Recap: Utilities and Rationality

- Utilities and Rationality
- Rational Preferences

**MEU Principle:**

Given Rational Preferences, Exists $U(X)$ s.t.

$U(A) \geq U(B) \iff A \geq B$

$U([p_1, S_1; \ldots; p_n, S_n]) = p_1 U(S_1) + \ldots + p_n U(S_n)$

**Orderability:** $(A > B) \lor (B > A) \lor (A \sim B)$

**Transitivity:** $(A > B) \land (B > C) \Rightarrow (A > C)$

**Continuity:** $(A > B > C) \Rightarrow \exists p \ [p, A; 1-p, C] \sim B$

**Substitutability:** $(A \sim B) \Rightarrow [p, A; 1-p, C] \sim [p, B; 1-p, C]$

**Monotonicity:** $(A > B) \Rightarrow$

$(p \geq q) \iff [p, A; 1-p, B] \geq [q, A; 1-q, B]$
Decision Networks
Decision Networks

Umbrella

Weather

Forecast

U
Decision Networks

- **MEU**: choose the action which maximizes the expected utility given the evidence

  - Can directly operationalize this with decision networks
    - Bayes nets, with new node types for utilities and actions
    - Lets us calculate the expected utility for each action

  - New node types:
    - Chance nodes (just like Bayes Nets)
    - Actions (rectangles, cannot have parents, act as observed evidence)
    - Utility node (diamond, depends on action and chance nodes)
Decision Networks

- **Action selection**
  - Instantiate all evidence
  - Set action node(s) each possible way
  - Calculate posterior for all parents of utility node, given the evidence
  - Calculate expected utility for each action
  - Choose maximizing action
Maximum Expected Utility

Umbrella = leave

$$EU(\text{leave}) = \sum_w P(w)U(\text{leave, } w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

Umbrella = take

$$EU(\text{take}) = \sum_w P(w)U(\text{take, } w)$$

$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

Optimal decision = leave

$$\text{MEU}(\emptyset) = \max_a EU(a) = 70$$
Decisions as Outcome Trees

- Almost exactly like expectimax
Maximum Expected Utility Given Evidence

Umbrella = leave

\[ EU(\text{leave}|\text{bad}) = \sum_w P(w|\text{bad})U(\text{leave}, w) \]

\[ P(W) = \frac{P(F|W)}{P(W|F)} \]

\[ P(W|F) = \frac{P(W, F)}{\sum_w P(w, F)} = \frac{P(F|W)P(W)}{\sum_w P(F|w)P(w)} \]

**Table:**

<table>
<thead>
<tr>
<th>Weather</th>
<th>Forecast</th>
<th>U(A,W)</th>
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<tbody>
<tr>
<td>leave</td>
<td>sun</td>
<td>100</td>
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<tr>
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Maximum Expected Utility Given Evidence

Umbrella = leave

$$EU(\text{leave}\mid \text{bad}) = \sum_w P(w\mid \text{bad})U(\text{leave}, w)$$

$$= 0.34 \cdot 100 + 0.66 \cdot 0 = 34$$

Umbrella = take

$$EU(\text{take}\mid \text{bad}) = \sum_w P(w\mid \text{bad})U(\text{take}, w)$$

$$= 0.34 \cdot 20 + 0.66 \cdot 70 = 53$$

Optimal decision = take

$$\text{MEU}(F = \text{bad}) = \max_a EU(a\mid \text{bad}) = 53$$

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<tr>
<td>rain</td>
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Decisions as Outcome Trees

Weather Forecast = bad

Umbrella

U(t,s)
U(t,r)
U(l,s)
U(l,r)
Value of Information
Value of Information

- **Idea:** compute value of acquiring evidence
  - Can be done directly from decision network

- **Example:** buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - You can drill in one location
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has EU = k/2, MEU = k/2

