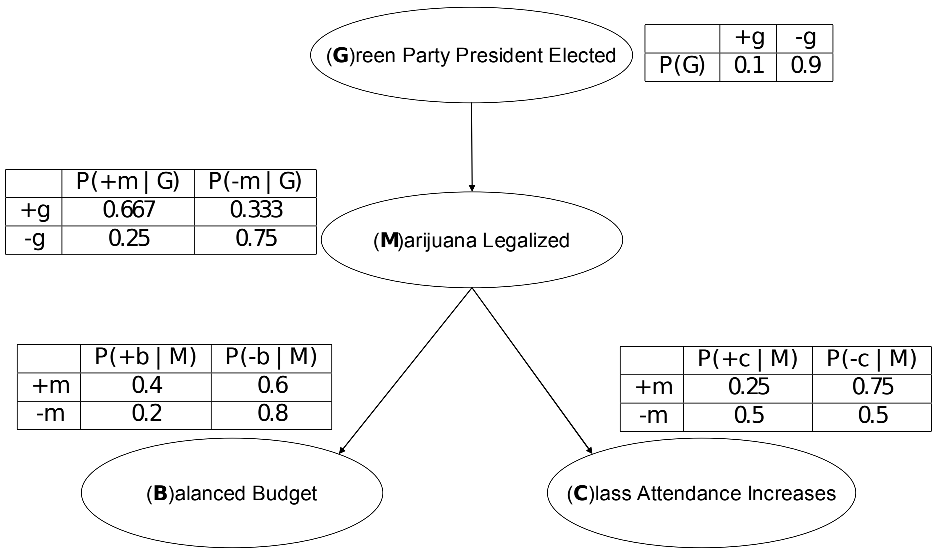
Regular Discussion 3b Solutions

Q1. Bayes Nets: Green Party President

In a parallel universe the Green Party is running for presidency. Whether a Green Party President is elected (G) will have an effect on whether marijuana is legalized (M), which then influences whether the budget is balanced (B), and whether class attendance increases (C). Armed with the power of probability, the analysts model the situation with the Bayes Net below.

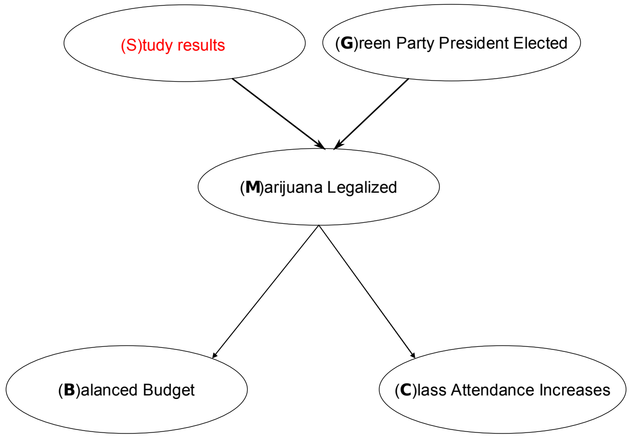


1. The full joint distribution is given below. Fill in the missing values.

G	M	B	C	$P(G, M, B, C)$	G	M	B	C	$P(G, M, B, C)$
+	+	+	+	1/150	-	+	+	+	9/400
+	+	+	-	1/50	-	+	+	-	27/400
+	+	-	+	1/100	-	+	-	+	27/800
+	+	-	-	3/100	-	+	-	-	81/800
+	-	+	+	1/300	-	-	+	+	27/400
+	-	+	-	1/300	-	-	+	-	27/400
+	-	-	+	1/75	-	-	-	+	27/100
+	-	-	-	1/75	-	-	-	-	27/100

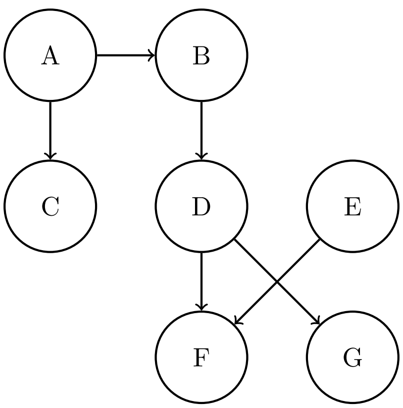
2. Now, add a node S to the Bayes net that reflects the possibility that a new scientific study could influence the probability that marijuana is legalized. Assume that the study does not directly influence B or C. Draw the new Bayes net below. Which CPT or CPT's need to be modified?

   $P(M|G)$ will become $P(M|G, S)$, and will contain 8 entries instead of 4.



2 Bayes' Nets: Representation and Independence

Parts (a) and (b) pertain to the following Bayes' Net.



