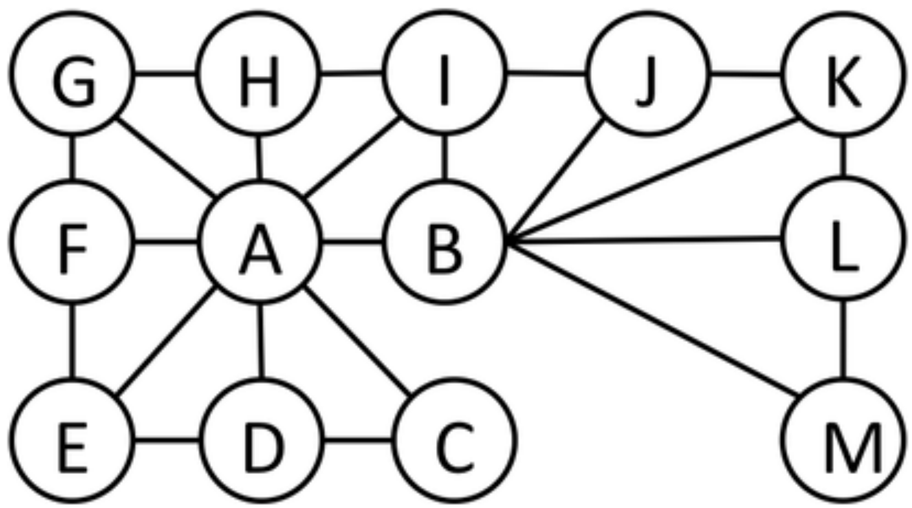
Compute residual CSP for each assignment

Solve the residual CSPs (tree structured)



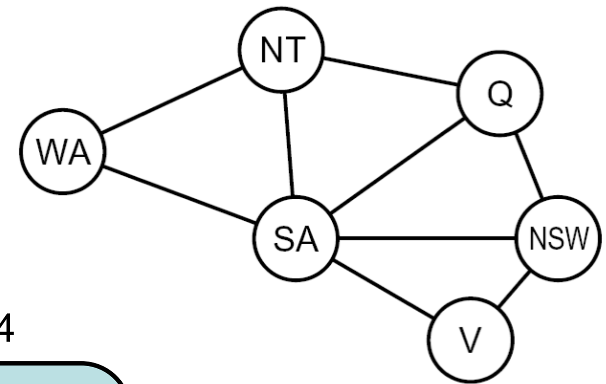
Cutset Quiz

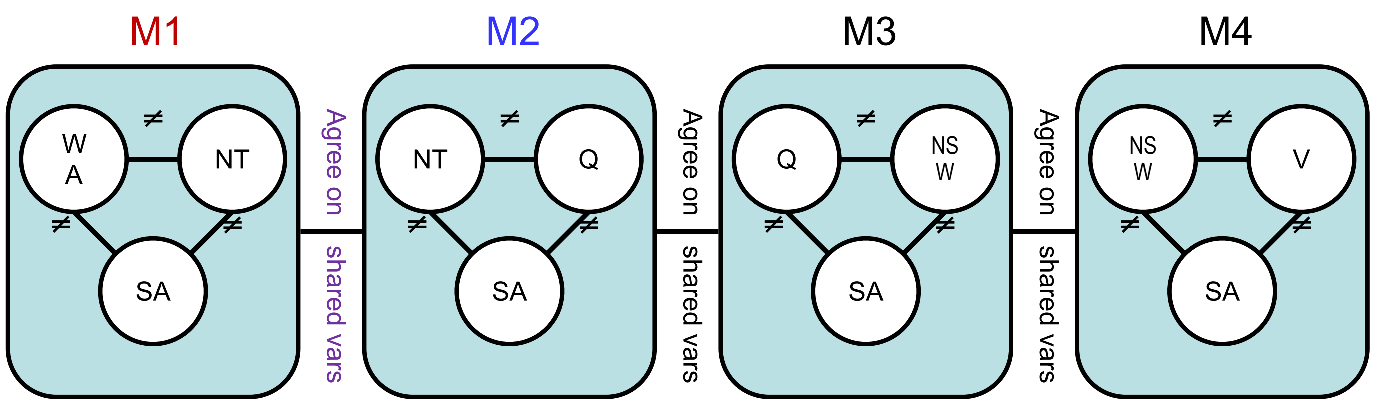
- Find the smallest cutset for the graph below.



Bonus: Tree Decomposition

- Idea: create a tree-structured graph of mega-variables
- Each mega-variable encodes part of the original CSP
- Subproblems overlap to ensure consistent solutions



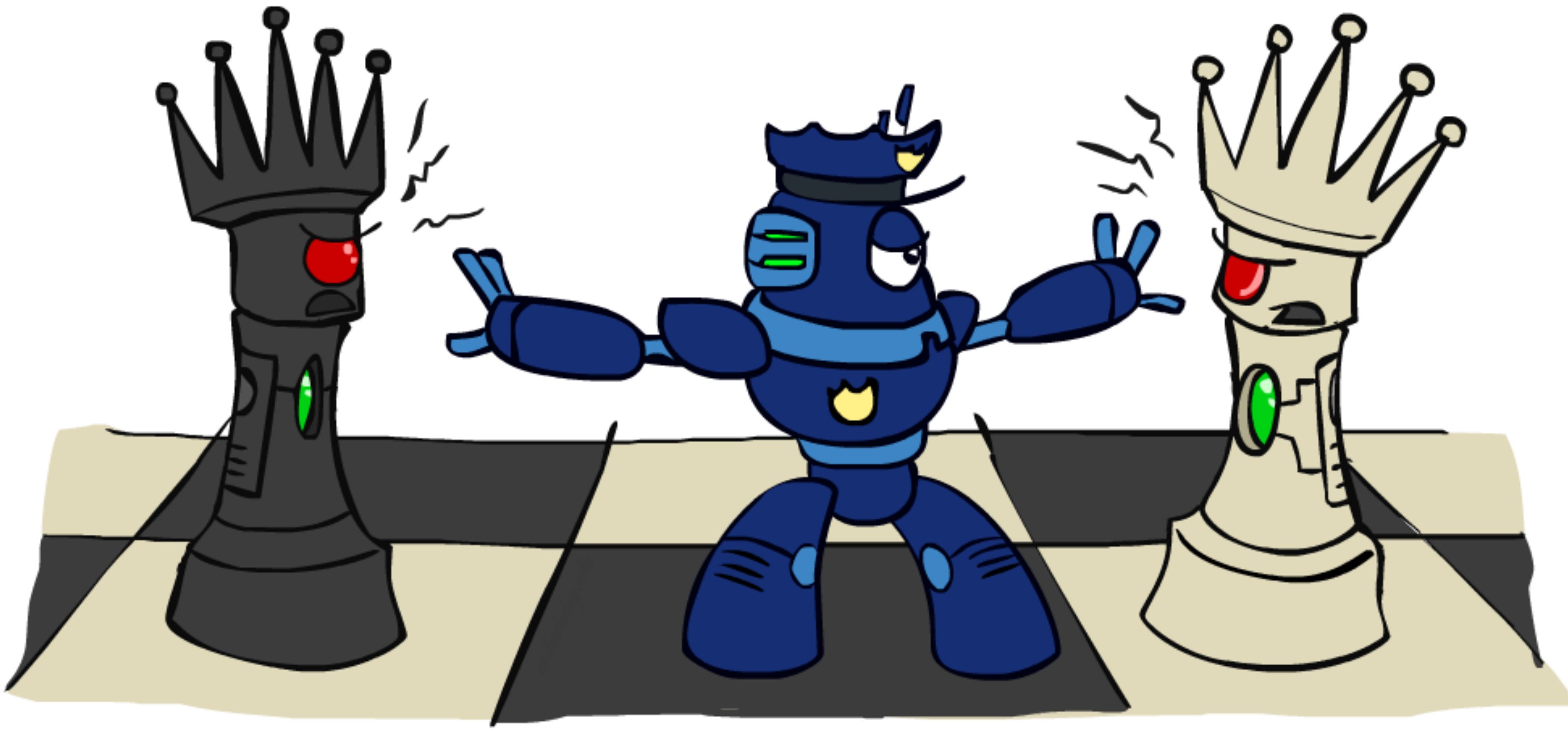


{(WA=r,SA=g,NT=b),
(WA=b,SA=r,NT=g),
…}

{(NT=r,SA=g,Q=b),
(NT=b,SA=g,Q=r),
…}

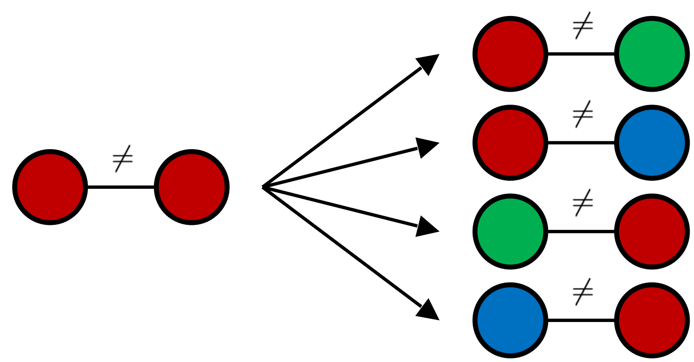
Agree: (M1,M2) $\in$
{((WA=g,SA=g,NT=g), (NT=g,SA=g,Q=g)), …}

Iterative Improvement



Local Search

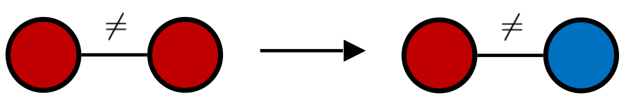
- Tree search keeps unexplored alternatives on the fringe (ensures completeness)
- Local search: improve a single option until you can't make it better (no fringe!)
- New successor function: local changes



- Generally much faster and more memory efficient (but incomplete and suboptimal)

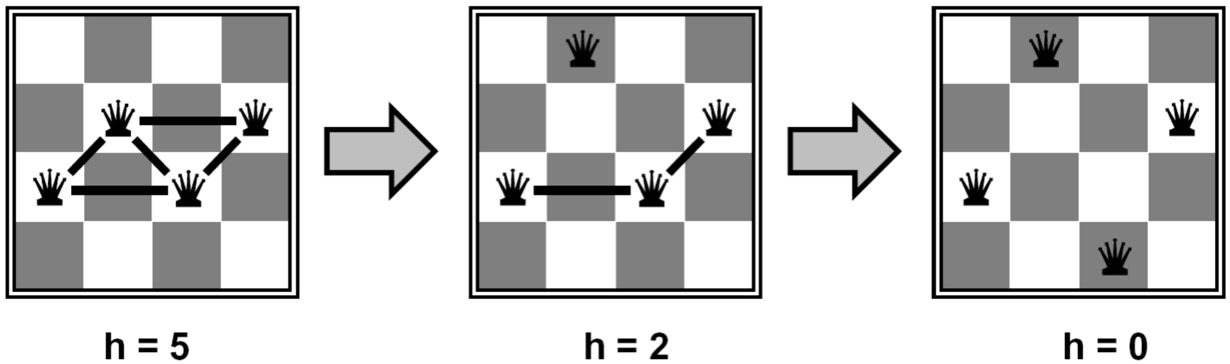
Iterative Algorithms for CSPs

- Local search methods typically work with "complete" states, i.e., all variables assigned
- To apply to CSPs:
  - Take an assignment with unsatisfied constraints
  - Operators *reassign* variable values
  - No fringe! Live on the edge.