- **Question:** what’s the value of information of O?
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say “oil in a” or “oil in b,” prob 0.5 each
  - If we know OilLoc, MEU is k (either way)
  - Gain in MEU from knowing OilLoc?
  - VPI(OilLoc) = k/2
  - Fair price of information: k/2
Value of Perfect Information

MEU with no evidence

\[ \text{MEU}(\emptyset) = \max_a \text{EU}(a) = 70 \]

MEU if forecast is bad

\[ \text{MEU}(F = \text{bad}) = \max_a \text{EU}(a|\text{bad}) = 53 \]

MEU if forecast is good

\[ \text{MEU}(F = \text{good}) = \max_a \text{EU}(a|\text{good}) = 89.4 \]

Forecast distribution

<table>
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<tbody>
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<td>bad</td>
<td>0.35</td>
</tr>
<tr>
<td>good</td>
<td>0.65</td>
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Forecast distribution

\[ P(F = \text{bad} | W) \]

\[ P(F = \text{sun} | W) = 0.7 \]

\[ P(F = \text{rain} | W) = 0.77 \]

\[ \text{VPI} = 0.35 \times 53 + 0.65 \times 89.4 - 70 \]

\[ \text{VPI} = 6.66 \]

\[ \text{VPI}(E'|e) = \left( \sum_{e'} P(e'|e) \text{MEU}(e, e') \right) - \text{MEU}(e) \]
Value of Information

- Assume we have evidence $E = e$. Value if we act now:
  \[ \text{MEU}(e) = \max_a \sum_s P(s|e) \, U(s, a) \]

- We see new evidence $E' = e'$. Value if we act then:
  \[ \text{MEU}(e, e') = \max_a \sum_s P(s|e, e') \, U(s, a) \]

- BUT $E'$ is a random variable whose value is unknown, so we don’t know what $e'$ will be.

- Expected value if $E'$ is revealed and then we act:
  \[ \text{MEU}(e, E') = \sum_{e'} P(e'|e) \text{MEU}(e, e') \]

- Value of information: how much MEU goes up by revealing $E'$ first then acting, as opposed to acting now:
  \[ \text{VPI}(E'|e) = \text{MEU}(e, E') - \text{MEU}(e) \]
VPI Properties

- Nonnegative
  \[ \forall E', e : \text{VPI}(E'|e) \geq 0 \]

- Nonadditive
  (think of observing \( E_j \) twice)
  \[ \text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e) \]

- Order-independent
  \[ \text{VPI}(E_j, E_k|e) = \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j) \]
  \[ = \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k) \]
Ghostbusters Decision Network

Ghost Location

Sensor (1,1) → Ghost Location
Sensor (1,2) → Ghost Location
Sensor (1,3) → Ghost Location
Sensor (1,n) → Ghost Location
Sensor (2,1) → Ghost Location
Sensor (m,1) → Ghost Location

U → Ghost Location

Bust → Ghost Location

Sensor (1,1) → Ghost Location
Sensor (1,2) → Ghost Location
Sensor (1,3) → Ghost Location
Sensor (1,n) → Ghost Location
Sensor (2,1) → Ghost Location
Sensor (m,1) → Ghost Location

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Quick VPI Questions

- The soup of the day is either clam chowder or split pea, but you wouldn’t order either one. What’s the value of knowing which it is?

- There are two kinds of plastic forks at a picnic. One kind is slightly sturdier. What’s the value of knowing which?

- You’re playing the lottery. The prize will be $0 or $100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?
Value of Imperfect Information?

- No such thing
- Information corresponds to the observation of a node in the decision network
- If data is “noisy” that just means we don’t observe the original variable, but another variable which is a noisy version of the original one
VPI Question

- VPI(OilLoc) ?
- VPI(ScoutingReport) ?
- VPI(Scout) ?
- VPI(Scout \mid ScoutingReport) ?

Generally:

If Parents(U) \models Z \mid CurrentEvidence
Then VPI( Z \mid CurrentEvidence) = 0