CS 188 Summer 2024 Introduction to Artificial Intelligence HW1

- **Due:** Friday 6/21 at 11:59pm.
- **Policy:** Can be solved in groups (acknowledge collaborators) but must be submitted individually.
- **Make sure to show all your work and justify your answers**.
- **Note:** This is a typical exam-level question. On the exam, you would be under time pressure, and have to complete this question on your own. We strongly encourage you to first try this on your own to help you understand where you currently stand. Then feel free to have some discussion about the question with other students and/or staff, before independently writing up your solution.
- Your submission on Gradescope should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question begins on page 2.). **Do not reorder, split, combine, or add extra pages**. The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	
Last name	
SID	
Collaborators	

Q1. [20 pts] A Comprehensive Search

It is training day for Pacbabies, also known as Hungry Running Maze Games day. Each of k Pacbabies starts in its own assigned start location s_i in a large maze of size $M \times N$ and must return to its own Pacdad who is waiting patiently but proudly at g_i along the way, the Pacbabies must, between them, eat all the dots in the maze.

At each step, all k Pacbabies move one unit to any open adjacent square. The only legal actions are Up, Down, Left, or Right. It is illegal for a Pacbaby to wait in a square, attempt to move into a wall, or attempt to occupy the same square as another Pacbaby. To set a record, the Pacbabies must find an optimal collective solution.

1.1) (3 pts) Define a minimal state space representation for this problem.

The minimal state space is defined by the current locations of k Pacbabies and, for each square of the grid, a Boolean variable that indicates whether there is food there or not.

1.2) (2 pts) How large is the state space?

Given the minimal state representation defined above an upper bound on the size of the state space is $(MN)^k \cdot 2^{MN}$. The first part is $(MN)^k$ as the pacbabies can move to any state in the state-space. The second term 2^{MN} accounts for all the possible food configurations on the grid. You could also point out that given that two pacbabies cannot be on the same place at the same time the first term is $(MN) * (MN-1) * \cdots (MN-(k-1))$. Both approaches are considered correct.

1.3) (2 pts) What is the maximum branching factor for this problem?

A) 4^k

B) 8^k

C) $4^k 2^{MN}$

D) $4^k 2^4$

A) 4^k

For each distinct action of a pacbaby we will end up in a possibly different child node. Given that we have k pacbabies then the answer is 4^k as each of the k Pacbabies has a choice of 4 actions.

1.4 - 1.9) (8 pts) Let $MH(p,q)$ be the Manhattan distance between positions p and q and F be the set of all positions of remaining food pellets and p_i be the current position of Pacbaby i. Which of the following are admissible heuristics?

1.4. $\frac{\sum_{i=1}^{k} MH(p_i,g_i)}{k}$

1.5. $\max_{1\le i\le k} MH(p_i, g_i)$

1.6. $\max_{1\le i\le k}[\max_{f\in F} MH(p_i, f)]$

1.7. $\max_{1\le i\le k}[\min_{f\in F} MH(p_i, f)]$

1.8. $\min_{1\le i\le k}[\min_{f\in F} MH(p_i, f)]$

1.9. $\min_{f\in F}[\max_{1\le i\le k} MH(p_i, f)]$

1.4, 1.5, and 1.8 are admissible. 1.6, 1.7, and 1.9 are inadmissible.

- 1.4 is admissible because the total Pacbaby–Pacdad distance can be reduced by at most k at each time step.
- 1.5 is admissible because it will take at least this many steps for the furthest Pacbaby to reach its Pacdad.

- 1.6 is inadmissible because it looks at the distance from each Pacbaby to its most distant food square and in the optimal solution we might have another Pacbaby, that is closer, going to that square so this heuristic is inadmissible.
- 1.7 same logic as C).
- 1.8 is admissible because some Pacbaby will have to travel at least this far to eat one piece of food.
- 1.9 is inadmissible because it connects each food square to the most distant Pacbaby, which may not be the one who eats it.

