

Q1. Search Party

Annie is throwing a party tonight, but she only has a couple hours to get ready. Luckily, she was recently gifted 4 one-armed robots! She will use them to rearrange her room for the guests. Here are the specifications:

- Her room is modeled as a W -by- L -by- H 3D grid in which there are N objects (which could be anywhere in the grid to start with) that need rearrangement.
- Each object occupies one grid cell, and no two objects can be in the same grid cell. Do not consider any part of the robot an "object."
- At each time-step, one robot may take an action $\in \{\text{move gripper to legal grid cell, close gripper, open gripper}\}$. Moving the gripper does not change whether the gripper was closed/open.
- A robot can move an object by
 1. Moving an open gripper into the object's grid cell
 2. Closing the gripper to grasp the object
 3. Moving to desired location
 4. Opening the gripper to release the object in-hand.
- The robots do not have unlimited range. The arm can move to any point *within* the room that is strictly less than R grid cells from its base per direction along each axis. Explicitly, if $R = 2$ and a robot's base is at $(0,0,0)$, the robot cannot reach $(0,0,2)$ but can reach $(1,1,1)$. Assume $R < W, L, H$.

(a) Annie stations one robot's stationary base at each of the 4 *corners* of the room. Thankfully, she knows where each of the N objects in the room should be and uses that to define the robots' goal. Complete the following expression such that it evaluates to the size of the minimal state space. Please approximate permutations as follows: X permute $Y \approx X^Y$. You may use scalars and the variables: W, L, H, R , and N in your answers.

$$2^{(a)} \cdot N^{(b)} \cdot R^{(c)} \cdot W^{(d)} \cdot L^{(e)} \cdot H^{(f)}$$

(a): <input style="width: 100%; height: 30px;" type="text"/>	(b): <input style="width: 100%; height: 30px;" type="text"/>	(c): <input style="width: 100%; height: 30px;" type="text"/>
(d): <input style="width: 100%; height: 30px;" type="text"/>	(e): <input style="width: 100%; height: 30px;" type="text"/>	(f): <input style="width: 100%; height: 30px;" type="text"/>

(b) Each of the following describes a modification to the scenario previously described and depicted in the figure. **Consider each modification independently (that is, the modifications introduced in (i) are *not* present in (ii)).** For each scenario, give the size of the new minimal state space.

(i) The robots are given wheels, so each base is able to slide along the floor (they still can't jump) from their original corners. That is, at each time-step, a robot has a new action that allows them to move its (once stationary) base arbitrarily far across the floor. When the robot slides its base, the relative arm position and status of the gripper remain the same.

- (ii) *One* robot is defective and can move for a maximum of T timesteps before it must rest for at least S timesteps. You may use S or T in your expression.

Q2. SpongeBob and Pacman (Search Formulation)

Pacman bought a car called the Invisible Boat Mobile, was speeding in Pac-City, and SpongeBob wasn't able to catch him. Now Pacman has run out of gas, his car has stopped, and he is currently hiding out at an undisclosed location. In this problem, you are on the SpongeBob side, tryin' to catch Pacman!

There are still p SpongeBob cars in the Pac-city of dimension m by n . In this problem, **all SpongeBob cars can move, with two distinct integer controls: throttle and steering, but Pacman has to stay stationary**. Once one SpongeBob car takes an action which lands him in the same grid as Pacman, Pacman will be arrested and the game ends.

Throttle: $t_i \in \{1, 0, -1\}$, corresponding to {Gas, Coast, Brake}. This controls the **speed** of the car by determining its acceleration. The integer chosen here will be added to his velocity for the next state. For example, if a SpongeBob car is currently driving at 5 grid/s and chooses Gas (1) he will be traveling at 6 grid/s in the next turn.

Steering: $s_i \in \{1, 0, -1\}$, corresponding to {Turn Left, Go Straight, Turn Right}. This controls the **direction** of the car. For example, if a SpongeBob car is facing North and chooses Turn Left, it will be facing West in the next turn.

- (a) Suppose you can **only control 1 SpongeBob car**, and have absolutely no information about the remainder of $p - 1$ SpongeBob cars, or where Pacman stopped to hide. Also, the SpongeBob cars can travel up to 6 grid/s so $0 \leq v \leq 6$ at all times.

- (i) What is the **tightest upper bound** on the size of state space, if your goal is to use search to plan a sequence of actions that guarantees Pacman is caught, no matter where Pacman is hiding, or what actions other SpongeBob cars take. Please note that your state space representation must be able to represent **all** states in the search space.

- (ii) What is the maximum branching factor? Your answer may contain integers, m, n .

- (iii) Which algorithm(s) is/are guaranteed to return a path passing through all grid locations on the grid, if one exists?

- Depth First Tree Search Breadth First Tree Search
 Depth First Graph Search Breadth First Graph Search

- (iv) Is Breadth First Graph Search guaranteed to return the path with the shortest number of **time steps**, if one exists?

- Yes No

- (b) Now let's suppose you can control **all** p SpongeBob cars at the same time (and know all their locations), but you still have no information about where Pacman stopped to hide

- (i) Now, you still want to search a sequence of actions such that the paths of p SpongeBob car combined **pass through all $m * n$ grid locations**. Suppose the size of the state space in part (a) was N_1 , and the size of the state space in this part is N_p . Please select the correct relationship between N_p and N_1

- $N_p = p * N_1$ $N_p = p^{N_1}$ $N_p = (N_1)^p$ None of the above

- (ii) Suppose the maximum branching factor in part (a) was b_1 , and the maximum branching factor in this part is b_p . Please select the correct relationship between b_p and b_1

- $b_p = p * b_1$ $b_p = p^{b_1}$ $b_p = (b_1)^p$ None of the above