- Algorithm: While not solved,
  - Variable selection: randomly select any conflicted variable
  - Value selection: min-conflicts heuristic:
    - Choose a value that violates the fewest constraints
    - I.e., hill climb with h(x) = total number of violated constraints

Example: 4-Queens



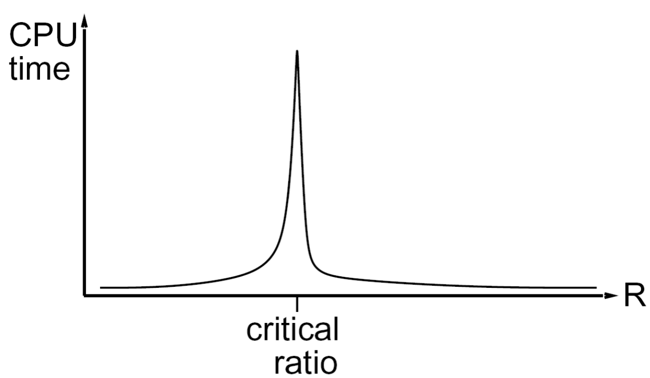
- States: 4 queens in 4 columns ($4^4 = 256$ states)
- Operators: move queen in column
- Goal test: no attacks
- Evaluation: c(n) = number of attacks

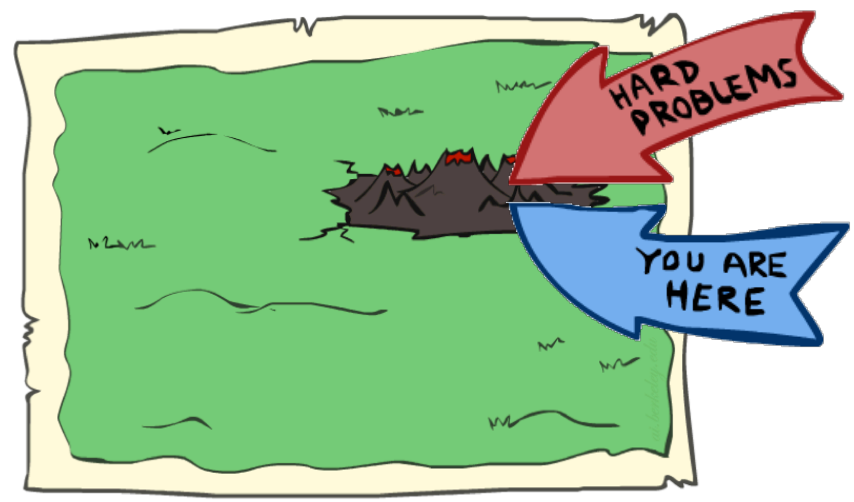
Performance of Min-Conflicts

- Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000)!

- The same appears to be true for any randomly-generated CSP *except* in a narrow range of the ratio

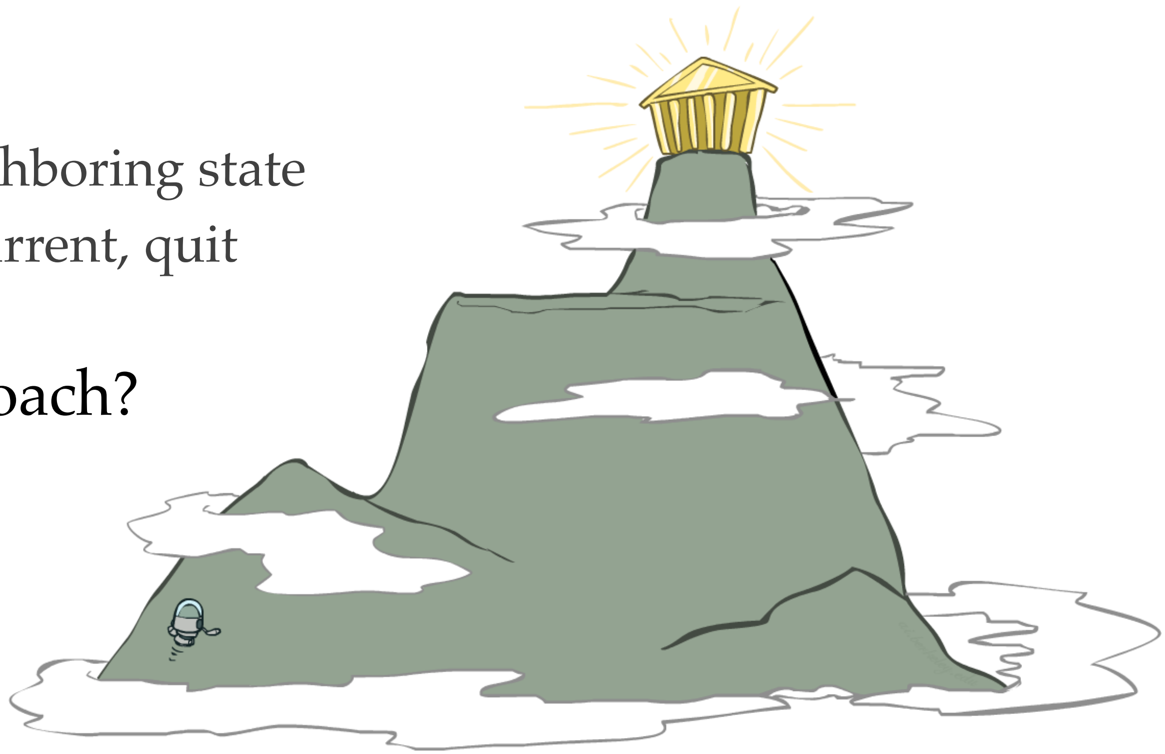
$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$



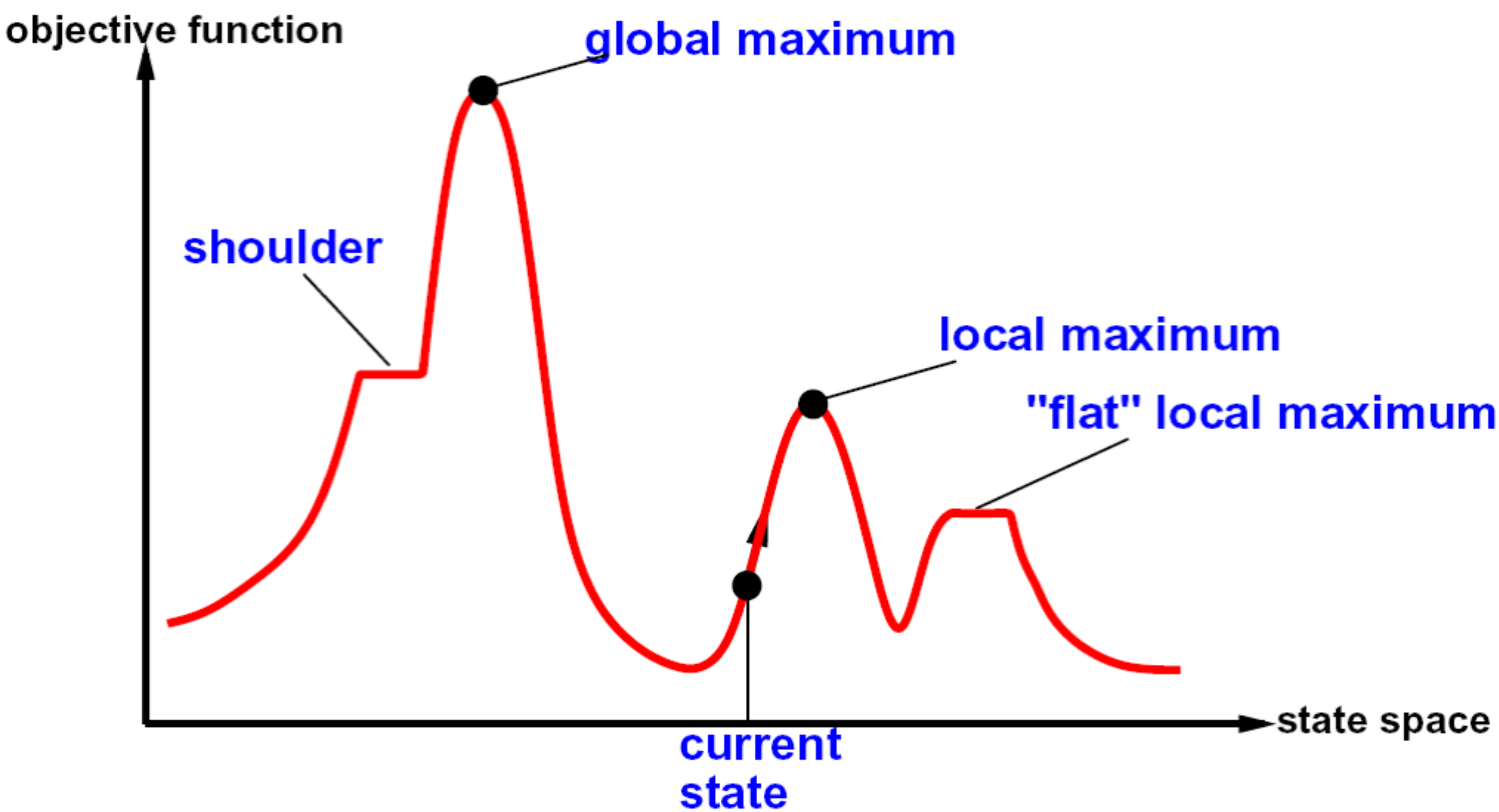


Hill Climbing

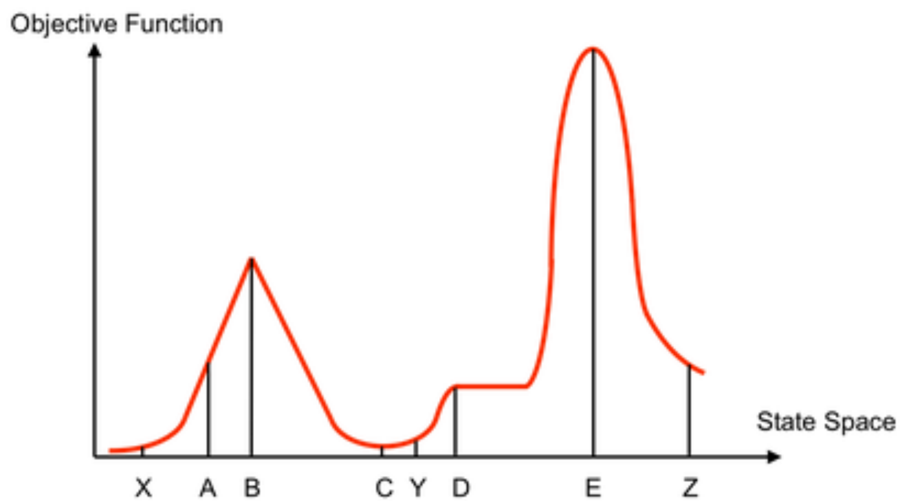
- Simple, general idea:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit
- What's bad about this approach?
- What's good about it?



