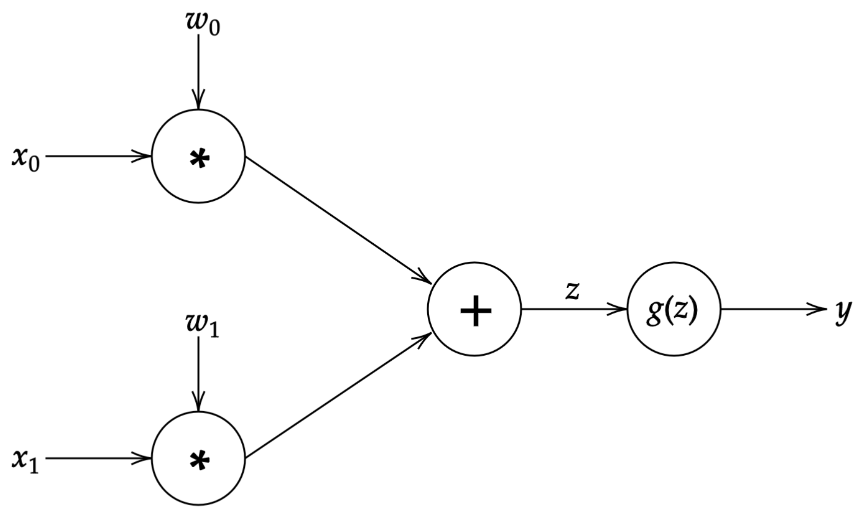
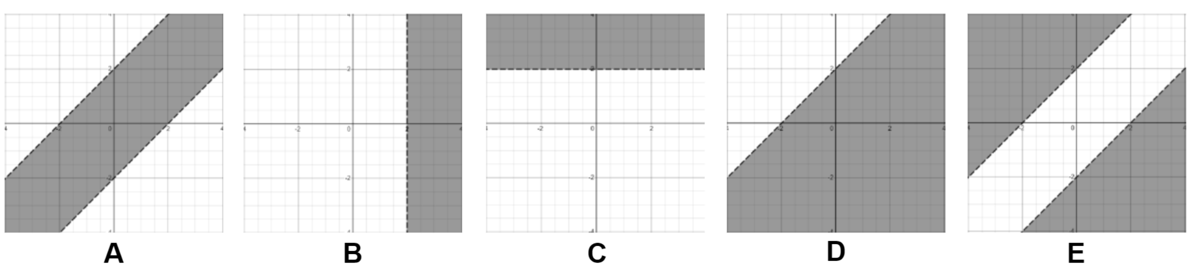
Q7. [8 pts] So Many Derivatives

Consider the neural network configuration below.



(a) [2 pts] Which of the following decision boundaries can be learned by the neural network? Assume $w_0, w_1, x_0, x_1, T_a \in \mathbb{R}^n$ and $z = w_0 x_0 + w_1 x_1$. Let $g(z)$ be the binary step activation function with T_a as the decision threshold, which is defined as follows:

$$g(z) = \begin{cases} 1 \text{ if } z \geq T_a \\ 0 \text{ if } z < T_a \end{cases}$$



[A] Graph A
[B] Graph B
[C] Graph C
[D] Graph D
[E] Graph E
(F) None of the above

(b) Now let $g(z)$ be the sigmoid activation function and y be a real number value between 0 and 1 (we will ignore the threshold T_a for this part). Recall that the derivative of the sigmoid function is $\frac{\partial}{\partial z} g(z) = g(z) \cdot (1 - g(z))$. You can represent your answers in terms of x_0, x_1, w_0, w_1, z, or y.

(i) [2 pts] Calculate the following partial derivatives for backpropagation.

(1) $\frac{\partial y}{\partial z} = \boxed{}$

(2) $\frac{\partial z}{\partial w_0} = \boxed{}$

(ii) [2 pts] Suppose we are running gradient **descent** on the neural network above. We are trying to minimize the upstream loss L using learning rate α. Given the upstream gradient $\frac{\partial L}{\partial y}$ and the two partial derivatives that you computed in the previous part ($\frac{\partial y}{\partial z}$ and $\frac{\partial z}{\partial w_0}$), determine the gradient descent update rule for w_0.

$w_0 \leftarrow$ $\boxed{}$

(c) [2 pts] The Binary Perceptron is defined as the following:

$$y = \text{classify}(x) = \begin{cases} +1 & \text{if } w \cdot f(x) + b \geq 0 \\ -1 & \text{if } w \cdot f(x) + b < 0 \end{cases}$$

where w is a vector of real-valued weights, $w \cdot f(x)$ is the dot product $\sum_{i=1}^{m} w_i f_i(x)$ where m is the number of features, $f_i(x)$ is the ith feature of x, and b is the bias.

Which of the following are true about the binary perceptron as defined above?

[A] It is possible that the perceptron learns a decision boundary that is nonlinear in terms of the features $f(x)$.

[B] It is possible that the perceptron learns a decision boundary that is nonlinear in terms of the data x.

