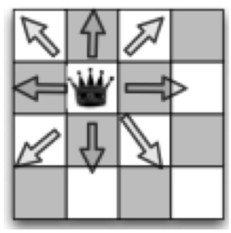
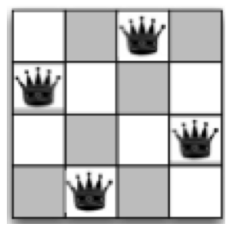
CS 188
Summer 2024

Regular Discussion 1 Solutions

1 n-Queens

Max Friedrich William Bezzel invented the "eight queens puzzle" in 1848: place 8 queens on an 8×8 chess board such that none of them can capture any other. The problem, and the generalized version with n queens on an $n \times n$ chess board, has been studied extensively (a Google Scholar search turns up over 3500 papers on the subject).





Queens can move any number of squares along rows, columns, and diagonals (left); An example solution to the 4-queens problem (right).

a) Formulate n-queens as a search problem. Have each search state be a board, where each square on the board may or may not contain a queen. To get started, we'll allow boards in our state-space to have any configuration of queens (including boards with more or less than n queens, or queens that are able to capture each other).

Start State: An empty board

Goal Test: Returns True iff n queens are on the board such that no two can attack each other

Successor Function: Return all boards with one more queen placed anywhere. Another possibility (see part d) - place queens left to right (i.e. in the first column, then the second column, etc.)

b) How large is the state-space in this formulation? There are n^2 squares, each of which may or may not contain a queen. Therefore there are 2^{n^2} possible states, or 1.8×10^{19} for 8-queens.

c) One way to limit the size of your state space is to limit what your successor function returns. Reformulate your successor function to reduce the effective state-space size. The successor function is limited to return legal boards. Then, the goal test need only check if the board has n queens.

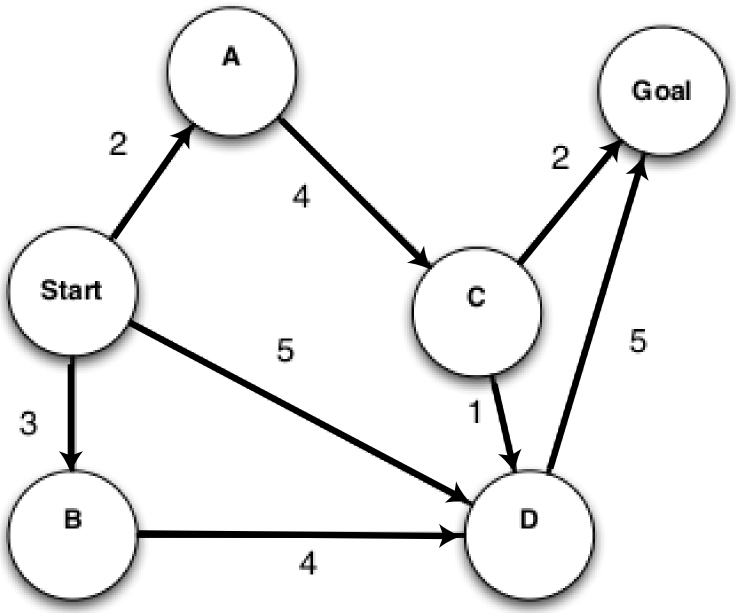
d) Give a more efficient state space representation. How many states are in this new state space? A more effective representation is to have a fixed ordering of queens, such that the queen in the first column is placed first, the queen in the second column is placed second, etc. The representation could be a n-length vector, in which each entry takes a value from 1 to n, or "null". The i-th entry in this vector then represents the row that the queen in column i is in. A "null" entry means that the queen has not been placed.

Since each of the n entries in the vector can take on $n+1$ values, the state space size is $(n+1)^n \approx 4.3 \times 10^7$ for $n = 8$.

To further limit the state space, we can require that queens are placed on the board in order (in this case, left to right). Now we know that if k queens have been placed on the board, the first k entries in the state space are non-null and the last $n-k$ entries are null. This creates a total state space size of $\sum_{k=0}^{n} n^k = \frac{n^{n+1}-1}{n-1} \approx 1.9 \times 10^7$ for $n = 8$.

Finally, by combining this idea with the successor function in part (c), we can further limit the *effective* state size.

2 Search Algorithms in Action



For each of the following graph search strategies, work out the order in which states are expanded, as well as the path returned by graph search. In all cases, assume ties resolve in such a way that states with earlier alphabetical order are expanded first. Remember that in graph search, a state is expanded only once.

a) Depth-first search.
States Expanded: Start, A, C, D, Goal
Path Returned: Start-A-C-D-Goal

b) Breadth-first search.
States Expanded: Start, A, B, D, C, Goal
Path Returned: Start-D-Goal

c) Uniform cost search.
States Expanded: Start, A, B, D, C, Goal
Path Returned: Start-A-C-Goal

3 Utilities

1. What is the expected monetary value (EMV) of the lottery $L(\frac{2}{3}, \$3; \frac{1}{3}, \$6)$?
   $\frac{2}{3} \cdot \$3 + \frac{1}{3} \cdot \$6 = \$4$

2. For each of the following types of utility function, state how the utility of the lottery $U(L)$ compares to the utility of the amount of money equal to the EMV of the lottery, $U(EMV(L))$. Write $<, >, =$, or ? for can't tell.

   (a) U is an arbitrary function.
   $U(L) \;\; ? \;\; U(EMV(L))$

   (b) U is monotonically increasing and its rate of increase is increasing (its second derivative is positive).
   $U(L) > U(EMV(L))$.
   As an example, consider $U = x^2$ from Q2. Then $U(L) = 18$ and $U(EMV(L)) = 4^2 = 16$.

   (c) U is monotonically increasing and linear (its second derivative is zero).
   $U(L) = U(EMV(L))$

   (d) U is monotonically increasing and its rate of increase is decreasing (its second derivative is negative).
   $U(L) < U(EMV(L))$.
   Consider $U = \sqrt{x}$. Then $U(L) = \frac{2}{3} \cdot \sqrt{3} + \frac{1}{3} \cdot \sqrt{6} \approx 1.97$, and $U(EMV(L)) = \sqrt{4} = 2$.