Hill Climbing Diagram



Hill Climbing Quiz



Starting from X, where do you end up ?

Starting from Y, where do you end up ?

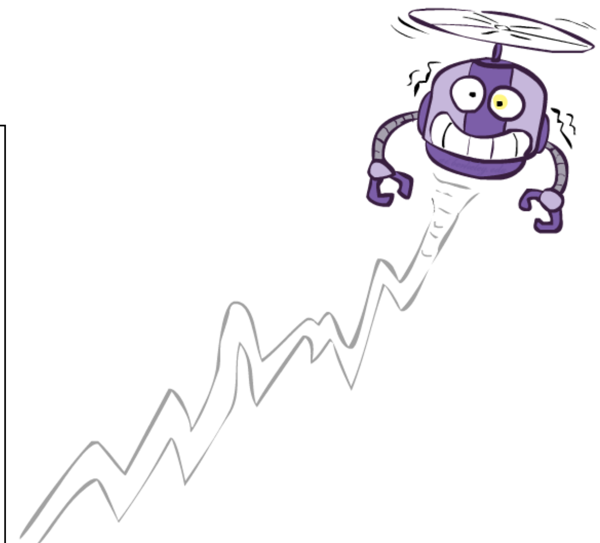
Starting from Z, where do you end up ?

Simulated Annealing

- Idea: Escape local maxima by allowing downhill moves
  - But make them rarer as time goes on

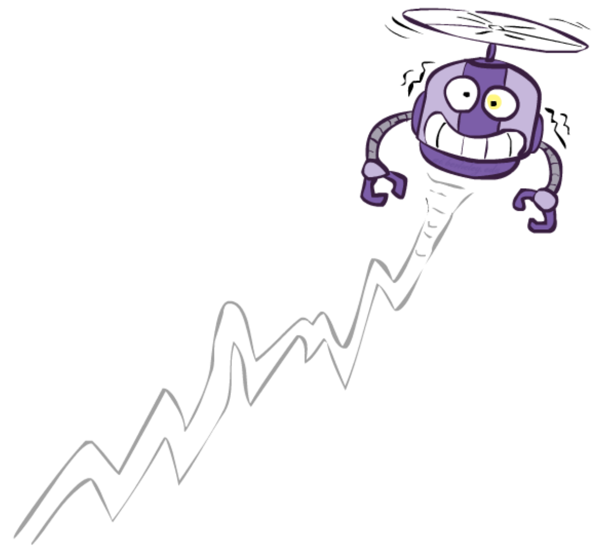
```
function SIMULATED-ANNEALING( problem, schedule) returns a solution state
    inputs: problem, a problem
            schedule, a mapping from time to "temperature"
    local variables: current, a node
                     next, a node
                     T, a "temperature" controlling prob. of downward steps

    current ← MAKE-NODE(INITIAL-STATE[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← VALUE[next] – VALUE[current]
        if ΔE > 0 then current ← next
        else current ← next only with probability e^(Δ E/T)
```



Simulated Annealing

- Theoretical guarantee:
  - Stationary distribution: $p(x) \propto e^{\frac{E(x)}{kT}}$
  - If T decreased slowly enough,
    will converge to optimal state!



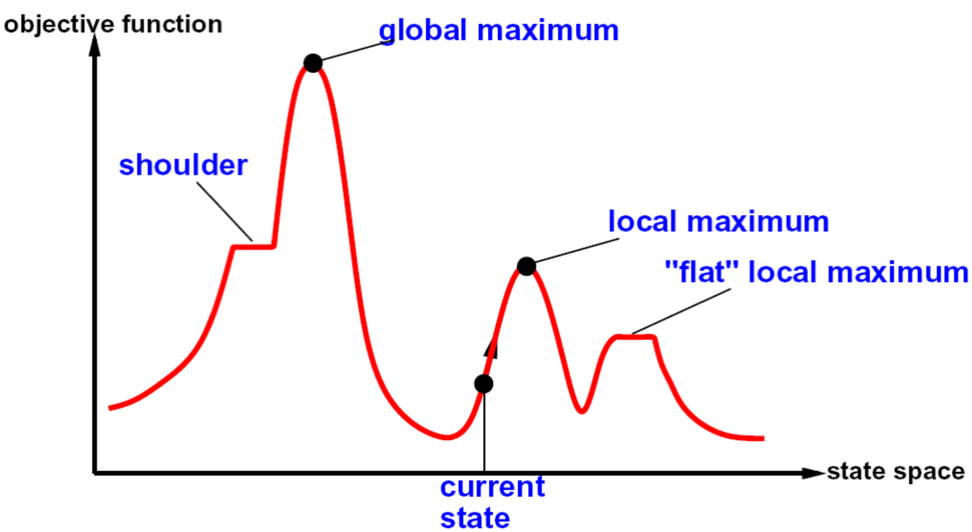
- Is this an interesting guarantee?

- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape a local optimum, the less likely you are to ever make them all in a row
  - People think hard about *ridge operators* which let you jump around the space in better ways

Gradient Descent as Simulated Annealing

- Q: Can we do better than randomly guessing?

- A: Yes, if the function is continuous and differentiable.

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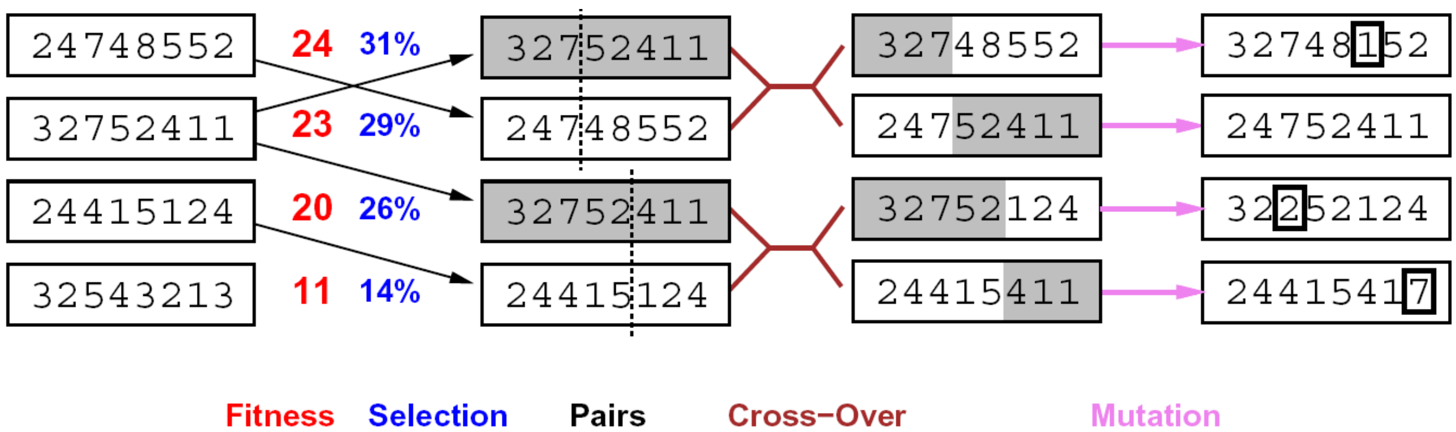
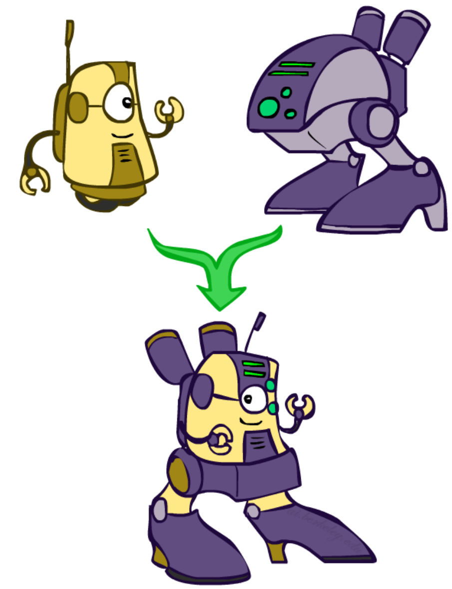
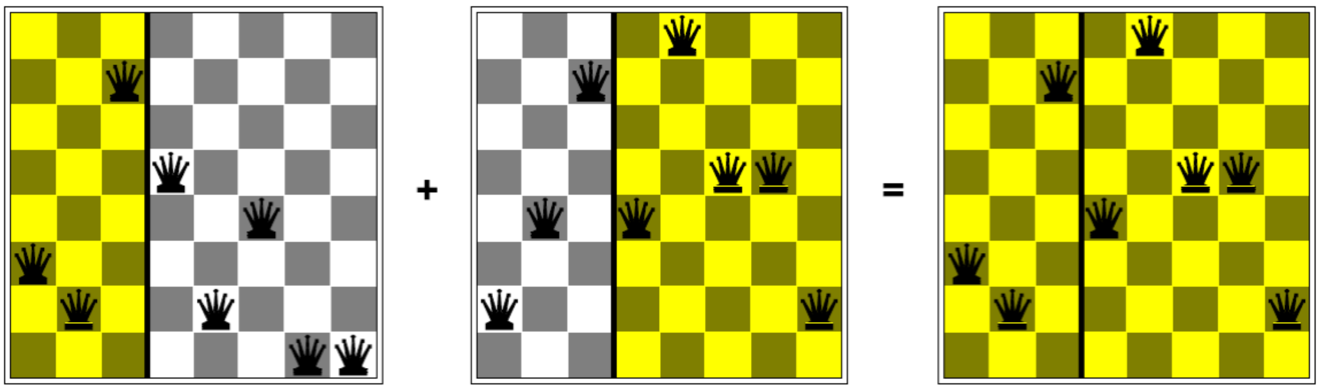
Genetic Algorithms





- Genetic algorithms use a natural selection metaphor
  - Keep best N hypotheses at each step (selection) based on a fitness function
  - Also have pairwise crossover operators, with optional mutation to give variety
- Possibly the most misunderstood, misapplied (and even maligned) technique around

Example: N-Queens



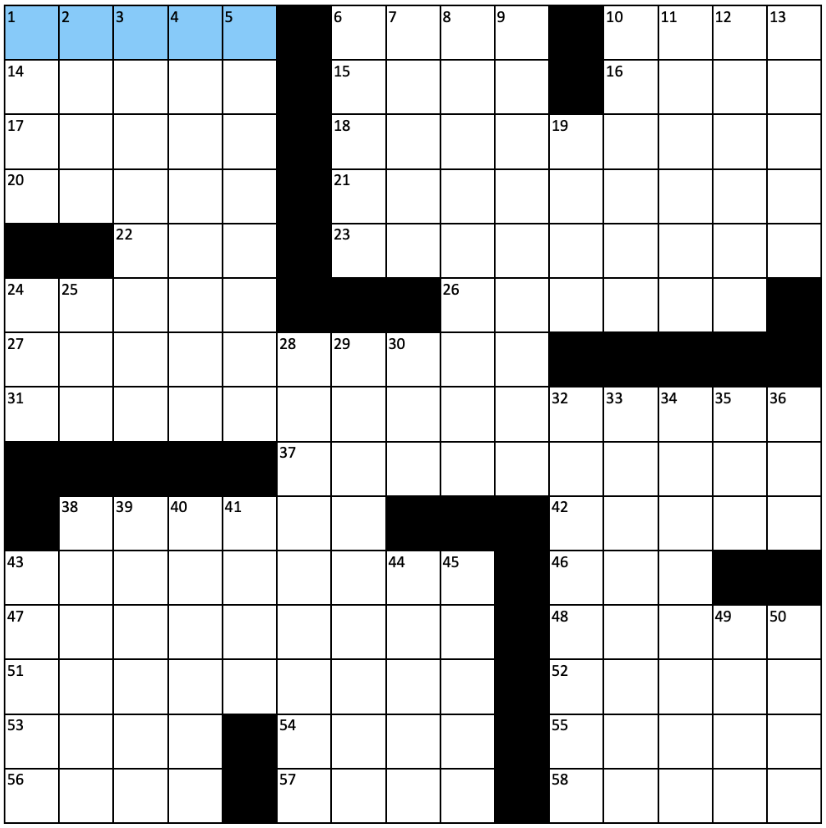
- Why does crossover make sense here?
- When wouldn't it make sense?
- What would mutation be?
- What would a good fitness function be?

