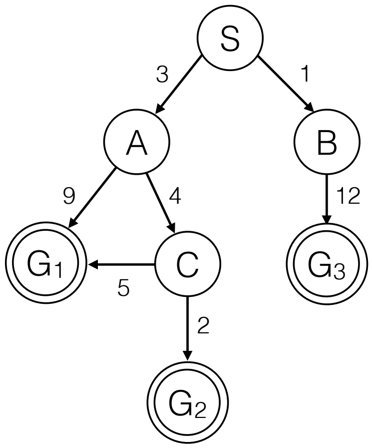
Q1. Search: Algorithms



	A	B	C	S
H-1	0	0	0	0
H-2	6	7	1	7
H-3	7	7	1	7
H-4	4	7	1	7

(a) Consider the search graph and heuristics shown above. Select **all** of the goals that **could** be returned by each of the search algorithms below. For this question, if there is a tie on the fringe, assume the tie is broken **randomly**.

(i) DFS — G_1 ◯ G_2 ◯ G_3 ◯

(ii) BFS — G_1 ◯ G_2 ◯ G_3 ◯

(iii) UCS — G_1 ◯ G_2 ◯ G_3 ◯

(iv) Greedy with H-1 — G_1 ◯ G_2 ◯ G_3 ◯

(v) Greedy with H-2 — G_1 ◯ G_2 ◯ G_3 ◯

(vi) Greedy with H-3 — G_1 ◯ G_2 ◯ G_3 ◯

(vii) A* with H-2 — G_1 ◯ G_2 ◯ G_3 ◯

(viii) A* with H-3 — G_1 ◯ G_2 ◯ G_3 ◯

(b) For each heuristic, indicate whether it is consistent, admissible, or neither (select more than one option if appropriate):

(i) H-1 — Consistent ◯ Admissible ◯ Neither ◯

(ii) H-2 — Consistent ◯ Admissible ◯ Neither ◯

(iii) H-3 — Consistent ◯ Admissible ◯ Neither ◯

(iv) H-4 — Consistent ◯ Admissible ◯ Neither ◯

Q2. Searching with Heuristics

Consider the A* searching process on the connected undirected graph, with starting node S and the goal node G. Suppose the cost for each connection edge is **always positive**. We define $h^*(X)$ as the shortest (optimal) distance to G from a node X.

Answer Questions (a), (b) and (c). You may want to solve Questions (a) and (b) at the same time.

(a) Suppose h is an **admissible** heuristic, and we conduct A* **tree search** using heuristic h' and finally find a solution. Let C be the cost of the found path (directed by h', defined in part (a)) from S to G

(i) Choose **one best** answer for each condition below.

1. If $h'(X) = \frac{1}{2}h(X)$ for all Node X, then ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
2. If $h'(X) = \frac{h(X)+h^*(X)}{2}$ for all Node X, then ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
3. If $h'(X) = h(X) + h^*(X)$ for all Node X, then ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
4. If we define the set $K(X)$ for a node X as all its neighbor nodes Y satisfying $h^*(X) > h^*(Y)$, and the following always holds

$$h'(X) \leq \begin{cases} \min_{Y \in K(X)} h'(Y) - h(Y) + h(X) & \text{if } K(X) \neq \emptyset \\ h(X) & \text{if } K(X) = \emptyset \end{cases}$$

then, ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$

5. If K is the same as above, we have

$$h'(X) = \begin{cases} \min_{Y \in K(X)} h(Y) + cost(X,Y) & \text{if } K(X) \neq \emptyset \\ h(X) & \text{if } K(X) = \emptyset \end{cases}$$

where $cost(X,Y)$ is the cost of the edge connecting X and Y,
then, ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$

6. If $h'(X) = \min_{Y \in K(X)+\{X\}} h(Y)$ (K is the same as above), ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$

(ii) In which of the conditions above, h' is still **admissible** and for sure to dominate h? Check all that apply. Remember we say h_1 dominates h_2 when $h_1(X) \geq h_2(X)$ holds for all X. ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

(b) Suppose h is a **consistent** heuristic, and we conduct A* **graph search** using heuristic h' and finally find a solution.

(i) Answer exactly the same questions for each conditions in Question (a)(i).

1. ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
2. ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
3. ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
4. ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
5. ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
6. ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$

(ii) In which of the conditions above, h' is still **consistent** and for sure to dominate h? Check all that apply.

☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

(c) Suppose h is an **admissible** heuristic, and we conduct A* **tree search** using heuristic h' and finally find a solution. If $\epsilon > 0$, and X_0 is a node in the graph, and h' is a heuristic such that

$$h'(X) = \begin{cases} h(X) & \text{if } X = X_0 \\ h(X) + \epsilon & \text{otherwise} \end{cases}$$

- Alice claims h' can be inadmissible, and hence $C = h^*(S)$ does not always hold.
- Bob instead thinks the node expansion order directed by h' is the same as the heuristic h'', where

$$h''(X) = \begin{cases} h(X) - \epsilon & \text{if } X = X_0 \\ h(X) & \text{if otherwise} \end{cases}$$

  Since h'' is admissible and will lead to $C = h^*(S)$, and so does h'. Hence, $C = h^*(S)$ always holds.

The two conclusions (underlined) apparently contradict with each other, and **only exactly one of them are correct and the other is wrong**. Choose the **best** explanation from below - which student's conclusion is wrong, and why are they wrong?

◯ Alice's conclusion is wrong, because the heuristic h' is always admissible.

◯ Alice's conclusion is wrong, because an inadmissible heuristics does not necessarily always lead to the failure of the optimality when conducting A* tree search.

◯ Alice's conclusion is wrong, because of another reason that is not listed above.

◯ Bob's conclusion is wrong, because the node visiting expansion ordering of h'' during searching might not be the same as h'.

◯ Bob's conclusion is wrong, because the heuristic h'' might lead to an incomplete search, regardless of its optimally property.

◯ Bob's conclusion is wrong, because of another reason that is not listed above.