Q7. [12 pts] Games and ML: BeatBlue

Pacman wants to design an agent that can play chess and beat his older (more successful) brother, DeepBlue.

First, Pacman would like to design a machine learning algorithm that takes in a board state and outputs a real number between 0.00 (for states where Pacman is losing) and 1.00 (for states where Pacman is winning).

(a) [2 pts] Pacman starts by asking experts to manually assign values to board states. Select all true statements.

- We can use the manually-assigned values as our training dataset.
- We can use the manually-assigned values as our testing dataset.
- Since we have values assigned by experts, the machine learning algorithm provides no additional benefit.
- None of the above

(b) [1 pt] Pacman suggests using a perceptron for this problem. Is it reasonable to use a perceptron for this problem?

- Yes, because this is a classification problem.
- Yes, because the training data is linearly separable.
- No, because this is a regression problem, and perceptrons output discrete classes, not continuous numbers.
- No, because perceptrons should never be used when the data is not linearly separable.

Pacman decides to represent the chess board state as a 64-dimensional vector. Pacman designs a fully-connected, feed-forward neural network with the following architecture:

\[
\begin{align*}
h &= \text{ReLU}(x \cdot W_1 + b_1) \\
\hat{y} &= \text{ReLU}(h \cdot W_2 + b_2)
\end{align*}
\]

This network takes in a 1 × 64 vector, \(x\), and outputs a scalar real number, \(\hat{y}\).

Pacman sets the hidden layer size to be 128. In other words, \(h\) has dimensions 1 × 128.

(c) [1 pt] What are the dimensions of \(W_1\)?

- \(1 \times 1\)
- \(64 \times 1\)
- \(128 \times 1\)
- \(8192 \times 1\)
- \(128 \times 128\)

(d) [1 pt] What are the dimensions of \(W_2\)?

- \(1 \times 1\)
- \(64 \times 1\)
- \(128 \times 1\)
- \(128 \times 128\)
- \(8192 \times 1\)
- \(64 \times 64\)
- \(64 \times 128\)

(e) [1 pt] Pacman considers changing the second layer of the neural network. Which of these proposed changes is best for this problem?

Reminders: 
\[
\begin{align*}
\text{ReLU}(x) &= \max(x, 0) \\
\sigma(x) &= \frac{1}{1+e^{-x}} \\
\text{sgn}(x) &= \begin{cases} 
1 & x > 0 \\
0 & x = 0 \\
-1 & x < 0 
\end{cases}
\end{align*}
\]

- \(\hat{y} = \text{ReLU}(h \cdot W_2 + b_2)\)
- \(\hat{y} = \sigma(h \cdot W_2 + b_2)\)
- \(\hat{y} = \text{sgn}(h \cdot W_2 + b_2)\)
- \(\hat{y} = h \cdot W_2 + b_2\)
Pacman now has a neural network $\hat{y} = f(x)$ that takes in board states $(x)$ and outputs values ($\hat{y}$), and would like to use the network to select actions in the chess game.

(f) [1 pt] Let $s' = g(s, a)$ represent the successor function that takes in a state $s$ and action $a$, and outputs a successor state $s'$. Which of the following expressions represents a reflex agent’s optimal action from state $s$, based on the values outputted by the neural network?

- $\text{arg max}_s f(s)$
- $\text{max}_a g(s, a)$
- $\text{arg max}_a f(g(s, a))$
- $\sum_a f(g(s, a))$

Finally, Pacman decides to run depth-limited minimax search and use the neural network as an evaluation function.

(g) [1 pt] Is it reasonable to use the neural network as an evaluation function?

- Yes, because the neural network maps states to real-numbered values.
- Yes, because the network is guaranteed to correctly identify terminal states where the game is over.
- No, because it would be more efficient and accurate to expand the entire minimax tree to the terminal states.
- No, because evaluation functions should map states to actions.

(h) [1 pt] Is it possible to use alpha-beta pruning in this problem?