1.10) (2 pts)You'd like to choose two heuristic functions f, g from the 6 heuristics listed above, such that their maximum, $h(n) = \max(f(n), g(n))$, is an admissible heuristic.

What is a sufficient condition on f and/or g for $h(n)$ to be admissible?

Your answer can be brief (<5 words).

Any pair between A), B), and E) would make $h(n)$ admissible as the max of two admissible heuristics is still admissible.

1.11) (3 pts) You'd like to choose two heuristic functions f, g from the 6 heuristics listed above, such that

$$h(n) = af(n) + (1 - a)g(n)$$

is an admissible heuristic for any value of a between 0 and 1.

Which is a sufficient condition for $h(n)$ to be admissible?

A) Any f and g is sufficient.

B) At least one of f and g is admissible.

C) Both f and g are admissible.

D) $h(n)$ is admissible for $a = 0.5$.

E) $h(n)$ is admissible for $a = 0$.

C) Both f and g are admissible.

Again, any pair between A), B), and E) would make $h(n)$ admissible as the convex combination of two functions is dominated by the max of those functions, $h(n) = \alpha h_i(n) + (1 - \alpha)h_j(n)) \leq \max(h_i(n), h_j(n))$ for any $\alpha \in [0, 1]$, and since from the previous part the max is admissible the same holds for the convex combination.

Q2. [9 pts] Rationality of Utilities

2.1) (3 pts) Consider a lottery $L = [0.2, A; 0.3, B; 0.4, C; 0.1, D]$, where the utility values of each of the outcomes are $U(A) = 1$, $U(B) = 3$, $U(C) = 5$, $U(D) = 2$. What is the utility of this lottery, $U(L)$?

$U(L) = 0.2 * U(A) + 0.3 * U(B) + 0.4 * U(C) + 0.1 * U(D) = 3.3$

2.2) (3 pts) Consider a lottery $L_1 = [0.5, A; 0.5, L_2]$, where $U(A) = 4$, and lottery $L_2 = [0.5, X; 0.5, Y]$, where $U(X) = 4$, $U(Y) = 8$. What is the utility of the the first lottery, $U(L_1)$?

$U(L_1) = 0.5 * U(A) + 0.5 * U(L2) = 2 + 3 = 5$

2.3) (3 pts) Assume $A \succ B$, $B \succ L$, where $L = [0.5, C; 0.5, D]$, and $D \succ A$. Assuming rational preferences, which of the following statements are guaranteed to be true?

A) $A \succ L$

B) $A \succ C$

C) $A \succ D$

D) $B \succ C$

E) $B \succ D$

A) $A \succ L$, B) $A \succ C$, D) $B \succ C$

- $A \succ B \succ L$, so by transitivity $A \succ L$
- $A \succ L \implies A \succ D \lor A \succ C$. Because $D \succ A$, then $A \succ C$ must be true.
- $D \succ A$ means this is false.
- $D \succ A \succ B \implies D \succ B$, so for the same reasoning as (b) this is true
- $D \succ A \succ B \implies D \succ B$, means this is false.

Q3. [14 pts] Preferences and Utilities

Our Pacman board now has food pellets of 3 different sizes - pellet P_1 of radius 1, P_2 of radius 2 and P_3 of radius 3. In different moods, Pacman has different preferences among these pellets. In each of the following questions, you are given Pacman's preference for the different pellets. From among the options pick the utility functions that are consistent with Pacman's preferences, where each utility function $U(r)$ is given as a function of the pellet radius r, and is defined over non-negative values of r.

3.1) (2 pts) $P_1 \sim P_2 \sim P_3$

A) $U(r) = 0$

B) $U(r) = 3$

C) $U(r) = r$

D) $U(r) = 2r + 4$

E) $U(r) = -r$

F) $U(r) = r^2$

G) $U(r) = -r^2$

H) $U(r) = \sqrt{r}$

I) $U(r) = -\sqrt{r}$

J) Irrational preferences!

A) and B)

Because all three sizes are preferred equally, $U(r)$ has to return the same value for $r = 1, 2, 3$. The only functions that do so from this list are those that do not depend on r.

