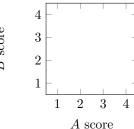
1 Perceptron

You want to predict if movies will be profitable based on their screenplays. You hire two critics A and B to read a script you have and rate it on a scale of 1 to 4. The critics are not perfect; here are five data points including the critics' scores and the performance of the movie:

#	Movie Name	A	В	Profit?
1	Pellet Power	1	1	-
2	Ghosts!	3	2	+
3	Pac is Bac	2	4	+
4	Not a Pizza	3	4	+
5	Endless Maze	2	3	-



- (a) First, you would like to examine the linear separability of the data. Plot the data on the 2D plane above; label profitable movies with + and non-profitable movies with and determine if the data are linearly separable.
- (b) Now you decide to use a perceptron to classify your data. Suppose you directly use the scores given above as features, together with a bias feature. That is $f_0 = 1$, $f_1 =$ score given by A and $f_2 =$ score given by B.

Run one pass through the data with the perceptron algorithm, filling out the table below. Go through the data points in order, e.g. using data point #1 at step 1.

step	Weights	Score	Correct?
1	[-1, 0, 0]	$-1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 = -1$	yes
2			
3			
4			
5			

Final weights:

(c) Have weights been learned that separate the data?

- (d) More generally, irrespective of the training data, you want to know if your features are powerful enough to allow you to handle a range of scenarios. Circle the scenarios for which a perceptron using the features above can indeed perfectly classify movies which are profitable according to the given rules:
 - (a) Your reviewers are awesome: if the total of their scores is more than 8, then the movie will definitely be profitable, and otherwise it won't be.

- (b) Your reviewers are art critics. Your movie will be profitable if and only if each reviewer gives either a score of 2 or a score of 3.
- (c) Your reviewers have weird but different tastes. Your movie will be profitable if and only if both reviewers agree.

2 Maximum Likelihood

A Geometric distribution is a probability distribution of the number X of Bernoulli trials needed to get one success. It depends on a parameter p, which is the probability of success for each individual Bernoulli trial. Think of it as the number of times you must flip a coin before flipping heads. The probability is given as follows:

$$P(X = k) = p(1 - p)^{k - 1}$$
(1)

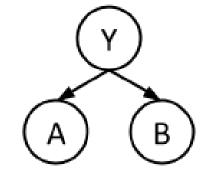
p is the parameter we wish to estimate.

We observe the following samples from a Geometric distribution: $x_1 = 5$, $x_2 = 8$, $x_3 = 3$, $x_4 = 5$, $x_5 = 7$. What is the maximum likelihood estimate for p?

3 Naive Bayes

In this question, we will train a Naive Bayes classifier to predict class labels Y as a function of input features A and B. Y, A, and B are all binary variables, with domains 0 and 1. We are given 10 training points from which we will estimate our distribution.

A										
B	1	0	0	1	1	1	1	0	1	1
Y	1	1	0	0	0	1	1	0	0	0



1. What are the maximum likelihood estimates for the tables P(Y), P(A|Y), and P(B|Y)?

Y	P(Y)
0	
1	

A	Y	P(A Y)
0	0	
1	0	
0	1	
1	1	

В	Y	P(B Y)
0	0	
1	0	
0	1	
1	1	

- 2. Consider a new data point (A = 1, B = 1). What label would this classifier assign to this sample?
- 3. Let's use Laplace Smoothing to smooth out our distribution. Compute the new distribution for P(A|Y) given Laplace Smoothing with k=2.

A	Y	P(A Y)
0	0	
1	0	
0	1	
1	1	