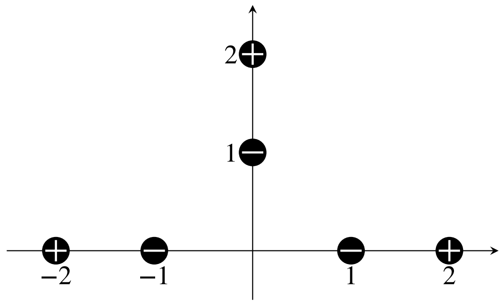
[C] The perceptron algorithm is guaranteed to converge if the data is linearly separable.

[D] The perceptron algorithm is trained using gradient descent.

(E) None of the above

Q9. [8 pts] Higher-Dimensional Perceptrons

Consider a dataset with 6 points on a 2D coordinate grid Each point belongs to one of two classes. Points $[-1, 0], [1, 0], [0, 1]$ belong to the negative class. Points $[-2, 0], [2, 0], [0, 2]$ belong to the positive class.



(a) [1 pt] Suppose we run the perceptron algorithm with the initial weight vector set to $[0, 5]$.

What is the updated weight vector after processing the data point $[0, 1]$?

(b) [1 pt] How many iterations of the perceptron algorithm will run before the algorithm converges? Processing one data point counts as one iteration. If the algorithm never converges, write ∞.

In the next few subparts, we'll consider *transforming* the data points by applying some modification to each of the data points. Then, we pass these modified data points into the perceptron algorithm.

For example, consider the transformation $[x, y] \to [x, y, x^2, 1]$. In this transformation, we add two extra dimensions: one whose value is always the square of the first coordinate, and one whose value is always the constant 1. For example, the point at $[2, 0]$ is transformed into a point at $[2, 0, 4, 1]$ in 4-dimensional space.

(c) [2 pts] Which of the following data transformations will cause the perceptron algorithm to converge, when run on the transformed data? Select all that apply.

- ☐ $[x, y] \to [y, x]$
- ☐ $[x, y] \to [x, y, 1]$
- ☐ $[x, y] \to [x, y, x^2, 1]$
- ☐ $[x, y] \to [x, y, x^2 + y^2]$
- ◯ None of the above.

(d) [2 pts] Suppose we transform $[x, y]$ to $[x, y, x^2 + y^2, 1]$, and pass the transformed data points into the perceptron.

Write one possible weight vector that the perceptron algorithm may converge to.

(e) [2 pts] Construct another transformation (not equal to the ones above) that will allow the perceptron algorithm to converge.

Hint: The transformation $[x, y] \to [x, y, x^2 + y^2, 1]$ allows the perceptron algorithm to converge.

Fill in the blank: $[x, y] \to [x, y, \_\_\_, 1]$.

Exam continues on next page.

Q10. [9 pts] Q-Networks

Consider running Q-learning on the following Pacman problem: the maze is an x-by-x square, and each position can contain a food pellet or no food pellet. There are no ghosts or walls. Pacman's only actions are $\{\text{up}, \text{down}, \text{left}, \text{right}\}$.

(a) [2 pts] How many Q-values do we need to learn for this problem?

Your answer should be an expression, possibly in terms of x.

To learn every Q-value, we could run standard Q-learning, but we decide to try a different approach:

Suppose somebody tells us N exact Q-values for N different state-action pairs: the exact Q-value for the state-action pair (s_i, a_i) is q_i, for $1 \leq i \leq N$. (N is less than the total number of state-action pairs.)

We decide to use these exact Q-values to train a neural network, so that we can estimate other Q-values we don't know. To train this neural network, we need to apply gradient descent to minimize the following loss function:

$$L(\theta) = \frac{1}{N} \sum_{i=1}^{N} (f(\theta, s_i, a_i) - q_i)^2$$

θ represents the weights of the neural network. $f(\theta, s_i, a_i)$ represents running the neural network with weights θ on state-action pair (s_i, a_i).

(b) [1 pt] What is the gradient $\frac{\partial L}{\partial \theta}$?

Your answer should be an expression, possibly in terms of N, $\frac{\partial f}{\partial \theta}$, and q_i.

$\frac{\partial L}{\partial \theta} =$

(c) [1 pt] After running t iterations of gradient descent, our current weights are θ_t. The learning rate is α.

What are the weights on the next iteration, θ_{t+1}?

Your answer should be an expression, possibly in terms of θ_t, α, and $\frac{\partial L}{\partial \theta}$.

$\theta_{t+1} =$

(d) [2 pts] Eventually, gradient descent converges to the weights θ^*.

We use the neural network with weights θ^* to compute Q-values, and extract a policy out of these Q-values:

$$\pi(s) = \arg\max_a f(\theta^*, s, a)$$

Is π the optimal policy for this problem?

◯ Yes ◯ No ◯ Not enough information

Instead of the Pacman problem, consider a different problem where the action space is continuous. In other words, there are infinitely many actions available from a given state.

(e) [3 pts] Can we still use the strategy from the previous subparts (without any modifications) to obtain a policy π?

◯ Yes ◯ No

Briefly explain why or why not.