Bonus: Weighted CSPs

- In the real world, many constraints are soft:
  - Scheduling:
    - With enough people attending a meeting, no times will work
    - Solution: some conflicts are more important than others, make sacrifices where necessary
  - Travel planning:
    - Budget: would like to keep things cheap, but willing to spend more if it's worth it
    - Distance: want to avoid long walks, but can make exceptions for really interesting places
  - Running example: crosswords!

- Different set of algorithms often used for WCSPs

Example Crossword



Across

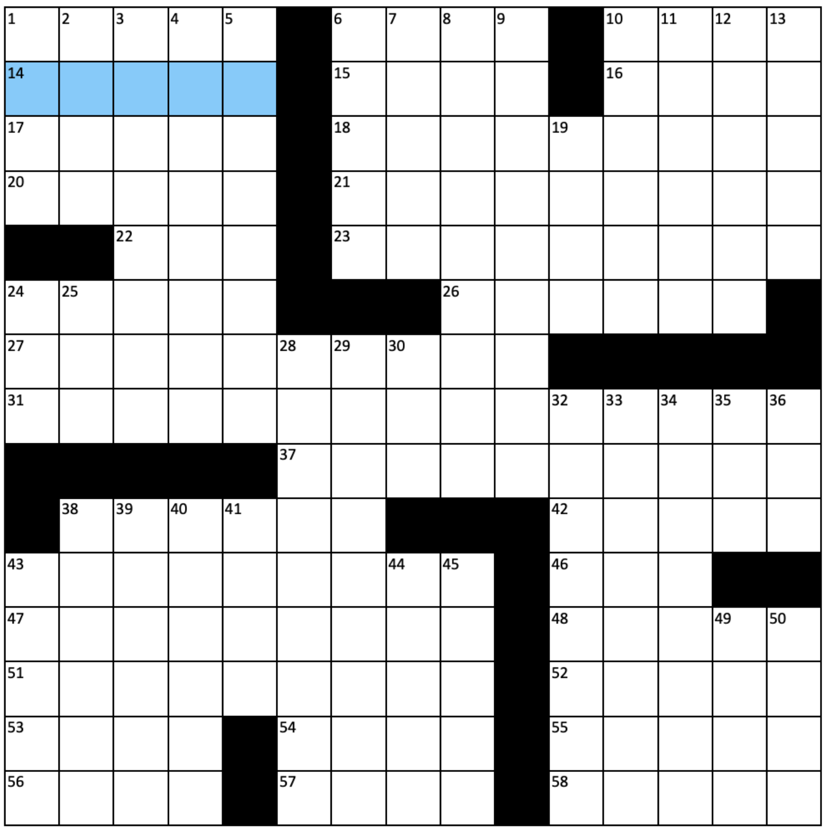
1	Regarding
6	Take back, in a way
10	Start of an aside
14	Omega competitor
15	Something hitting a nerve?
16	Papyrus, e.g.
17	"Take me with you!"
18	Begin flirting with someone, so to speak
20	Assign
21	Hoth, in "Star Wars"
22	___ rule
23	They don't hold water
24	Feudal figure
26	Panegyric, e.g.
27	Sci-fi enemy collective, perhaps
31	Ones born beginning in the early 2010s
37	They might cut to the chase
38	Electrically balanced, in chemistry
42	Leaders at the Kaaba

Down

1	Home to 41-Down
2	Convince
3	"I wanna know all the details"
4	Start of a modern inquiry
5	Drink similar to sarsaparilla
6	Co-star of 1984's "Ghostbusters"
7	Right on
8	Aces with aces?
9	Like some households
10	Designer with an eponymous hotel in the Burj Khalifa
11	Currency units in West Africa
12	Grievous
13	"Golden Boy" playwright
19	Actress Shawkat of "Arrested Development"
24	Assignment for an anchor
25	"___ noticed ..."
28	Great Hindu sage
29	Late-Triassic flier

The New York Times Crossword from Saturday, May 22, 2021. Constructed by Sid Sivakumar and Matthew Stock and edited by Will Shortz.

Example Crossword



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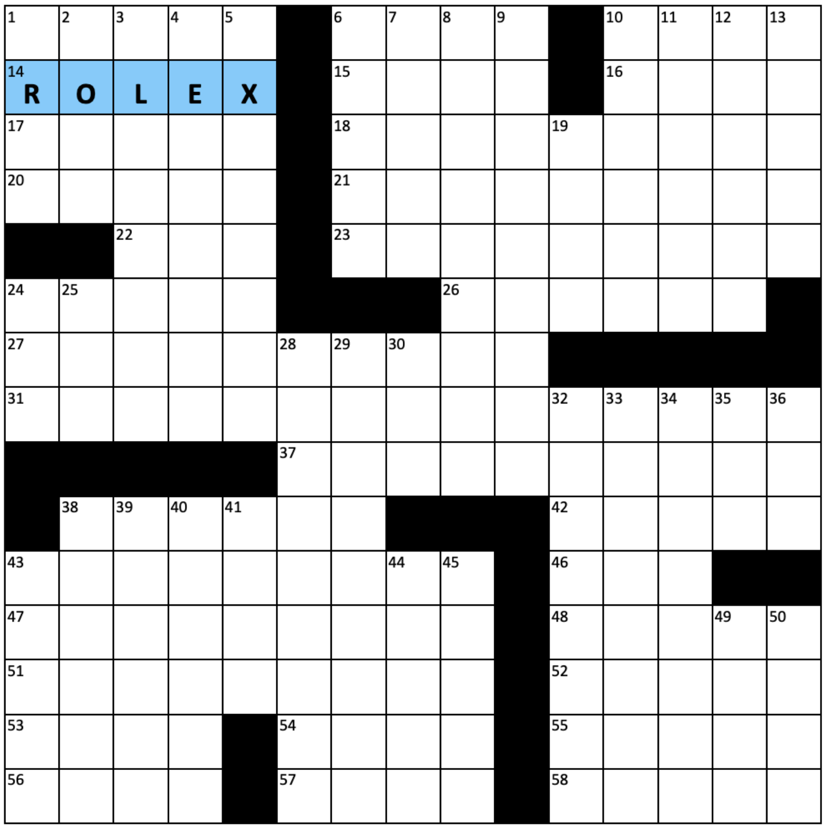

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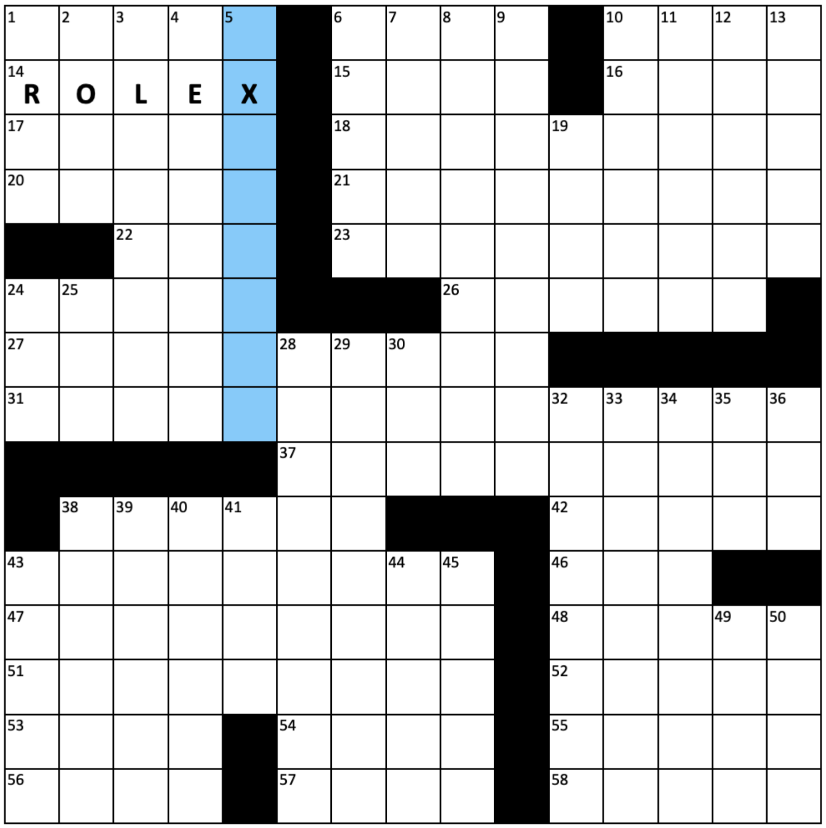

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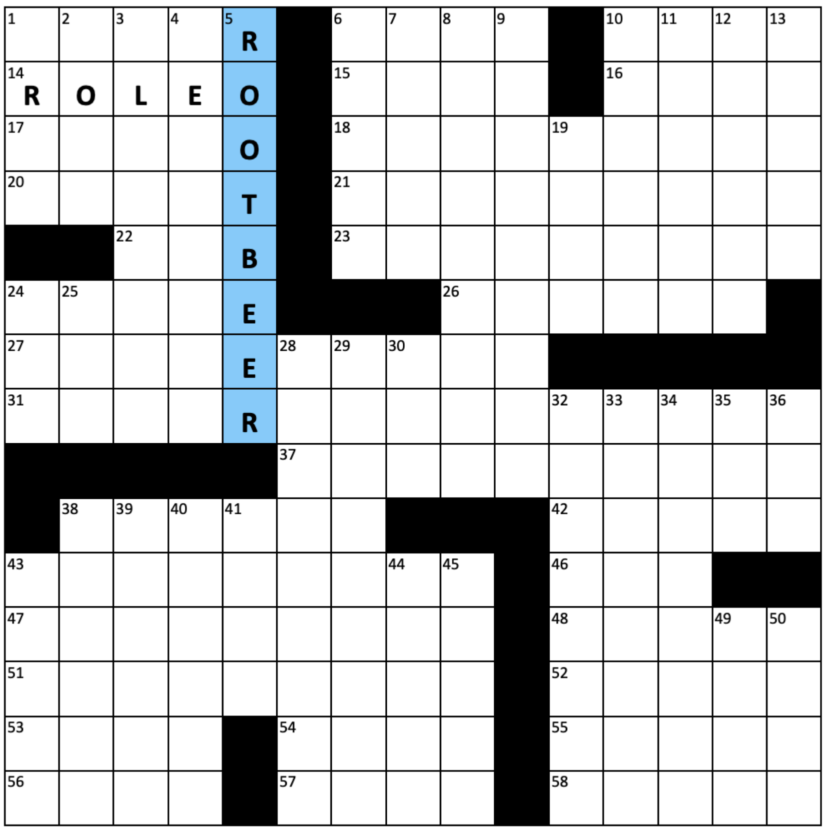