- Yes, pruning works even if the neural network output was unbounded.
- Yes, but only because the neural network only outputs numbers between 0 and 1.
- No, because pruning requires you to know the values at all leaf nodes in advance.
- No, because the values outputted by the neural network are not guaranteed to be correct.

(i) [1 pt] Is it better to spend more time on training the neural network, or expanding more layers of the game tree?

- Training the neural network
- Expanding the game tree
- Not enough information

(j) [2 pts] Recall that in Monte Carlo tree search, we allocate more rollouts to game states that are more promising (i.e. better utility for Pacman).

Inspired by this idea, Pacman considers running gradient descent for a longer time when using the neural network to evaluate game states that are more promising. Would this idea work?

- Yes, because this causes the neural network to focus its training on promising game states.
- Yes, because training for a longer time leads to better evaluations.
- No, because running gradient descent for too long always leads to overfitting.
- No, because gradient descent runs during training, not evaluation.
Q2. [15 pts] Search & Games: Shelving Books

At the library, Carolyn plans to place 36 books on a bookshelf. The bookshelf consists of 6 rows, labeled Row 0 to Row 5, and each row can hold exactly 6 books.

The 36 books are labeled from $b_0$ to $b_{35}$. Furthermore, each spot on the bookshelf is labeled from 0 to 35. Here is a diagram of what the bookshelf looks like with all the books in their correct positions:

<table>
<thead>
<tr>
<th>Row 0</th>
<th>b₀</th>
<th>b₁</th>
<th>b₂</th>
<th>b₃</th>
<th>b₄</th>
<th>b₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 1</td>
<td>b₆</td>
<td>b₇</td>
<td>b₈</td>
<td>b₉</td>
<td>b₁₀</td>
<td>b₁₁</td>
</tr>
<tr>
<td>Row 2</td>
<td>b₁₂</td>
<td>b₁₃</td>
<td>b₁₄</td>
<td>b₁₅</td>
<td>b₁₆</td>
<td>b₁₇</td>
</tr>
<tr>
<td>Row 3</td>
<td>b₁₈</td>
<td>b₁₉</td>
<td>b₂₀</td>
<td>b₂₁</td>
<td>b₂₂</td>
<td>b₂₃</td>
</tr>
<tr>
<td>Row 4</td>
<td>b₂₄</td>
<td>b₂₅</td>
<td>b₂₆</td>
<td>b₂₇</td>
<td>b₂₈</td>
<td>b₂₉</td>
</tr>
<tr>
<td>Row 5</td>
<td>b₃₀</td>
<td>b₃₁</td>
<td>b₃₂</td>
<td>b₃₃</td>
<td>b₃₄</td>
<td>b₃₅</td>
</tr>
</tbody>
</table>

Carolyn first places all 36 books onto the bookshelf in a random order.

Carolyn can move books around using these three actions (each action costs 1):

1. Shift all books in any row to the right by 1, with the rightmost book moving to the first spot on that row.
   - For instance, using this action on Row 3, $\{b_{18}, b_{19}, b_{20}, b_{21}, b_{22}, b_{23}\}$, turns the row into $\{b_{23}, b_{18}, b_{19}, b_{20}, b_{21}, b_{22}\}$.

2. Shift all books in any row to the left by 1, with the leftmost book moving to the last spot on that row.
   - For instance, using this action on Row 4, $\{b_{24}, b_{25}, b_{26}, b_{27}, b_{28}, b_{29}\}$, turns the row into $\{b_{25}, b_{26}, b_{27}, b_{28}, b_{29}, b_{24}\}$.

3. Swap any two books that are on different rows.
   - For instance, the books at positions 6 and 22 can be swapped, and so can the books at positions 25 and 31, but the books at positions 24 and 26 cannot be swapped.

Carolyn's goal is to sort the books by putting all the books in their correct positions. In other words, for $0 \leq i \leq 35$, Carolyn wants book $b_i$ to be placed on the $i$th position on the bookshelf.

Carolyn decides to model this as a search problem.

(a) [2 pts] Carolyn claims that the size of the state space for this problem is $2^{36}$, while a coworker Julia claims that it is $36!$ instead. Who is correct, and why? Explain your choice in two sentences or fewer.

