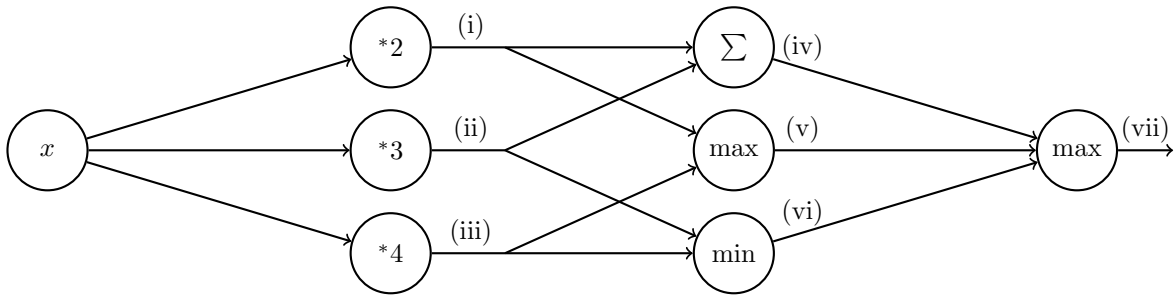


Q1. Backpropagation

- (a) Perform forward propagation on the neural network below for $x = 1$ by filling in the values in the table. Note that (i), ..., (vii) are outputs after performing the appropriate operation as indicated in the node.

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
2	3	4	5	4	3	5

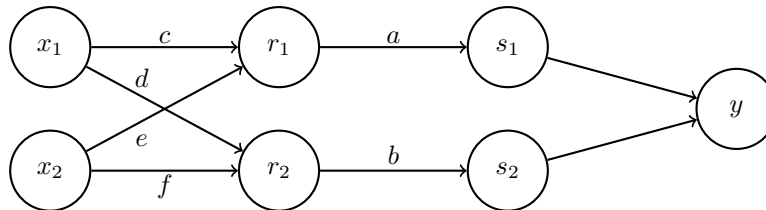


- (b) Below is a neural network with weights a, b, c, d, e, f . The inputs are x_1 and x_2 . The first hidden layer computes $r_1 = \max(c \cdot x_1 + e \cdot x_2, 0)$ and $r_2 = \max(d \cdot x_1 + f \cdot x_2, 0)$. The second hidden layer computes $s_1 = \frac{1}{1 + \exp(-a \cdot r_1)}$ and $s_2 = \frac{1}{1 + \exp(-b \cdot r_2)}$. The output layer computes $y = s_1 + s_2$. Note that the weights a, b, c, d, e, f are indicated along the edges of the neural network here.

Suppose the network has inputs $x_1 = 1, x_2 = -1$.

The weight values are $a = 1, b = 1, c = 4, d = 1, e = 2, f = 2$.

Forward propagation then computes $r_1 = 2, r_2 = 0, s_1 = 0.9, s_2 = 0.5, y = 1.4$. Note: some values are rounded.



Using the values computed from forward propagation, use backpropagation to numerically calculate the following partial derivatives. Write your answers as a single number (not an expression). You do not need a calculator. Use scratch paper if needed.

Hint: For $g(z) = \frac{1}{1 + \exp(-z)}$, the derivative is $\frac{\partial g}{\partial z} = g(z)(1 - g(z))$.

$\frac{\partial y}{\partial a}$	$\frac{\partial y}{\partial b}$	$\frac{\partial y}{\partial c}$	$\frac{\partial y}{\partial d}$	$\frac{\partial y}{\partial e}$	$\frac{\partial y}{\partial f}$
0.18	0	0.09	0	-0.09	0

$$\begin{aligned}
\frac{\partial y}{\partial a} &= \frac{\partial y}{\partial s_1} \frac{\partial s_1}{\partial a} \\
&= 1 \cdot \frac{\partial g(a \cdot r_1)}{\partial a} \\
&= r_1 \cdot g(a \cdot r_1)(1 - g(a \cdot r_1)) \\
&= r_1 \cdot s_1(1 - s_1) \\
&= 2 \cdot 0.9 \cdot (1 - 0.9) \\
&= 0.18
\end{aligned}$$

$$\begin{aligned}
\frac{\partial y}{\partial b} &= \frac{\partial y}{\partial s_2} \frac{\partial s_2}{\partial b} \\
&= 1 \cdot \frac{\partial g(b \cdot r_2)}{\partial b} \\
&= r_2 \cdot g(b \cdot r_2)(1 - g(b \cdot r_2)) \\
&= r_2 \cdot s_2(1 - s_2) \\
&= 0 \cdot 0.5(1 - 0.5) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial y}{\partial c} &= \frac{\partial y}{\partial s_1} \frac{\partial s_1}{\partial r_1} \frac{\partial r_1}{\partial c} \\
&= 1 \cdot [a \cdot g(a \cdot r_1)(1 - g(a \cdot r_1))] \cdot x_1 \\
&= [a \cdot s_1(1 - s_1)] \cdot x_1 \\
&= [1 \cdot 0.9(1 - 0.9)] \cdot 1 \\
&= 0.09
\end{aligned}$$

$$\begin{aligned}
\frac{\partial y}{\partial d} &= \frac{\partial y}{\partial s_2} \frac{\partial s_2}{\partial r_2} \frac{\partial r_2}{\partial d} \\
&= \frac{\partial y}{\partial s_2} \frac{\partial s_2}{\partial r_2} \cdot 0 \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial y}{\partial e} &= \frac{\partial y}{\partial s_1} \frac{\partial s_1}{\partial r_1} \frac{\partial r_1}{\partial e} \\
&= 1 \cdot [a \cdot g(a \cdot r_1)(1 - g(a \cdot r_1))] \cdot x_2 \\
&= [a \cdot s_1(1 - s_1)] \cdot x_2 \\
&= [1 \cdot 0.9(1 - 0.9)] \cdot -1 \\
&= -0.09
\end{aligned}$$

$$\begin{aligned}
\frac{\partial y}{\partial f} &= \frac{\partial y}{\partial s_2} \frac{\partial s_2}{\partial r_2} \frac{\partial r_2}{\partial f} \\
&= \frac{\partial y}{\partial s_2} \frac{\partial s_2}{\partial r_2} \cdot 0 \\
&= 0
\end{aligned}$$