3.2) (2 pts) $P_1 \prec P_2 \prec P_3$

A) $U(r) = 0$

B) $U(r) = 3$

C) $U(r) = r$

D) $U(r) = 2r + 4$

E) $U(r) = -r$

F) $U(r) = r^2$

G) $U(r) = -r^2$

H) $U(r) = \sqrt{r}$

I) $U(r) = -\sqrt{r}$

J) Irrational preferences!

C) , D), F), H)

Higher radii are preferred over lower ones, so increasing functions of r satisfy the constraints.

3.3) (2 pts) $P_1 \succ P_2 \succ P_3$

A) $U(r) = 0$

B) $U(r) = 3$

C) $U(r) = r$

D) $U(r) = 2r + 4$

E) $U(r) = -r$

F) $U(r) = r^2$

G) $U(r) = -r^2$

H) $U(r) = \sqrt{r}$

I) $U(r) = -\sqrt{r}$

J) Irrational preferences!

E), G), and I)

Lower radii are preferred over higher ones, so decreasing functions of r satisfy the constraints.

3.4) (2 pts) $\left(P_1 \prec P_2 \prec P_3\right)$ and $\left(P_2 \prec \left(\text{ 50-50 lottery among } P_1 \text{ and } P_3\right)\right)$

A) $U(r) = 0$

B) $U(r) = 3$

C) $U(r) = r$

D) $U(r) = 2r + 4$

E) $U(r) = -r$

F) $U(r) = r^2$

G) $U(r) = -r^2$

H) $U(r) = \sqrt{r}$

I) $U(r) = -\sqrt{r}$

J) Irrational preferences!

F) only.

The first constraint means that $U(r)$ must be increasing, and the second constraint means that the rate at which it is increasing must be increasing as well, and r^2 is the only function that is of the ones provided.

3.5) (2 pts) $\left(P_1 \succ P_2 \succ P_3\right)$ and $\left(P_2 \succ \left(\text{ 50-50 lottery among } P_1 \text{ and } P_3\right)\right)$

A) $U(r) = 0$

B) $U(r) = 3$

C) $U(r) = r$

D) $U(r) = 2r + 4$

E) $U(r) = -r$

F) $U(r) = r^2$

G) $U(r) = -r^2$

H) $U(r) = \sqrt{r}$

I) $U(r) = -\sqrt{r}$

J) Irrational preferences!

G) only.

The first constraint means that $U(r)$ must be decreasing, and the second constraint means that $U(2) > 0.5 * U(1) + 0.5 * U(3)$.

3.6) (2 pts) $(P_1 \prec P_2)$ and $(P_2 \prec P_3)$ and $(($ 50-50 lottery among P_2 and $P_3) \prec ($ 50-50 lottery among P_1 and $P_2))$

A) $U(r) = 0$

B) $U(r) = 3$

C) $U(r) = r$

D) $U(r) = 2r + 4$

E) $U(r) = -r$

F) $U(r) = r^2$

G) $U(r) = -r^2$

H) $U(r) = \sqrt{r}$

I) $U(r) = -\sqrt{r}$

J) Irrational preferences!

J)

$P_3 \succ P_1$ by transitivity, and since both lotteries have equal chances for P_2, it can be ignored when comparing the two. So the last constraint is essentially $P_3 \prec P_1$, which makes the preferences irrational.

3.7) (2 pts) Which of the following would be a utility function for a risk-seeking preference? That is, for which utility(s) would Pacman prefer entering a lottery for a random food pellet, with expected size s, over receiving a pellet of size s?

A) $U(r) = 0$

B) $U(r) = 3$

C) $U(r) = r$

D) $U(r) = 2r + 4$

E) $U(r) = -r$

F) $U(r) = r^2$

G) $U(r) = -r^2$

H) $U(r) = \sqrt{r}$

I) $U(r) = -\sqrt{r}$

F) and I)

Functions that are either decreasing slower than linearly, like $-\sqrt{r}$, or increasing faster than linearly, like r^2, satisfy this.