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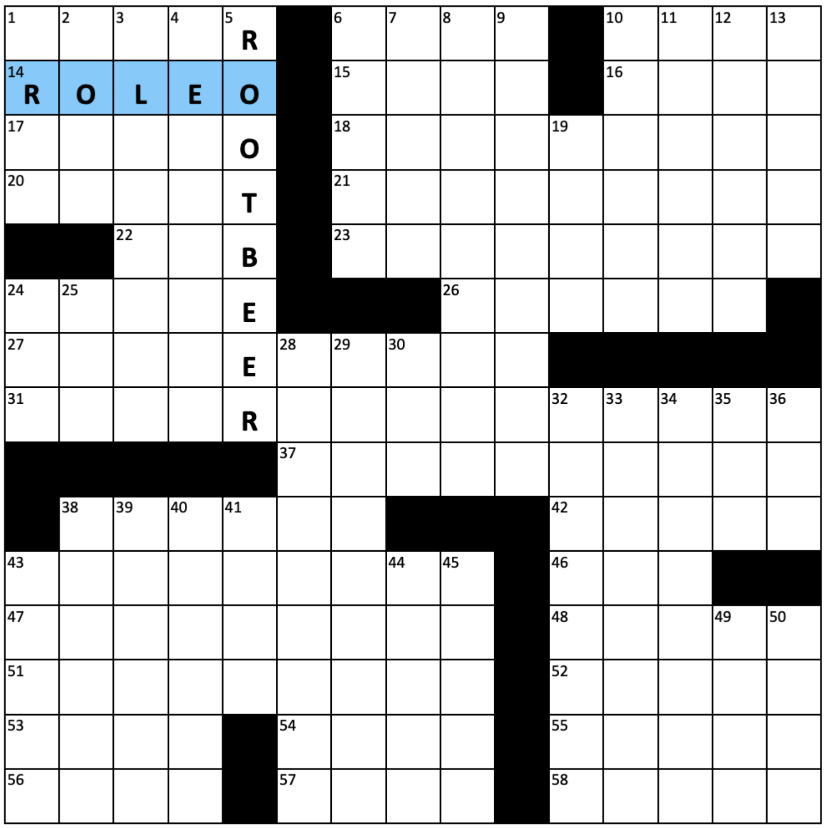

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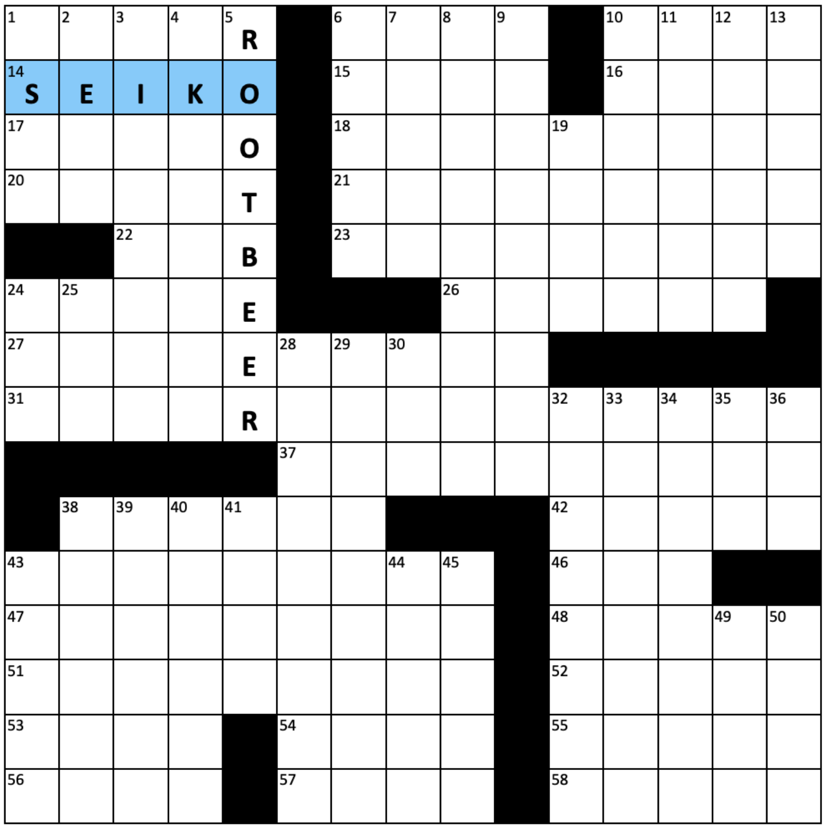

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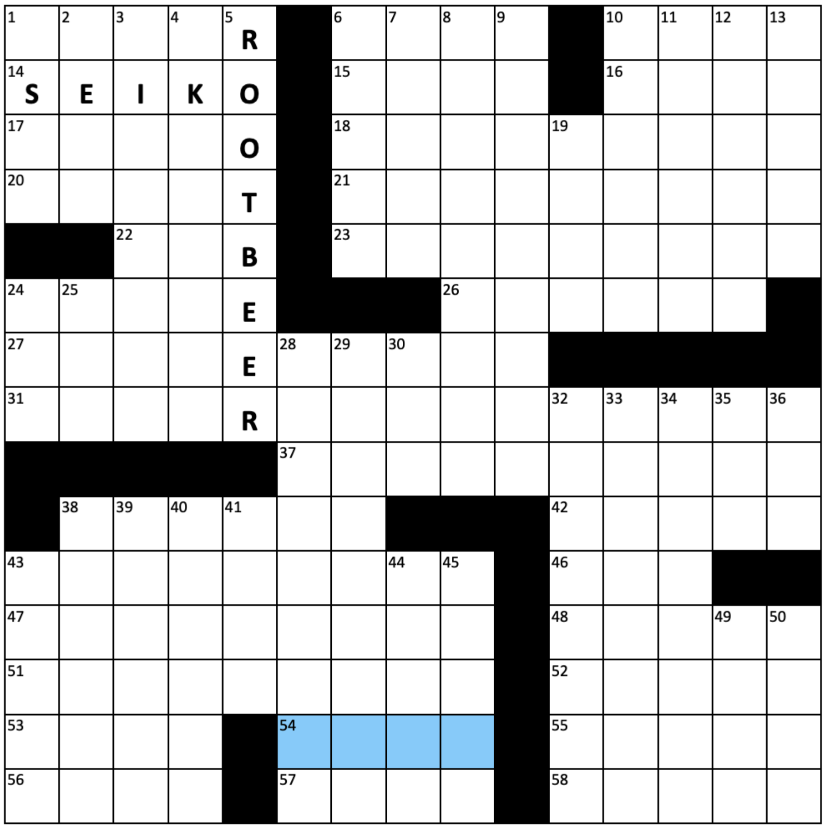

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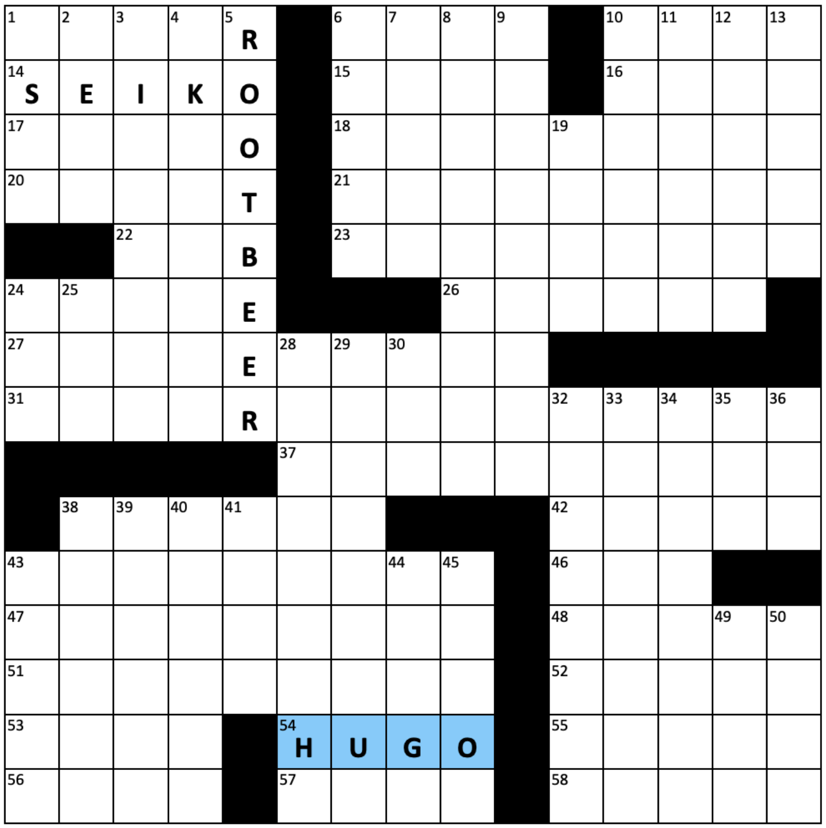

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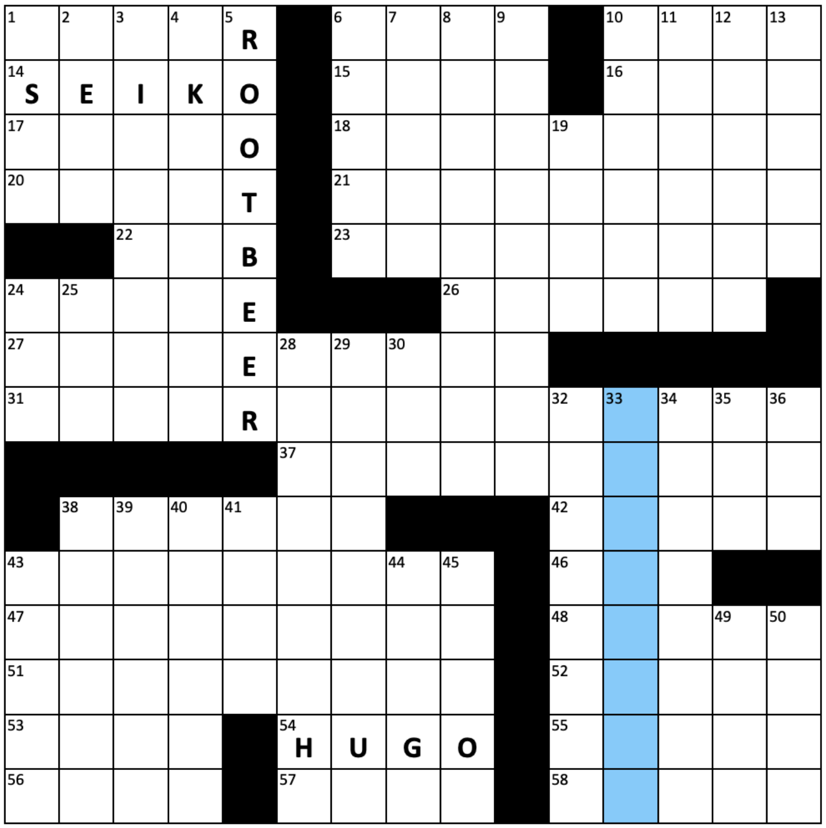

