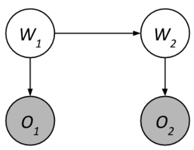
CS 188
Summer 2024

Regular Discussion 8 Solutions

1 Particle Filtering

Let's use Particle Filtering to estimate the distribution of $P(W_2|O_1 = a, O_2 = b)$. Here's the HMM again. O_1 and O_2 are supposed to be shaded.



W_1	$P(W_1)$
0	0.3
1	0.7

W_t	W_{t+1}	$P(W_{t+1}|W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t|W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

We start with two particles representing our distribution for W_1.
$P_1 : W_1 = 0$
$P_2 : W_1 = 1$
Use the following random numbers to run particle filtering:

$$[0.22, 0.05, 0.33, 0.20, 0.84, 0.54, 0.79, 0.66, 0.14, 0.96]$$

(a) **Observe**: Compute the weight of the two particles after evidence $O_1 = a$.

$w(P_1) = P(O_t = a|W_t = 0) = 0.9$
$w(P_2) = P(O_t = a|W_t = 1) = 0.5$

(b) **Resample**: Using the random numbers, resample P_1 and P_2 based on the weights.

We now sample from the weighted distribution we found above. Using the first two random samples, we find:
$P_1 = sample(weights, 0.22) = 0$
$P_2 = sample(weights, 0.05) = 0$

(c) **Predict**: Sample P_1 and P_2 from applying the time update.

$P_1 = sample(P(W_{t+1}|W_t = 0), 0.33) = 0$
$P_2 = sample(P(W_{t+1}|W_t = 0), 0.20) = 0$

(d) **Update**: Compute the weight of the two particles after evidence $O_2 = b$.

$w(P_1) = P(O_t = b|W_t = 0) = 0.1$
$w(P_2) = P(O_t = b|W_t = 0) = 0.1$

(e) **Resample**: Using the random numbers, resample P_1 and P_2 based on the weights.

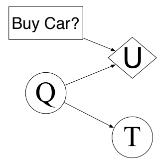
Because both of our particles have $X = 0$, resampling will still leave us with two particles with $X = 0$.
$P_1 = 0$
$P_2 = 0$

(f) What is our estimated distribution for $P(W_2|O_1 = a, O_2 = b)$?

$P(W_2 = 0|O_1 = a, O_2 = b) = 2/2 = 1$
$P(W_2 = 1|O_1 = a, O_2 = b) = 0/2 = 0$

2 Decision Networks and VPI

A buyer is deciding whether to buy a certain used car. The car may be good quality ($Q = +q$) or bad quality ($Q = \neg q$). A test (T) costs \$50 and can help to figure out the quality of the car. There are only two outcomes for the test: T = pass or T = fail. The car costs \$1,500, and its market value is \$2,000 if it is good quality; if not, \$700 in repairs will be needed to make it good quality. The buyer's estimate is that the car has 70% chance of being good quality.



1. Calculate the expected net gain from buying the car, given no test.

$$\begin{aligned} EU(\text{buy}) &= P(Q=+q)\cdot U(+q,\text{buy}) + P(Q=\neg q)\cdot U(\neg q, buy) \\ &= .7\cdot 500 + 0.3\cdot -200 = 290 \end{aligned}$$

2. Tests can be described by the probability that the car will pass or fail the test given that the car is in good or bad shape. We have the following information:

$$P(T=\text{pass}|Q=+q) = 0.9$$
$$P(T=\text{pass}|Q=\neg q) = 0.2$$

Calculate the probability that the car will pass (or fail) its test, and then the probability that it is in good (or bad) shape given each possible test outcome.

$$\begin{aligned} P(T=\text{pass}) &= \sum_q P(T=\text{pass}, Q=q) \\ &= P(T=\text{pass}|Q=+q)P(Q=+q) + P(T=\text{pass}|Q=\neg q)P(Q=\neg q) \\ &= 0.69 \\ P(T=fail) &= 0.31 \\ P(Q=+q|T=\text{pass}) &= \frac{P(T=\text{pass}|Q=+q)P(Q=+q)}{P(T=\text{pass})} \\ &= \frac{0.9\cdot 0.7}{0.69} = \frac{21}{23} \approx 0.91 \\ P(Q=+q|T=\text{fail}) &= \frac{P(T=\text{fail}|Q=+q)P(Q=+q)}{P(T=\text{fail})} \\ &= \frac{0.1\cdot 0.7}{0.31} = \frac{7}{31} \approx 0.22 \end{aligned}$$

3. Calculate the optimal decisions given either a pass or a fail, and their expected utilities.

$$\begin{aligned} EU(\text{buy}|T=pass) &= P(Q=+q|T=\text{pass})U(+q,\text{buy}) + P(Q=\neg q|T=\text{pass})U(\neg q,\text{buy}) \\ &\approx 0.91\cdot 500 + 0.09\cdot(-200) \approx 437 \\ EU(\text{buy}|T=\text{fail}) &= P(Q=+q|T=\text{fail})U(+q,\text{buy}) + P(Q=\neg q|T=\text{fail})U(\neg q,\text{buy}) \\ &\approx 0.22\cdot 500 + 0.78\cdot(-200) = -46 \\ EU(\neg\text{buy}|T=pass) &= 0 \\ EU(\neg\text{buy}|T=fail) &= 0 \end{aligned}$$

Therefore: $MEU(T = \text{pass}) = 437$ (with buy) and $MEU(T = \text{fail}) = 0$ (using $\neg$buy)

4. Calculate the value of (perfect) information of the test. Should the buyer pay for a test?

$$\begin{aligned} VPI(T) &= (\sum_t P(T = t) MEU(T = t)) - MEU(\phi) \\ &= 0.69 \cdot 437 + 0.31 \cdot 0 - 290 \approx 11.53 \end{aligned}$$

You shouldn't pay for it, since the cost is \$50.