(a) Express the joint probability distribution as a product of terms representing individual conditional probabilities tables associated with the Bayes Net.

$P(A)P(C|A)P(B|A)P(D|B)P(E)P(F|D,E)P(G|D)$

(b) Assume each node can take on 4 values. How many entries do the factors at A, D, and F have?

A: 4

D: 4^2

F: 4^3

Consider the following probability distribution tables. The joint distribution $P(A, B, C, D)$ is equal to the product of these probability distribution tables.

A	$P(A)$
+a	0.8
–a	0.2

A	B	$P(B\|A)$
+a	+b	0.9
+a	–b	0.1
–a	+b	0.6
–a	–b	0.4

B	C	$P(C\|B)$
+b	+c	0.8
+b	–c	0.2
–b	+c	0.8
–b	–c	0.2

C	D	$P(D\|C)$
+c	+d	0.25
+c	–d	0.75
–c	+d	0.5
–c	–d	0.5

(c) State all non-conditional independence assumptions that are implied by the probability distribution tables.

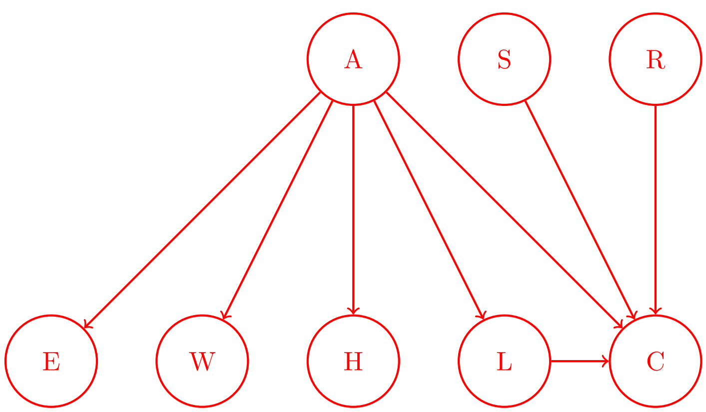
From the tables, we have $A \not\perp\!\!\!\perp B$ and $C \not\perp\!\!\!\perp D$. Then, we have every remaining pair of variables: $A \perp\!\!\!\perp C, A \perp\!\!\!\perp D, B \perp\!\!\!\perp C, B \perp\!\!\!\perp D$

You are building advanced safety features for cars that can warn a driver if they are falling asleep (A) and also calculate the probability of a crash (C) in real time. You have at your disposal 6 sensors (random variables):

- E: whether the driver's eyes are open or closed
- W: whether the steering wheel is being touched or not
- L: whether the car is in the lane or not
- S: whether the car is speeding or not
- H: whether the driver's heart rate is somewhat elevated or resting
- R: whether the car radar detects a close object or not

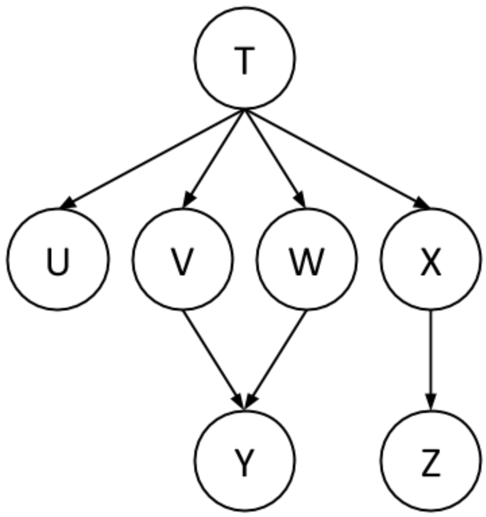
A influences $\{E, W, H, L, C\}$. C is influenced by $\{A, S, L, R\}$.

(d) Draw the Bayes Net associated with the description above by adding edges between the provided nodes where appropriate.



3 D-Separation

Indicate whether each of the following conditional independence relationships is guaranteed to be true in the Bayes Net below. If the independence relationship does not hold, identify all active (d-connected) paths in the graph.



1. $U \perp\!\!\!\perp X$

   Not guaranteed, path U-T-X is active

2. $U \perp\!\!\!\perp X | T$

   Guaranteed

3. $V \perp\!\!\!\perp W | Y$

   Not guaranteed, paths V-T-W and V-Y-W are both active

4. $V \perp\!\!\!\perp W | T$

   Guaranteed

5. $T \perp\!\!\!\perp Y | V$

   Not guaranteed, path T-W-Y is active

6. $Y \perp\!\!\!\perp Z | W$

   Not guaranteed, path Y-V-T-X-Z is active

7. $Y \perp\!\!\!\perp Z | T$

   Guaranteed, with T being observed, there are no active paths from Y to Z