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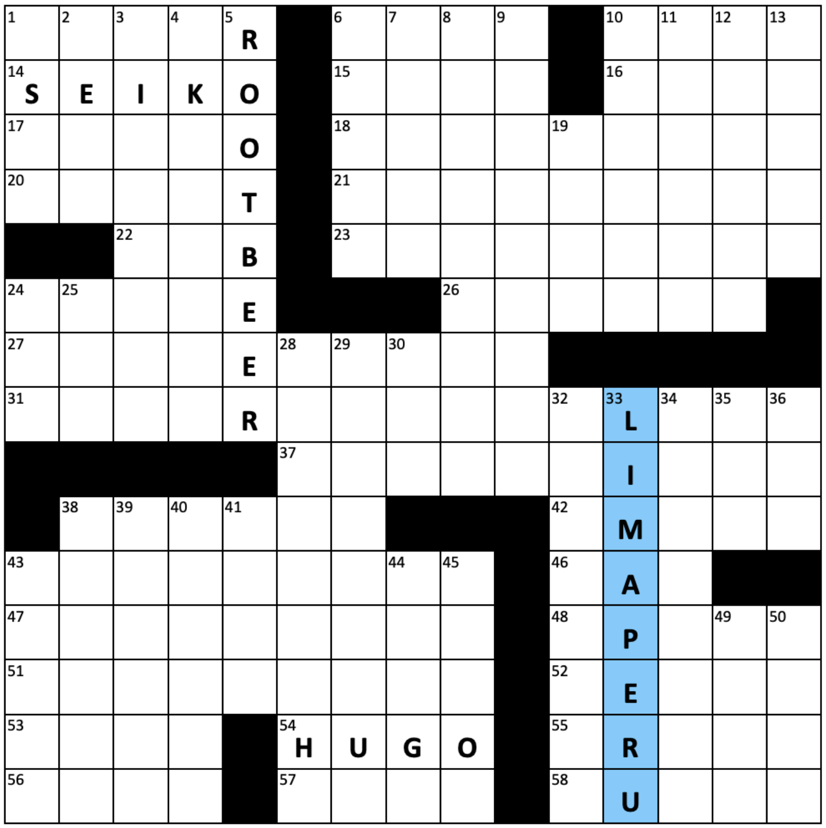

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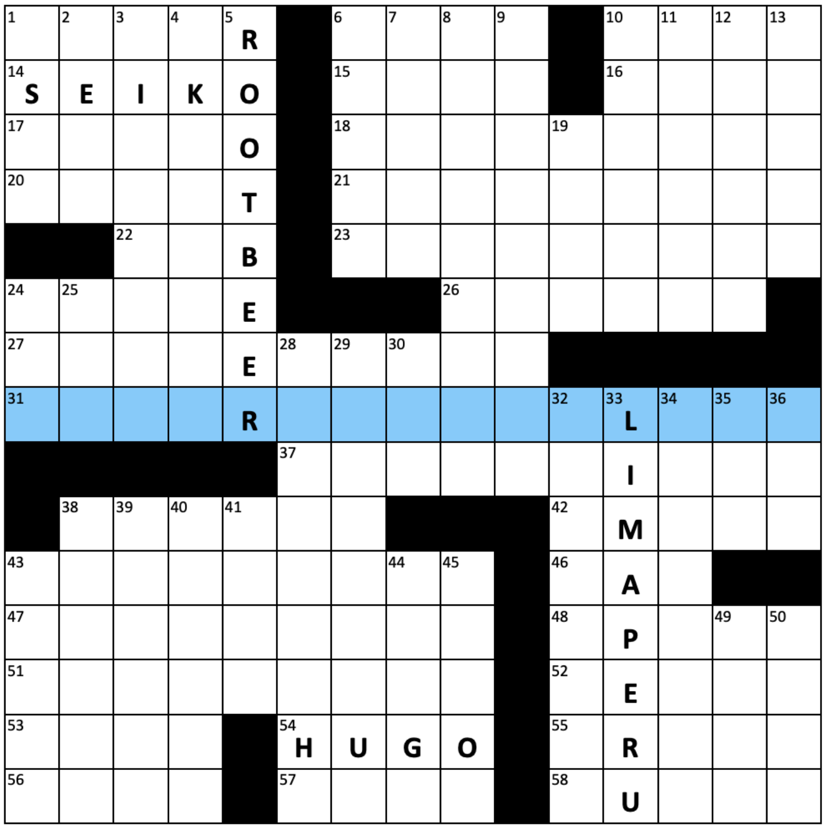

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Crosswords as a Weighted CSP

- Variables: entries in the grid
- Domains: all possible words (??)
- Constraints:
  - Hard constraints: intersecting words match
  - Scoring function: match the clues

- Questions:
  - Where does the scoring function come from?
  - What's the search algorithm?

Scoring Function

14. Omega competitor

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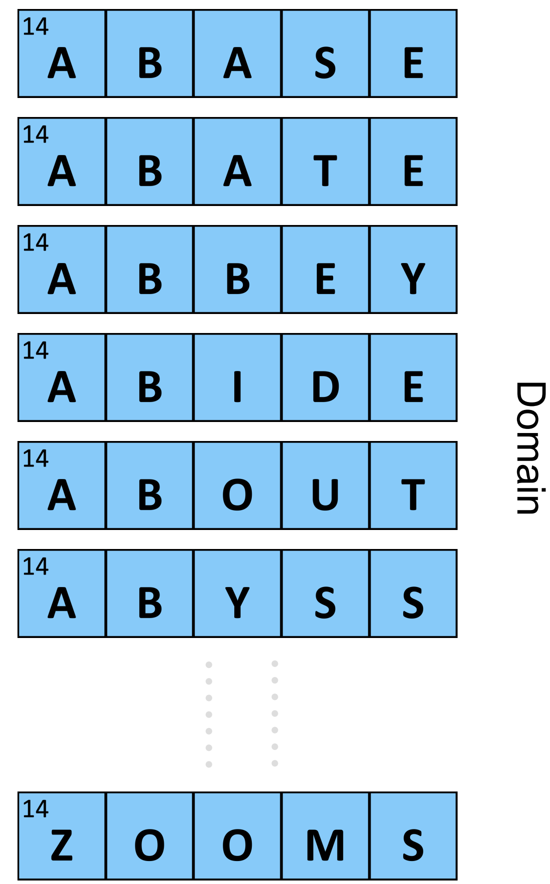

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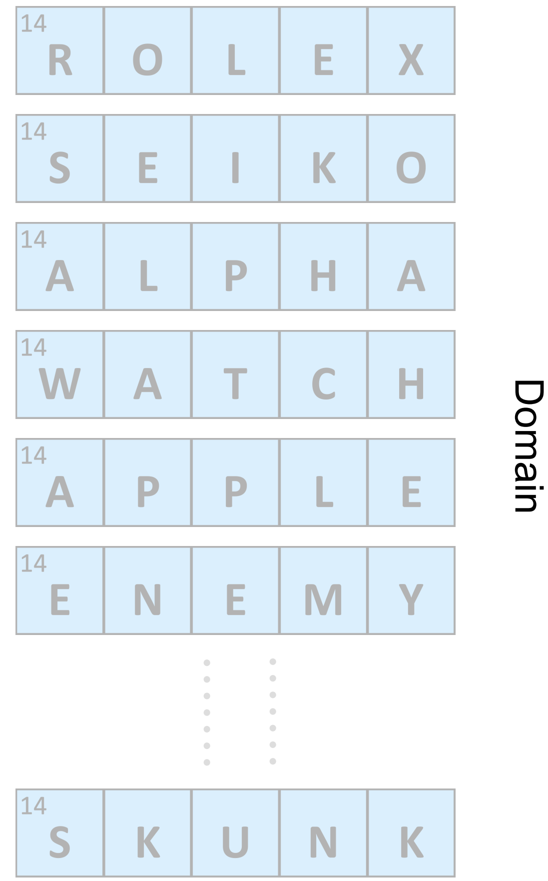

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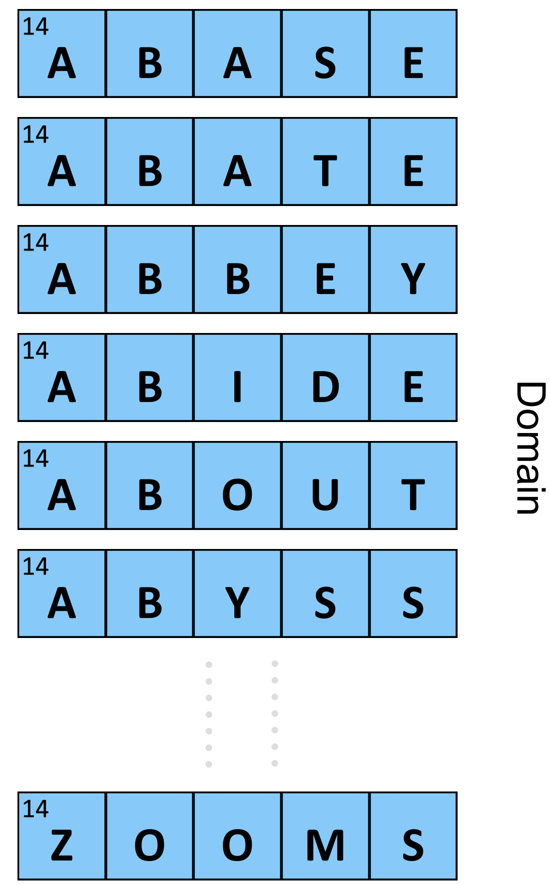