- Carolyn is correct
- Julia is correct
- Neither are correct
(b) [4 pts] Derive the maximum branching factor for this problem. Your answer must written as a single integer.

**Hint 1:** How many ways are there to swap two books on different rows? How many possible ways are there to shift any of the rows?

**Hint 2:** The choose function, \( \binom{n}{k} = \frac{n!}{k!(n-k)!} \), may be useful here (though you can also solve this question without it).

(c) [1 pt] Another coworker Nawoda claims that the following goal test is valid for this search problem.

(Reminder: the goal state is when every book is at its correct position on the bookshelf.)

- For the leftmost book \( b_i \) of every row, book \( b_{i+1} \) is to its direct right.
- For the rightmost book \( b_i \) of every row, book \( b_{i-1} \) is to its direct left.
- For each book \( b_i \) on neither end of a row, book \( b_{i-1} \) is to its direct left, and book \( b_{i+1} \) is to its direct right.

○ This is a valid goal test. ○ This is not a valid goal test.

(d) [3 pts] Which of the following heuristics are admissible for this search problem? Select all that apply.

- The minimum number of swaps needed to sort all the books, assuming you can swap any two books, regardless of whether they are on the same row.
- The number of books that are not in their correct position.
- For some book \( b_i \), define \( \text{dist}(b_i) \) to be the number of rows plus the number of columns book \( b_i \) is from its correct position. The heuristic is \( \max(\text{dist}(b_i)) \) for all books \( b \).
- None of the above.

Now consider the following game: Julia and Carolyn take turns making moves on the same bookshelf until all the books are in their correct position. The person who makes the move that results in the bookshelf being completely sorted wins. Assume both players are playing optimally.

(e) [2 pts] Julia and Carolyn decide to model this game with a game tree. Select all true statements.

- This game tree is infinite in depth.
- This game tree requires expectation nodes.
- This is a zero-sum game.
- Every non-terminal node’s branching factor will be equal to the game’s maximum branching factor.
- None of the above.

(f) [3 pts] Julia is trying to maximize her utility. Fill in the boxes below with integer values such that \( f(s) \) is an evaluation function that causes Julia to act optimally.

\[
f(s) = \begin{cases} 
\begin{array}{c}
\hline
\hline
\end{array} & \text{if state } s \text{ is move(s) away from a state where all the books are sorted.} \\
\begin{array}{c}
\hline
\end{array} & \text{otherwise.}
\end{cases}
\]
Q3  **Friend or Foe?**  
(13 points)

Robotron is a latest-generation robot. Robotron is loaded into a world where Robotron chooses one action, then a human chooses one action, then Robotron receives a reward. We can model this as a game tree, shown below.

```
   X
  / \  
(i)  (ii)
 / \  /  
Y   (iii)  (iv)  Z  (v)  (vi)
|   1  8   3   7 |
```

At node X, Robotron selects action (i) or (ii). At node Y, the human selects action (iii) or (iv). At node Z, the human selects action (v) or (vi). The circles do not necessarily represent chance nodes.

Q3.1 (1 point) Suppose the human acts adversarially, and Robotron knows this. If Robotron acts optimally, what reward will Robotron receive?

- (A) 1
- (B) 8
- (C) 3
- (D) 7
- (E) 4.5
- (F) 5

Q3.2 (1 point) Suppose the human acts cooperatively with Robotron to maximize Robotron’s reward, and Robotron knows this. If Robotron acts optimally, what reward will Robotron receive?

- (A) 1
- (B) 8
- (C) 3
- (D) 7
- (E) 4.5
- (F) 5

Q3.3 (3 points) For the rest of the question, suppose Robotron doesn’t know the human’s behavior. Robotron knows that with probability p, the human will act adversarially, and with probability 1 – p, the human will act cooperatively. For what p will Robotron be indifferent about Robotron’s choice of action?

- (A) 1/8
- (B) 1/6
- (C) 1/3
- (D) 1/2
- (E) 2/3
- (F) 5/6