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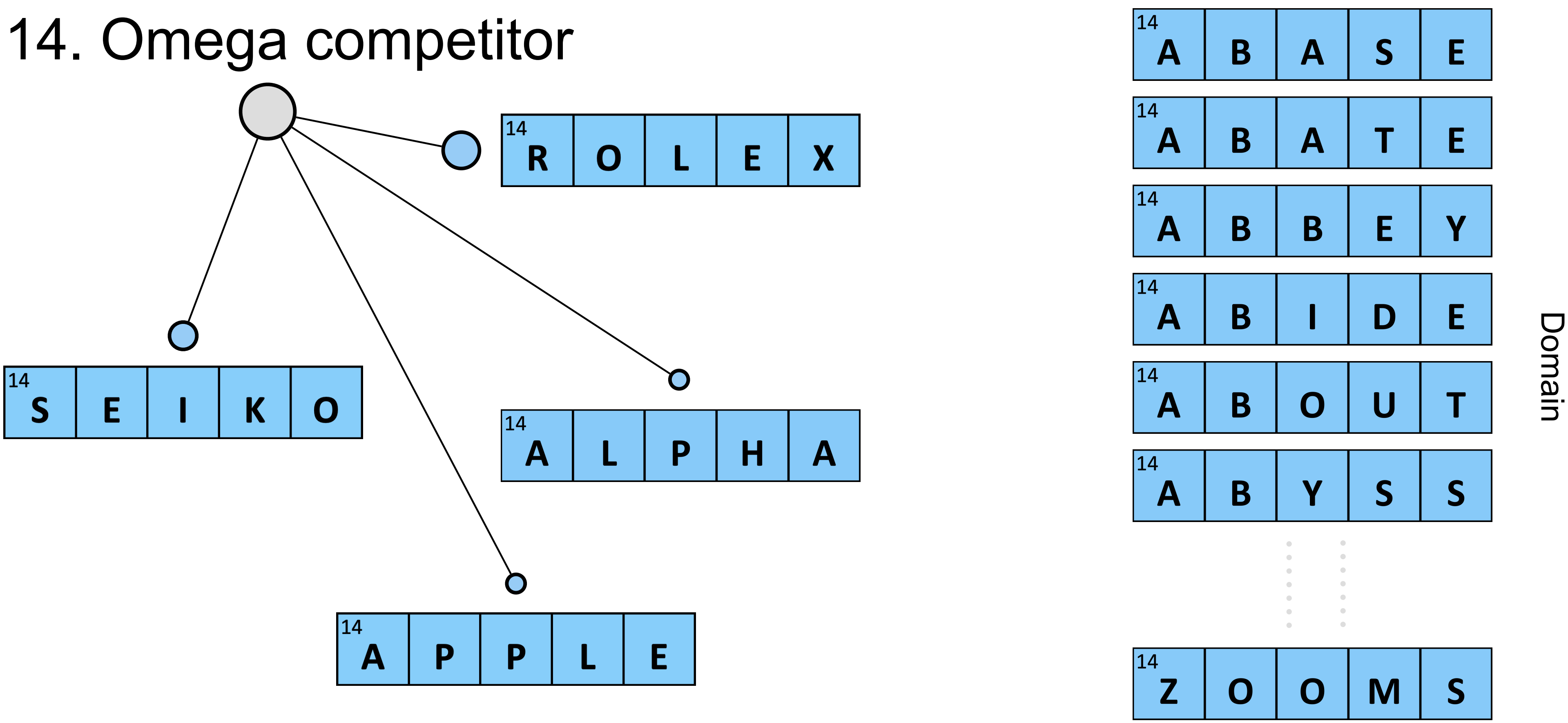

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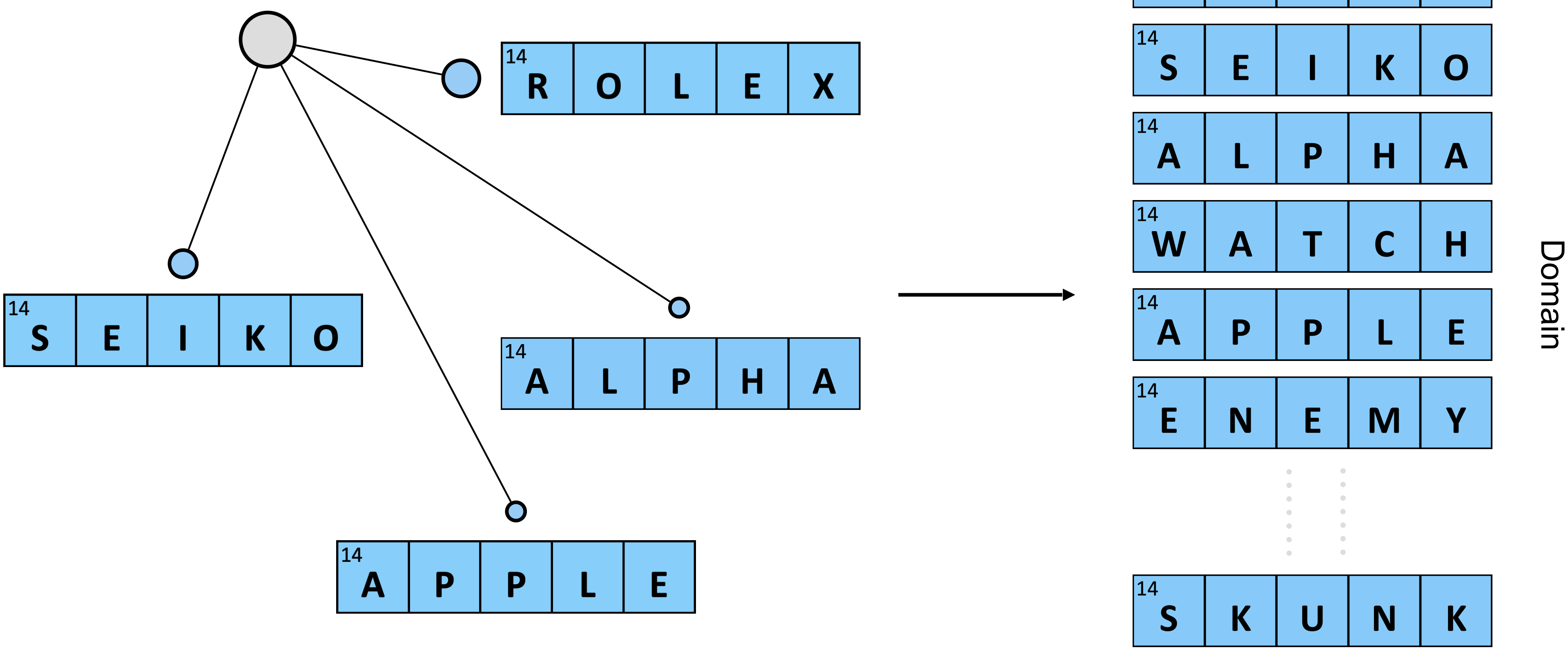

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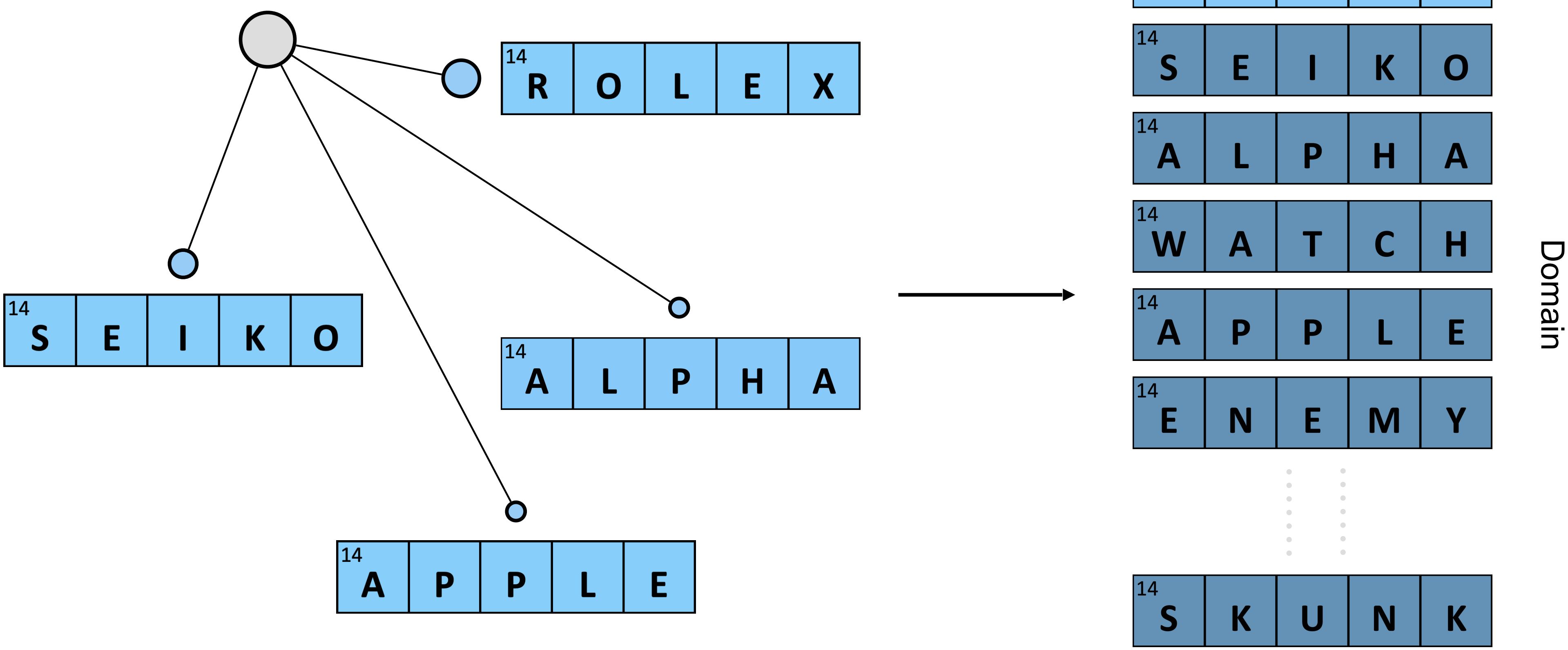

Search Algorithm for Crosswords

- Our approach:
  - Use QA model to get probability distribution over answers
  - Belief propagation (we'll talk more about this later in the class)
  - Initialize solution with greedy search
  - Iterative improvement on individual letters

Iterative Improvement

1 H	2 A	3 H	4 I	5 D		6 S	7 N	8 A	9 P		10 E	11 D	12 S	
13 A	M	O	R	E		14 C	A	N	I		15 V	E	T	16 S
17 M	O	T	E		18 N	I	S	T	S		19 I	C	E	E
20 M	E	T		21 A	C	E		22 I	M	23 A		O	N	E
	24 B	A	25 R	T	O	N			26 O	D	E	D	O	N
27 C	O	K	I	E		28 T	29 S	30 P		31 D	Y	E	S	
32 D	I	E	D		33 S	I	T	U	34 A	T	E			
35 C	D	S		36 Y	E	S	O	R	N	O		37 M	38 F	39 R
			40 T	A	S	T	B G	A	S		41 A	R	E	A
	42 A	43 C	H	Y		44 S	A	A		45 B	I	P	E	D
46 P	R	O	A	M	47 S			48 G	49 A	O	L	E	D	
50 D	E	G		E	E	51 S		52 A	U	G		53 A	B	54 E
55 A	N	N	A		56 T	U	57 R	N	S	I	58 G	N	A	L
59 S	T	A	D		60 I	M	A	C		61 E	A	U	C	I
	62 I	C	S		63 N	O	P	E		64 S	I	T	K	A

Across

1 Beloved, in Arabic

6 [Make it quick!]

10 Masthead listings, for short

13 Italian herbal liqueur

14 "Please, please, please?"

15 People calling the shots at the zoo?

17 Not radical

19 Drink with a dome-shaped lid

20 Where to find the Egyptian Temple of Dendur, with "the"

21 Crush, as a test

22 Cry from a survivor

24 Clara in the National Women's Hall of Fame

26 Stuffed oneself with, facetiously

27 Newswoman Roberts

28 Recipe amt.

31 Doesn't stay natural?

32 Fell off, as laughter

33 Place

Down

1 Soccer star on a 1999 Wheaties box

2 Bloblike

3 Ill-advised opinions

4 Fury

5 Feeling on a lo-o-ong car trip

6 Lab workers

7 Performer with the hit 2006 album "Hip Hop Is Dead"

8 Voting no

9 ___ Beach, Calif.

10 Satan, with "the"

11 Crack, as a secret message

12 Shorthand writers, for short

16 Dated

18 Cpl. or sgt.

23 Expand

25 Purge (of)

27 Atlanta-based health org.

29 Long fur scarfs

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13 A	M	O	R	E		14 C	A	N	I		15 V	E	T	16 S
17 M	O	D	E		18 N	I	S	T	S		19 I	C	E	E
20 M	E	T		21 A	C	E		22 I	M	23 A		O	N	E
	24 B	A	25 R	T	O	N			26 O	D	E	D	O	N
27 C	O	K	I	E		28 T	29 S	30 P		31 D	Y	E	S	
32 D	I	E	D		33 S	I	T	U	34 A	T	E			
35 C	D	S		36 Y	E	S	O	R	N	O		37 M	38 F	39 R
			40 C	A	S	T	B L	A	S		41 A	R	E	A
	42 A	43 C	H	Y		44 S	A	A		45 B	I	P	E	D
46 P	R	O	A	M	47 S			48 H	49 A	I	L	E	D	
50 D	E	G		E	E	51 S		52 A	U	G		53 A	B	54 E
55 A	N	N	A		56 T	U	57 R	N	S	I	58 G	N	A	L
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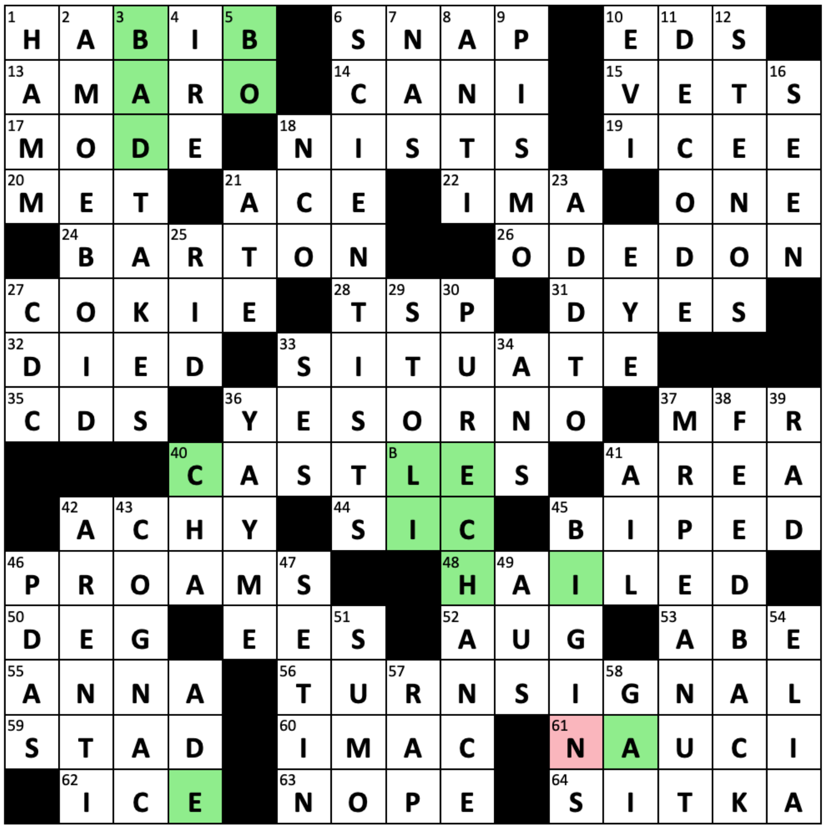

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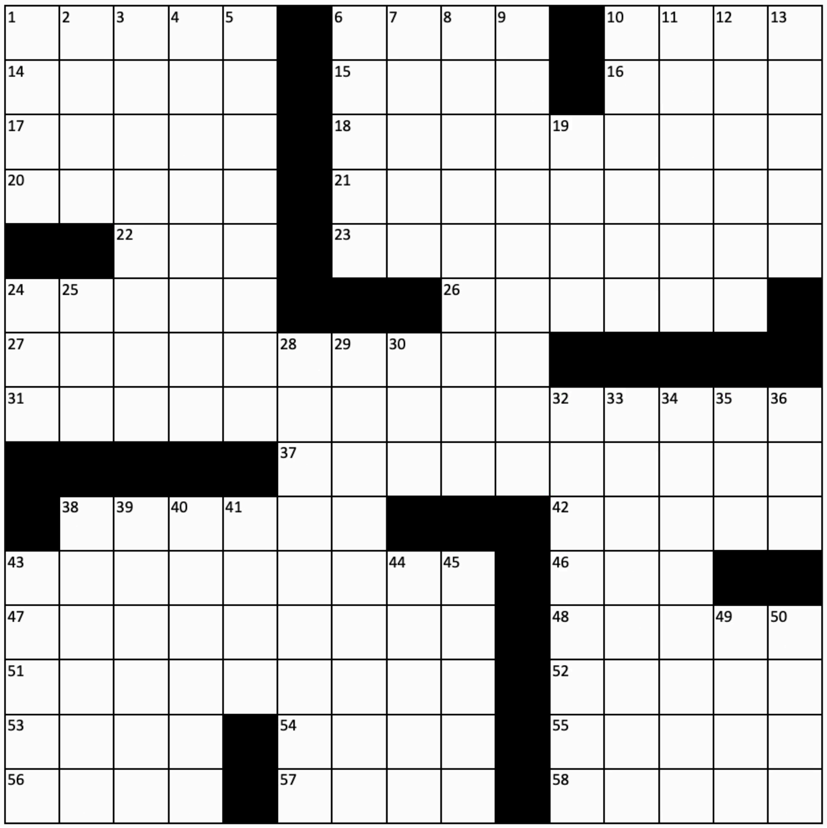
6 [Make it quick!]
10 Masthead listings, for short
13 Italian herbal liqueur
14 "Please, please, please?"
15 People calling the shots at the zoo?
17 Not radical
19 Drink with a dome-shaped lid
20 Where to find the Egyptian Temple of Dendur, with "the"
21 Crush, as a test
22 Cry from a survivor
24 Clara in the National Women's Hall of Fame
26 Stuffed oneself with, facetiously
27 Newswoman Roberts
28 Recipe amt.
31 Doesn't stay natural?
32 Fell off, as laughter
33 Place

Down

1 Soccer star on a 1999 Wheaties box
2 Bloblike
3 Ill-advised opinions
4 Fury
5 Feeling on a lo-o-ong car trip
6 Lab workers
7 Performer with the hit 2006 album "Hip Hop Is Dead"
8 Voting no
9 ___ Beach, Calif.
10 Satan, with "the"
11 Crack, as a secret message
12 Shorthand writers, for short
16 Dated
18 Cpl. or sgt.
23 Expand
25 Purge (of)
27 Atlanta-based health org.
29 Long fur scarfs

The New York Times Crossword from Thursday, January 21, 2021. Constructed by Daniel Mauer and edited by Will Shortz.

Demo



Across

1 Regarding
6 Take back, in a way
10 Start of an aside
14 Omega competitor
15 Something hitting a nerve?
16 Papyrus, e.g.
17 "Take me with you!"
18 Begin flirting with someone, so to speak
20 Assign
21 Hoth, in "Star Wars"
22 ___ rule
23 They don't hold water
24 Feudal figure
26 Panegyric, e.g.
27 Sci-fi enemy collective, perhaps
31 Ones born beginning in the early 2010s
37 They might cut to the chase
38 Electrically balanced, in chemistry
42 Leaders at the Kaaba

Down

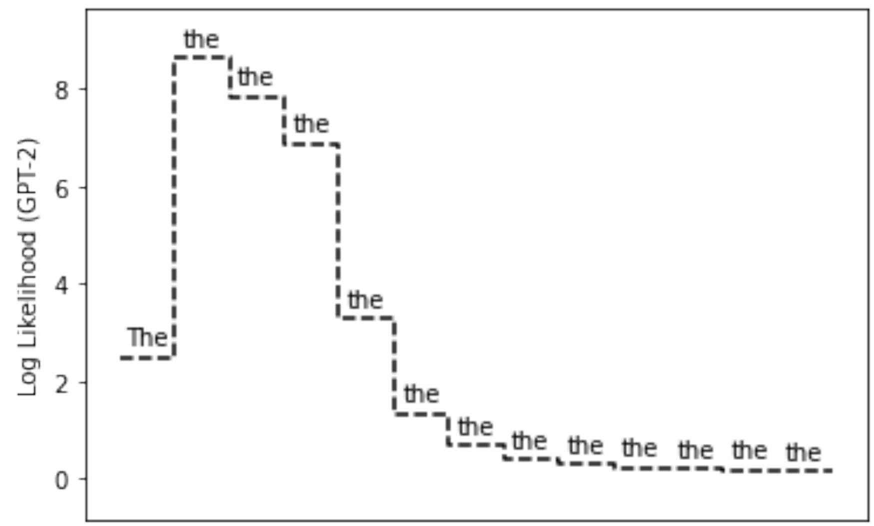
1 Home to 41-Down
2 Convince
3 "I wanna know all the details"
4 Start of a modern inquiry
5 Drink similar to sarsaparilla
6 Co-star of 1984's "Ghostbusters"
7 Right on
8 Aces with aces?
9 Like some households
10 Designer with an eponymous hotel in the Burj Khalifa
11 Currency units in West Africa
12 Grievous
13 "Golden Boy" playwright
19 Actress Shawkat of "Arrested Development"
24 Assignment for an anchor
25 "___ noticed ..."
28 Great Hindu sage
29 Late-Triassic flier

The New York Times Crossword from Saturday, May 22, 2021. Constructed by Sid Sivakumar and Matthew Stock and edited by Will Shortz.

Bonus: Fortuitous Search Errors

- Scoring function can be *incorrectly calibrated*

- Example: decoding from language models
  - Language models assign probabilities to strings of words
  - Decoding uses greedy or beam search
  - But what if you searched over all possible strings (exponentially many) and chose the highest scoring one?
  - Answer: you often end up generating the empty string, or "The the the the the…"



Summary: CSPs

- CSPs are a special kind of search problem:
  - States are partial assignments
  - Goal test defined by constraints
- Basic solution: backtracking search
- Speed-ups:
  - Ordering
  - Filtering
  - Structure – trees are easy!
- Local search often effective in practice